WHIR 

Proximity testing for Reed–Solomon+

Gal Arnon
WEIZMANN INSTITUTE OF SCIENCE

Giacomo Fenzi
EPFL

Alessandro Chiesa
EPFL

Eylon Yogev

Motivation

SNARKs

Succinct Non-interactive Arguments of Knowledge

SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show “knowledge” of w s.t. $(x, w) \in R$

SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

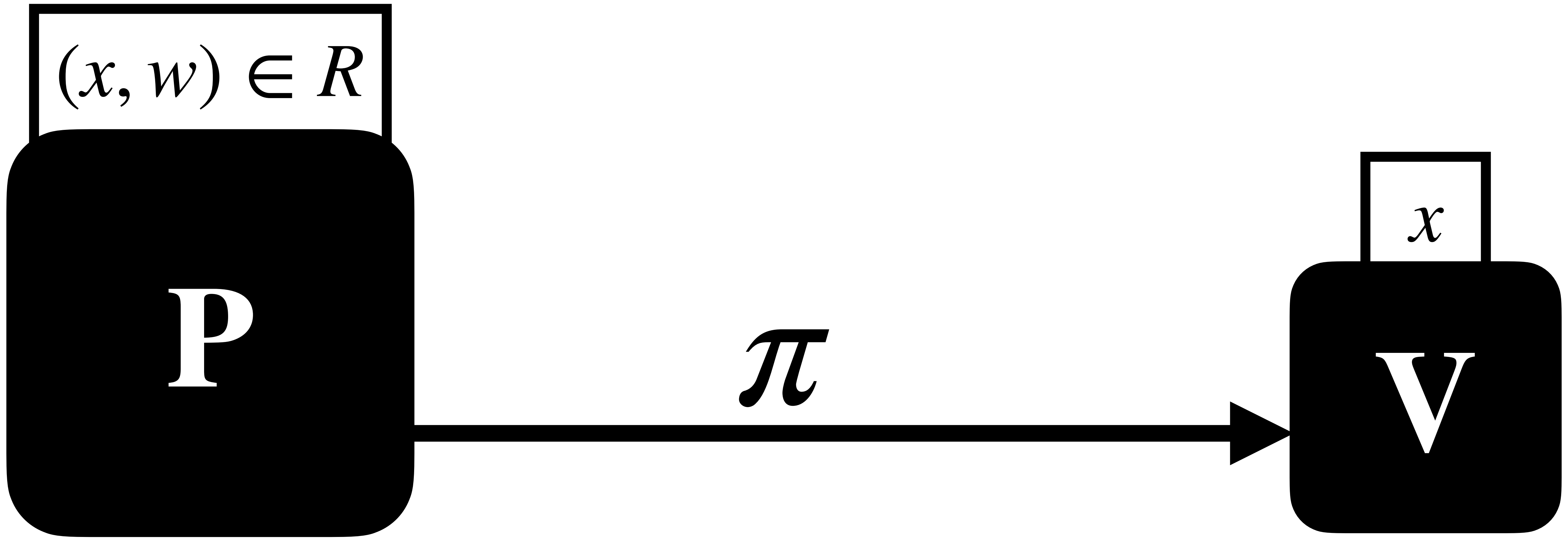
e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

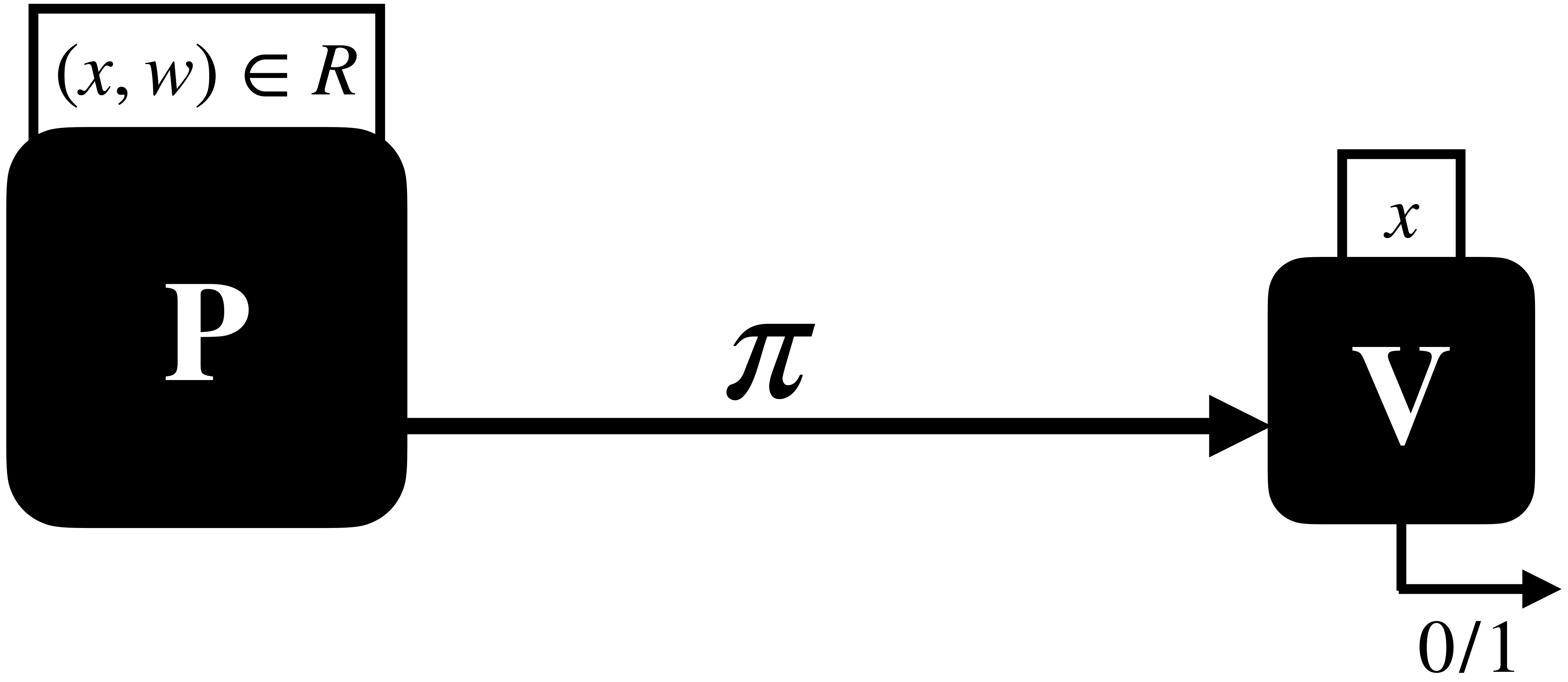


SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \mathsf{SHA3}(w) = x\}$



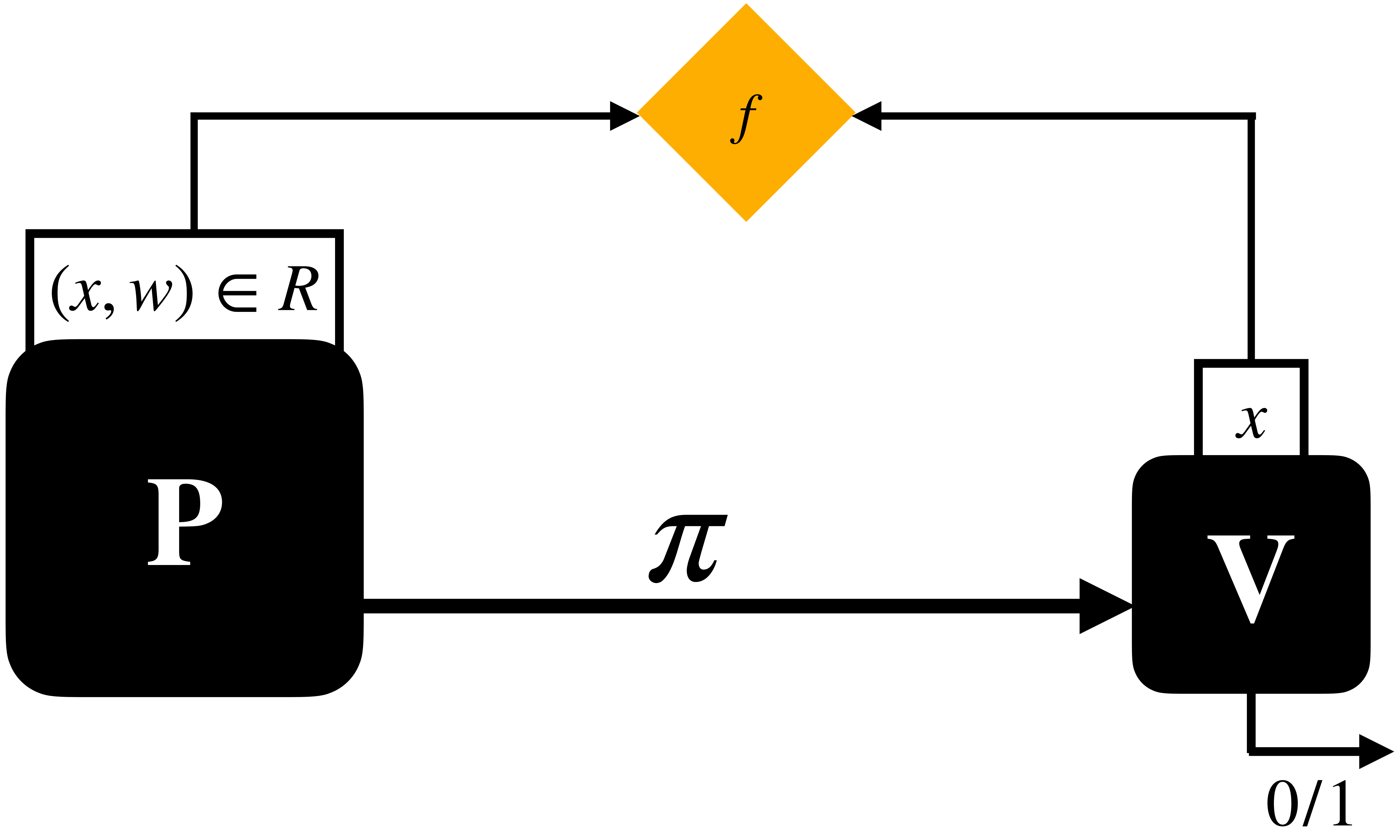
SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

- **Need*** to add a **random oracle.**



SNARKs

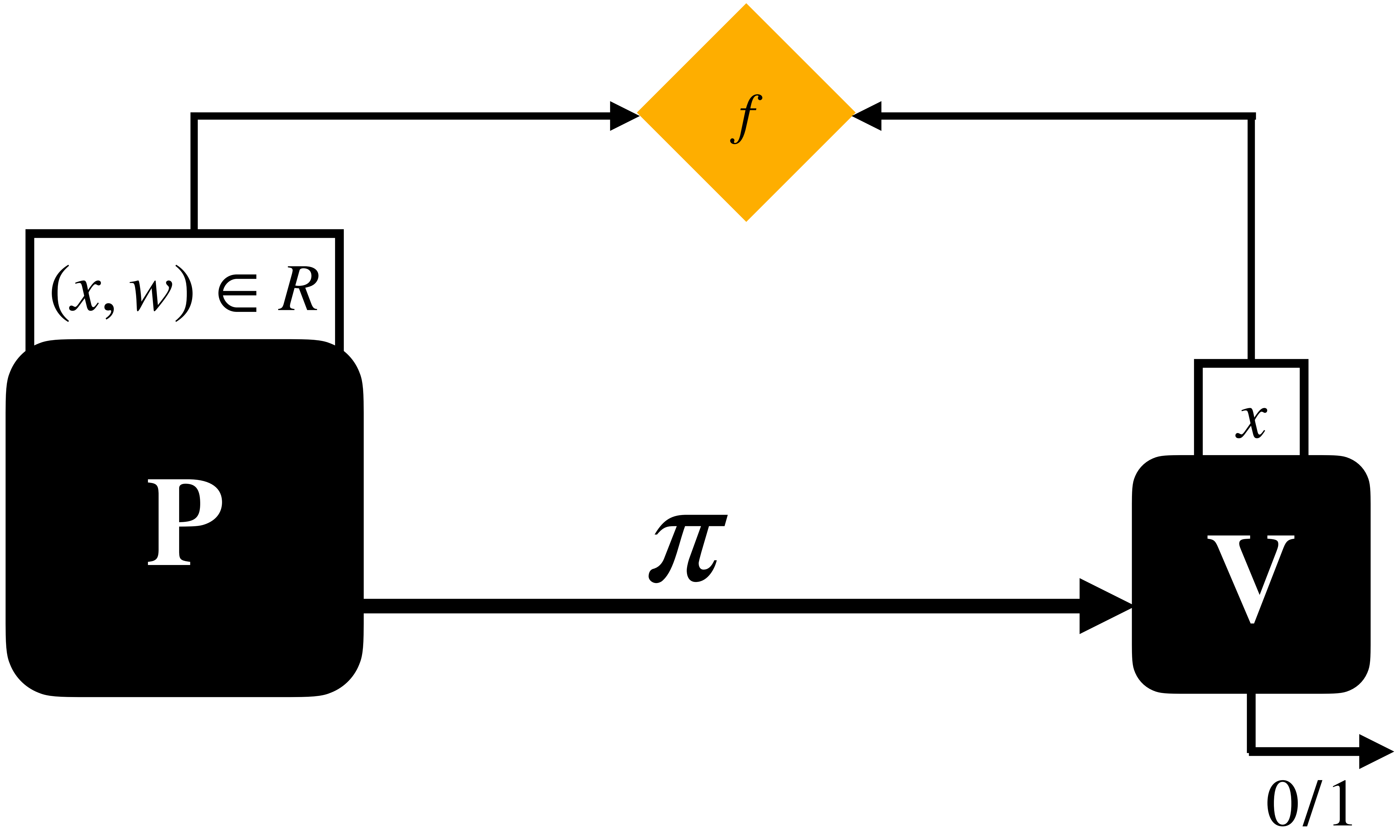
Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

- **Need*** to add a **random oracle.**

- Can be based on many computational assumptions.



SNARKs

Succinct Non-interactive Arguments of Knowledge

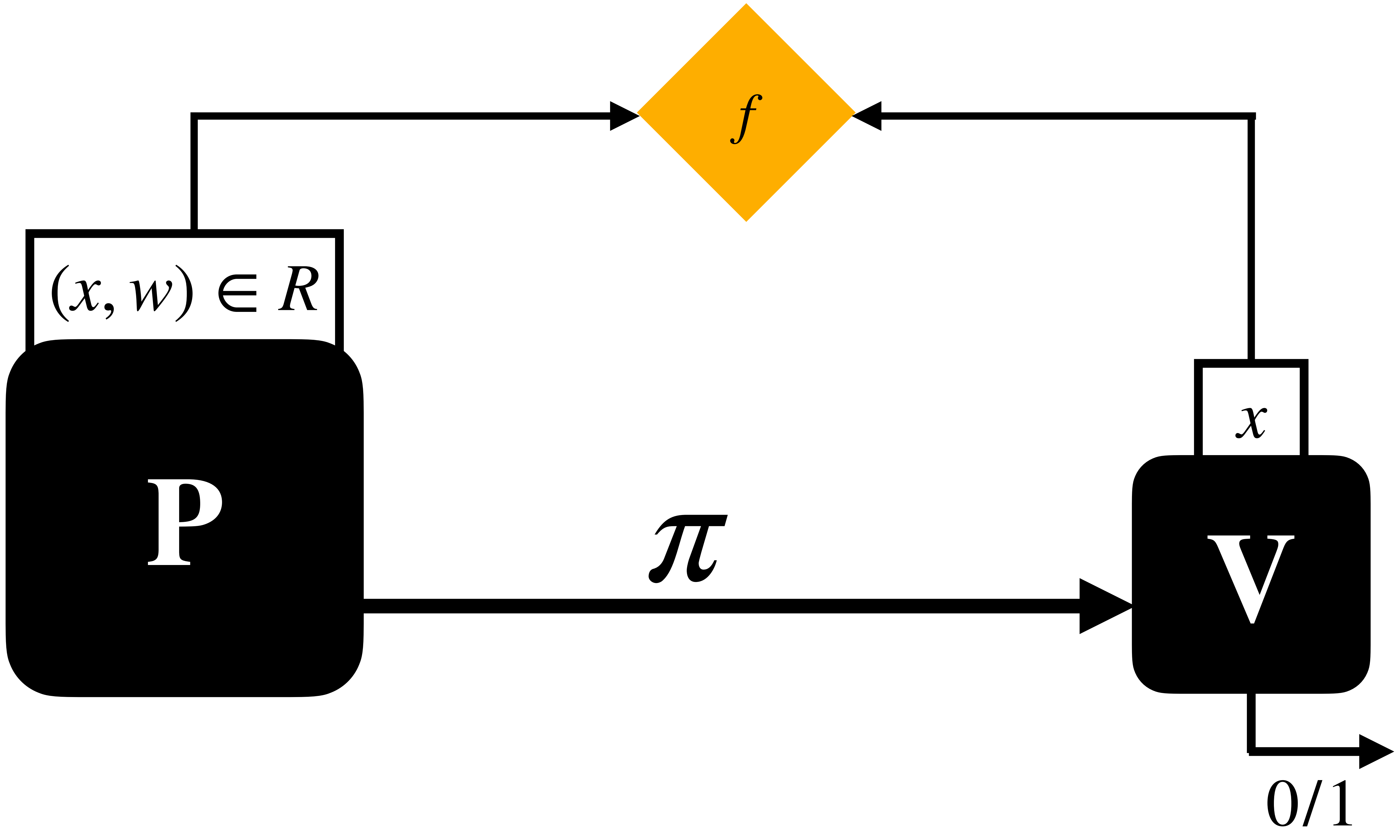
- Want to show "knowledge" of w s.t. $(x, w) \in R$

e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

- **Need*** to add a **random oracle.**

- Can be based on many computational assumptions.

- **Today:** we limit ourselves to **pure** ROM SNARKs



SNARKs

Succinct Non-interactive Arguments of Knowledge

- Want to show "knowledge" of w s.t. $(x, w) \in R$

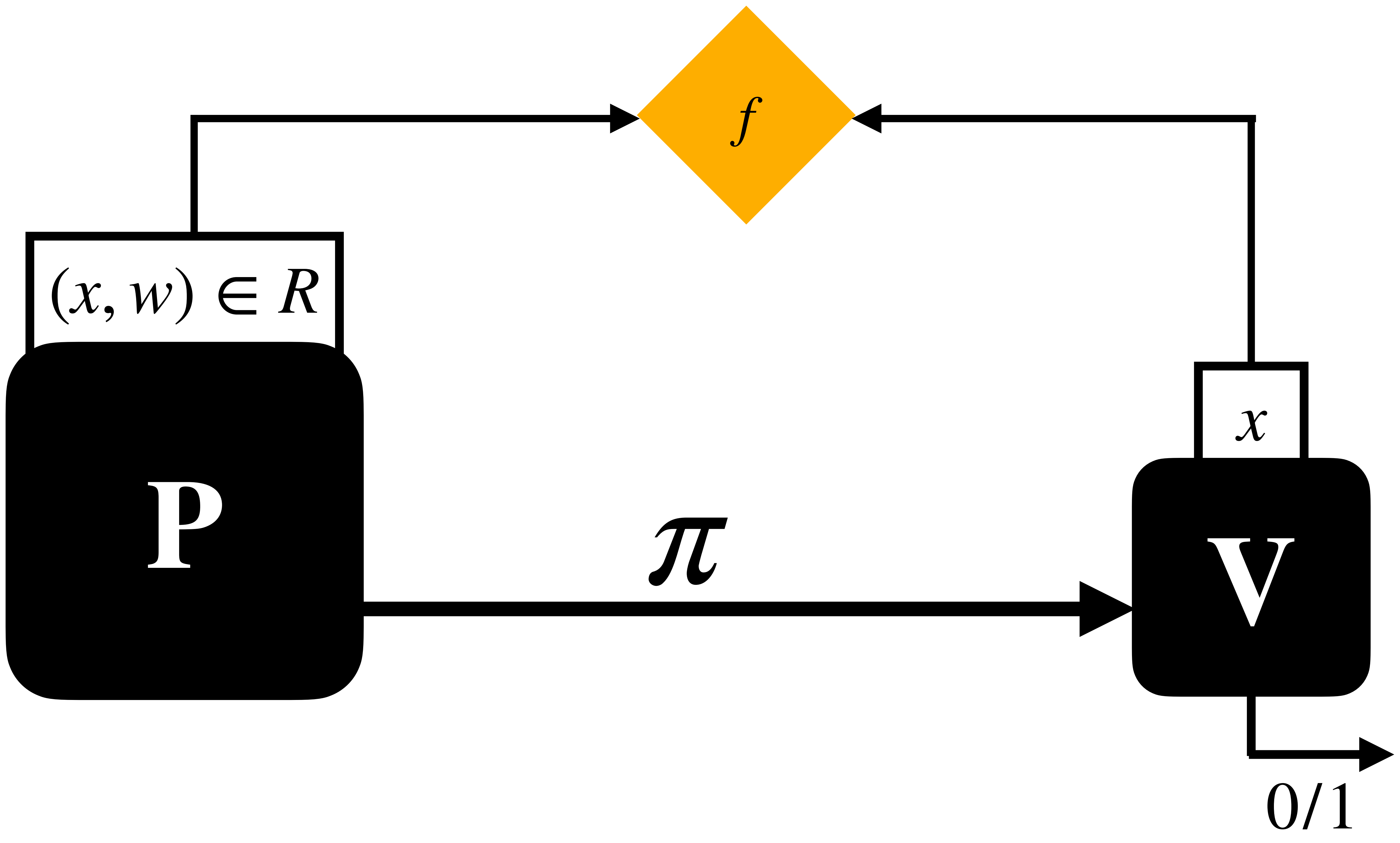
e.g. $R := \{(x, w) : \text{SHA3}(w) = x\}$

- **Need*** to add a **random oracle.**

- Can be based on many computational assumptions.

- **Today:** we limit ourselves to **pure** ROM SNARKs

- Will call these **hash-based SNARKs**.



Hash-based SNARKs

In practice

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

- Transparent setup (choice of hash)

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

- Transparent setup (choice of hash)
- Highly efficient implementations (no public-key crypto)

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

- Transparent setup (choice of hash)
- Highly efficient implementations (no public-key crypto)
- Plausibly post-quantum secure (secure in QROM)

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

- Transparent setup (choice of hash)
- Highly efficient implementations (no public-key crypto)
- Plausibly post-quantum secure (secure in QROM)

Used to secure billions of dollars in real-world blockchains:

Hash-based SNARKs

In practice

Instantiating random oracle gives amazing SNARKs:

- Transparent setup (choice of hash)
- Highly efficient implementations (no public-key crypto)
- Plausibly post-quantum secure (secure in QROM)

Used to secure billions of dollars in real-world blockchains:

STARKWARE polygon Irreducible LAMBDA CLASS zkSync dYdX Matter Labs RISC ZERO Succinct

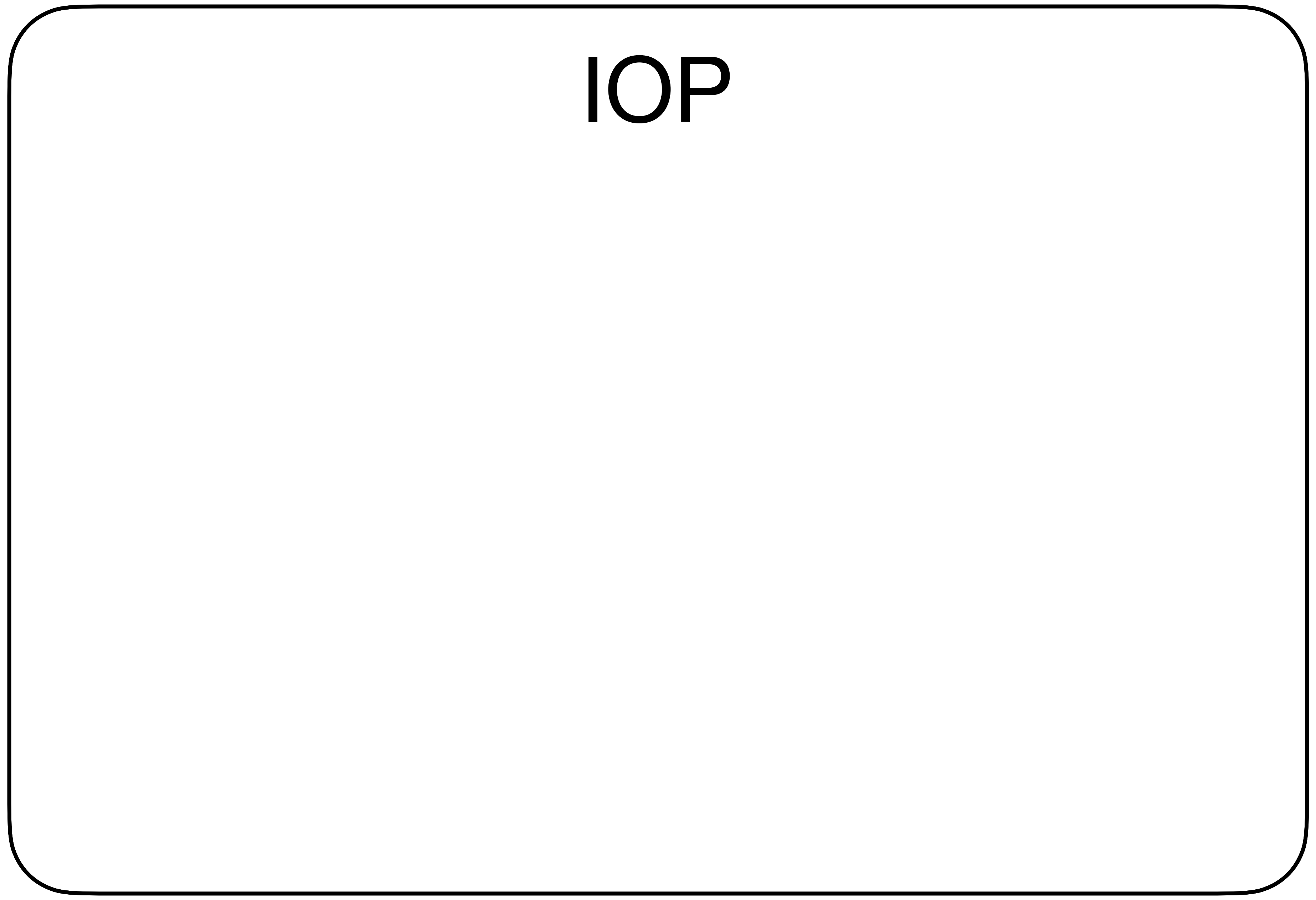
And many more…

Constructing SNARKs

[BCS16] Construction

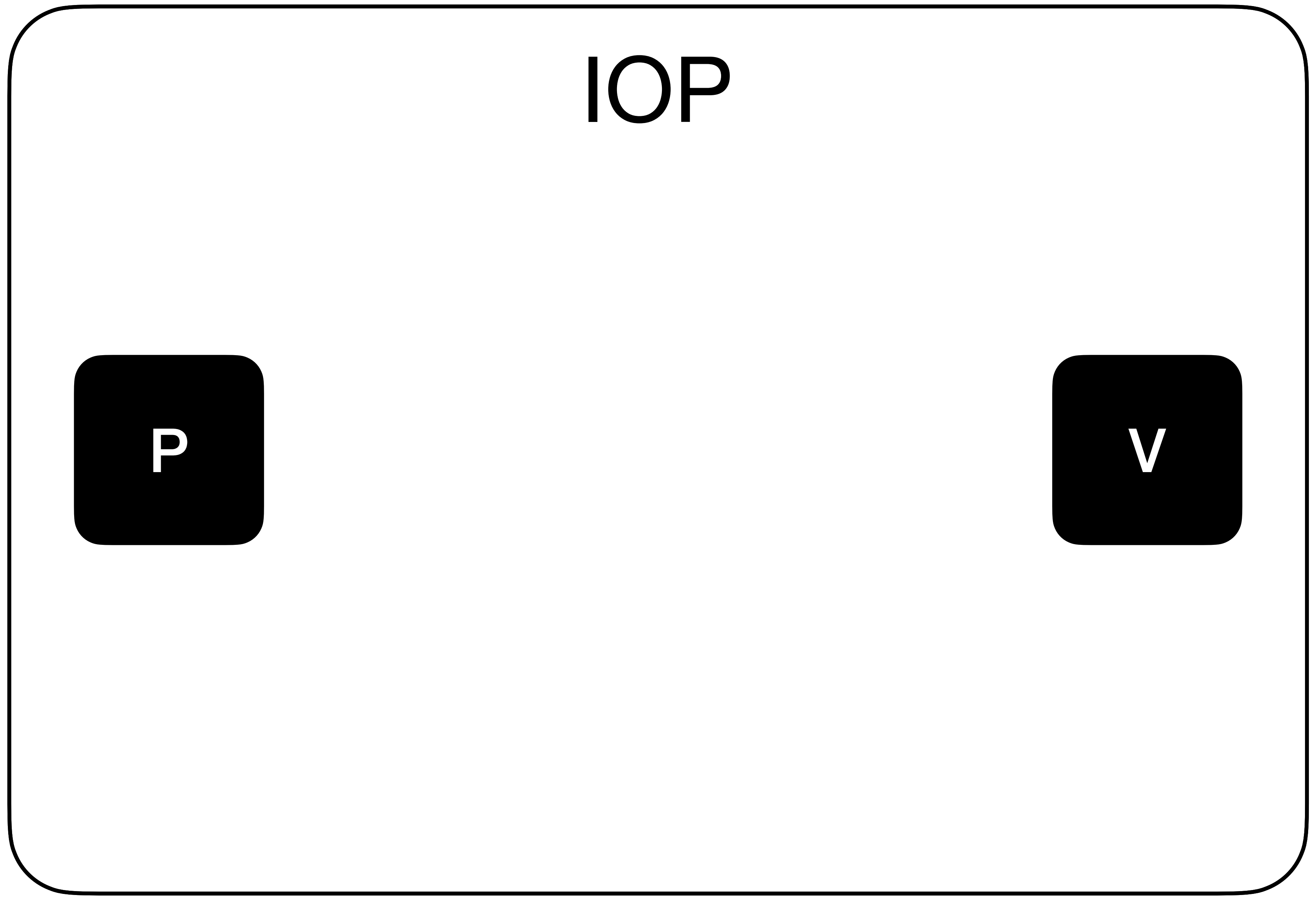
Constructing SNARKs

[BCS16] Construction



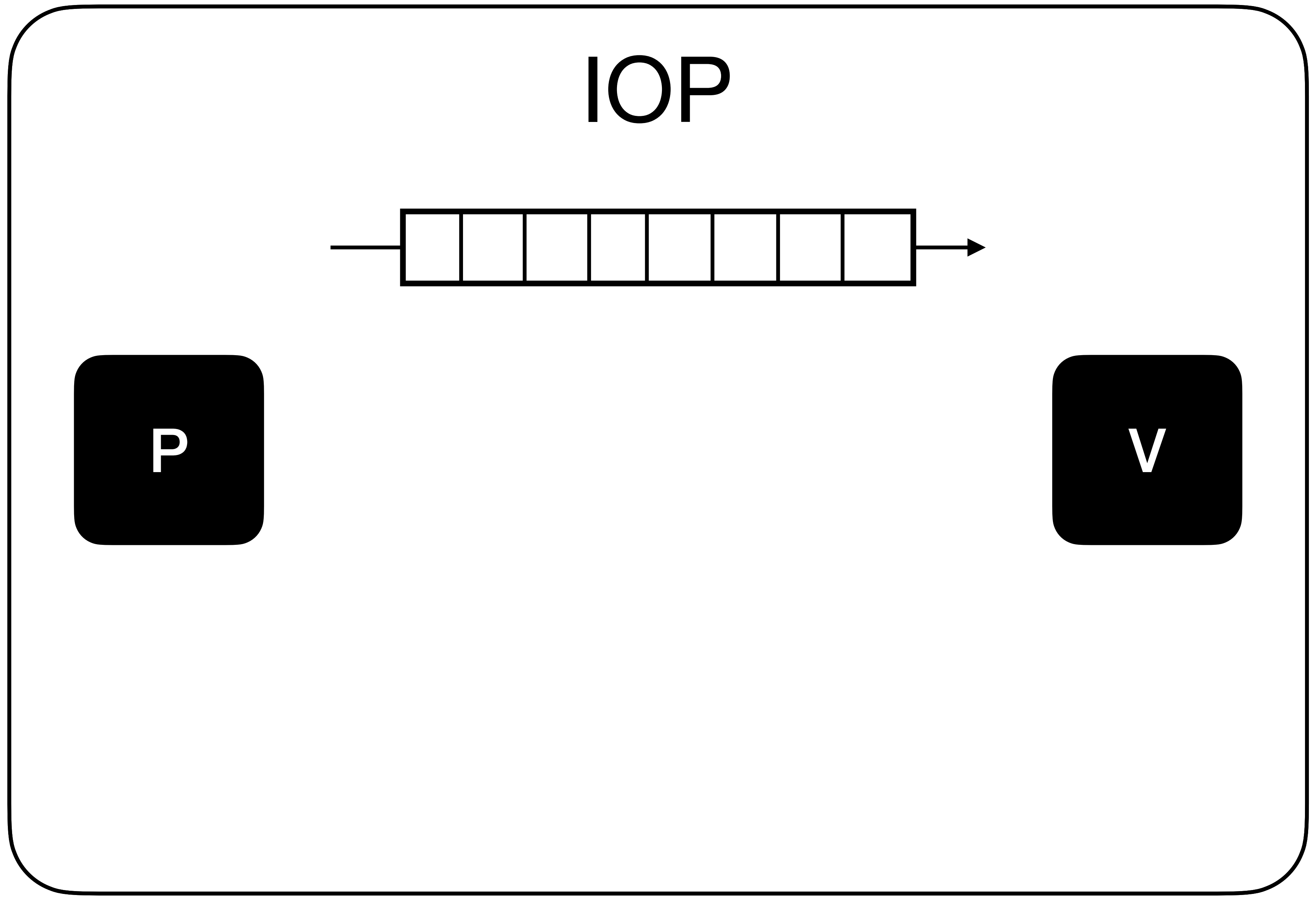
Constructing SNARKs

[BCS16] Construction



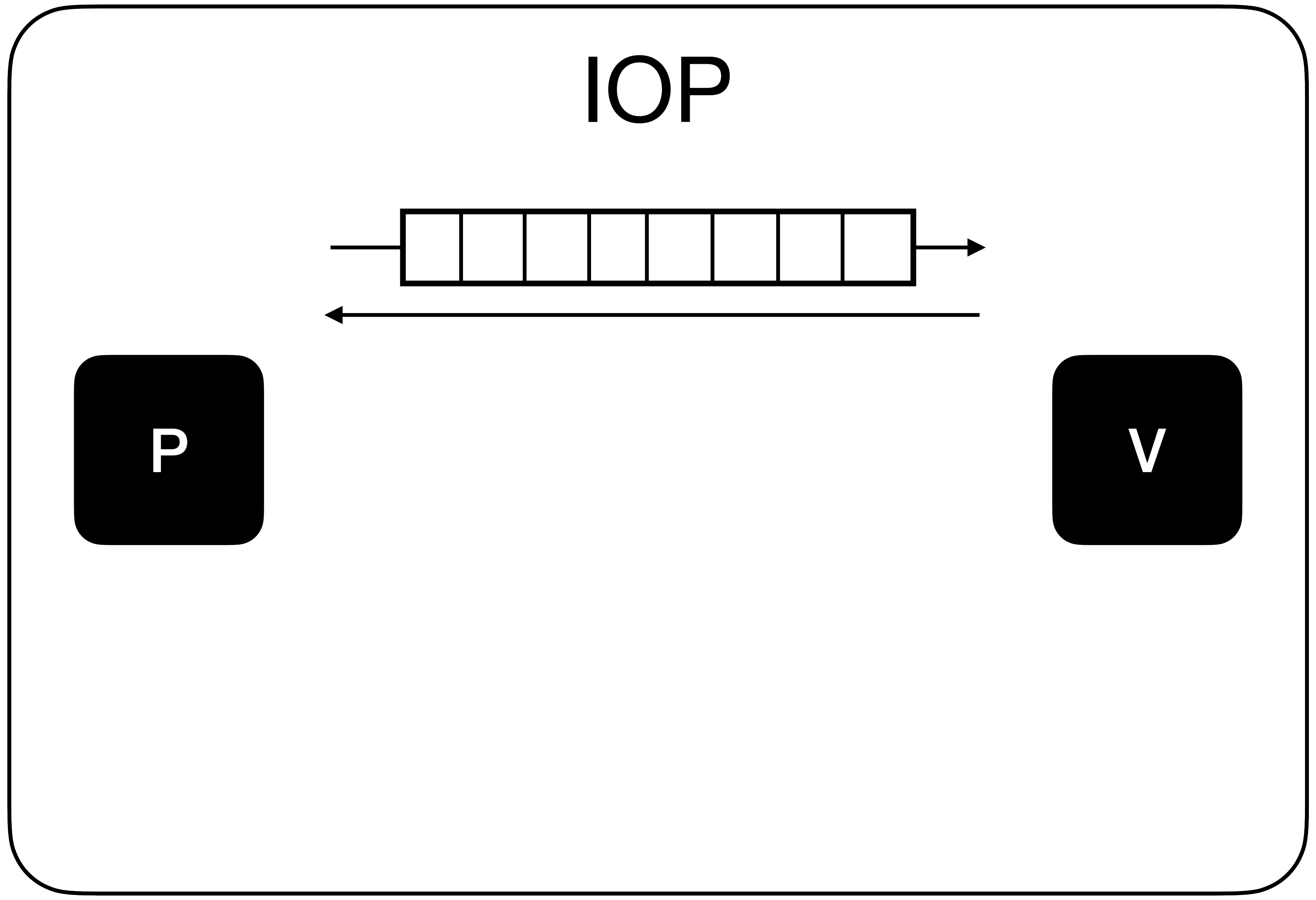
Constructing SNARKs

[BCS16] Construction



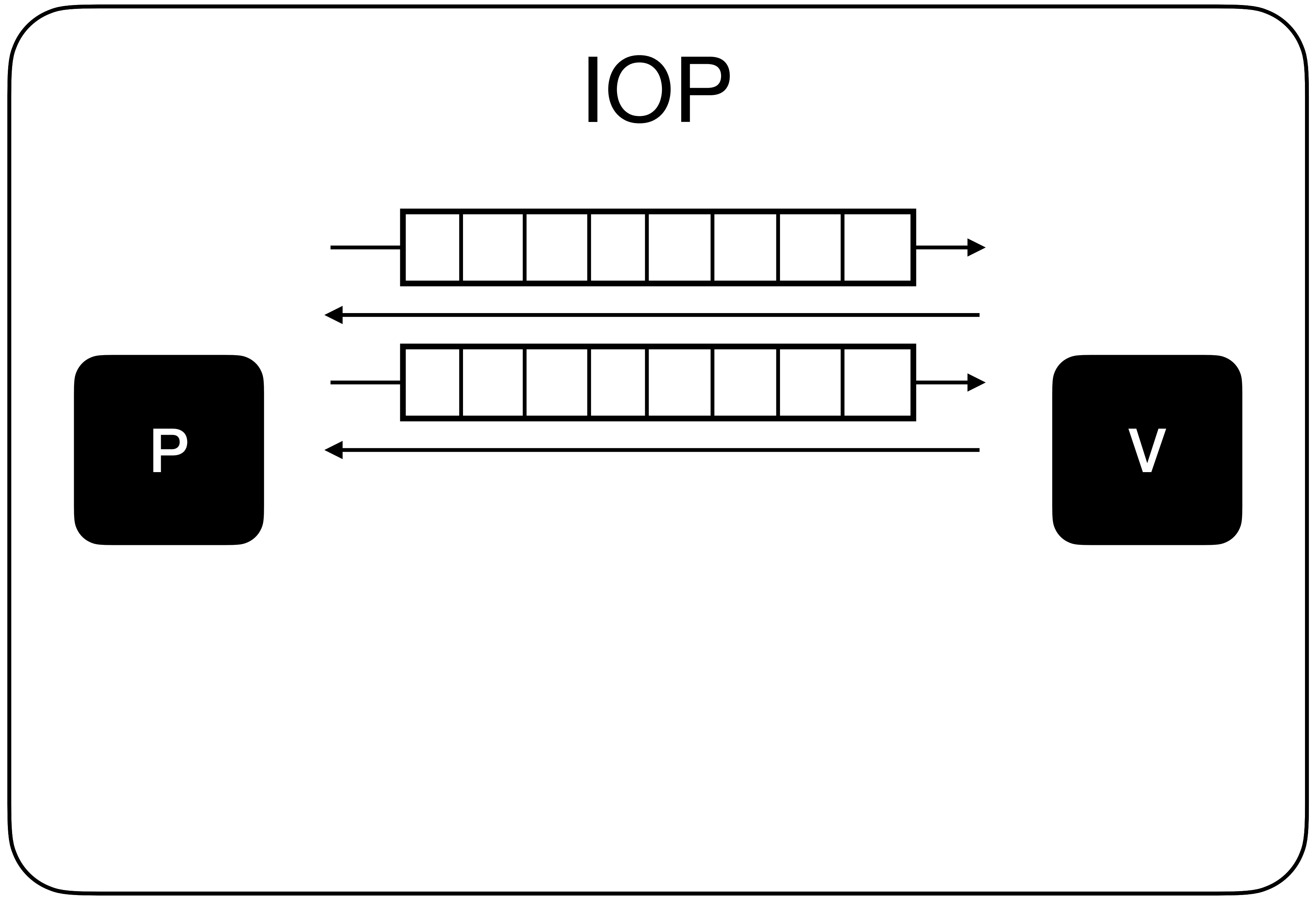
Constructing SNARKs

[BCS16] Construction



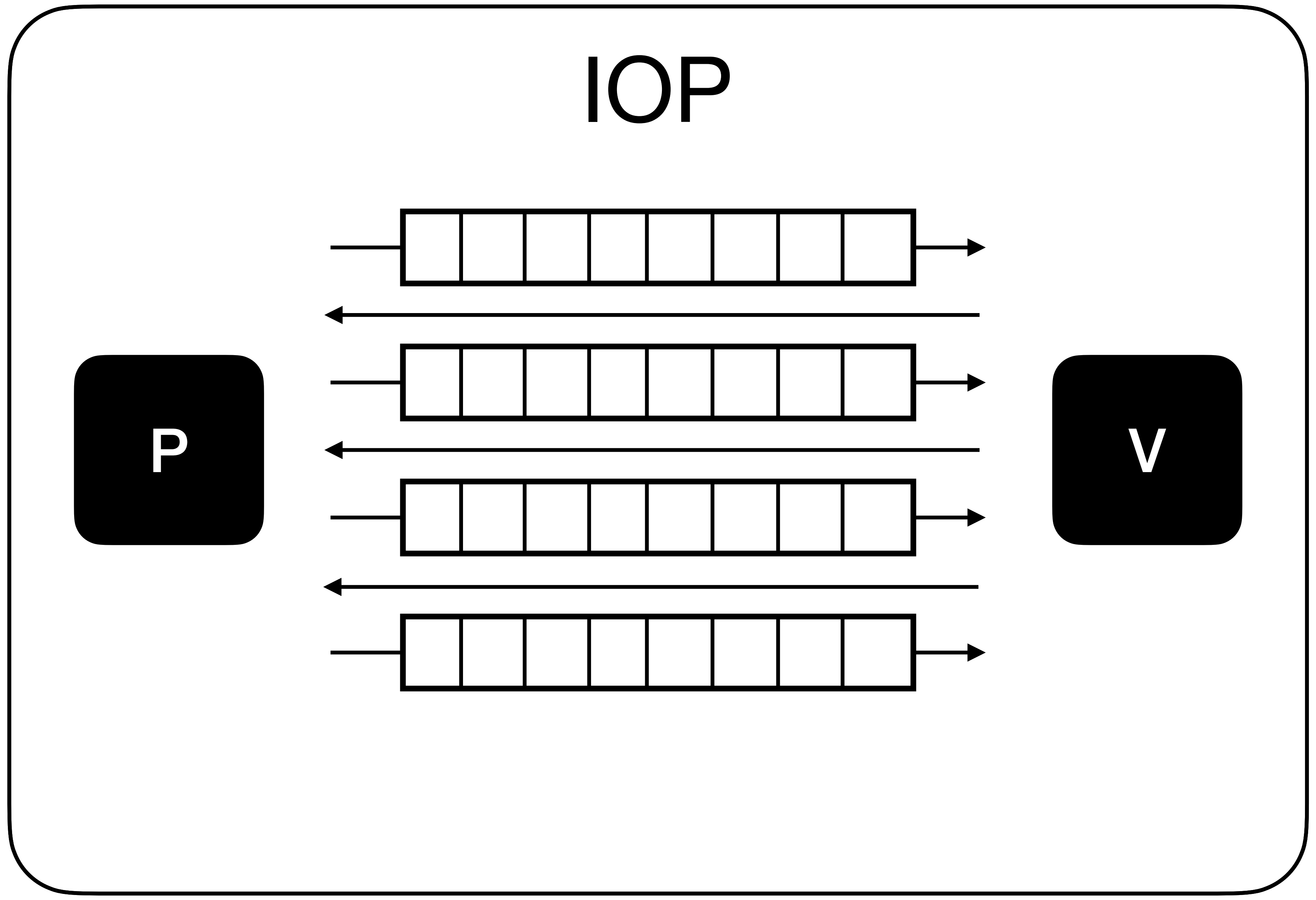
Constructing SNARKs

[BCS16] Construction



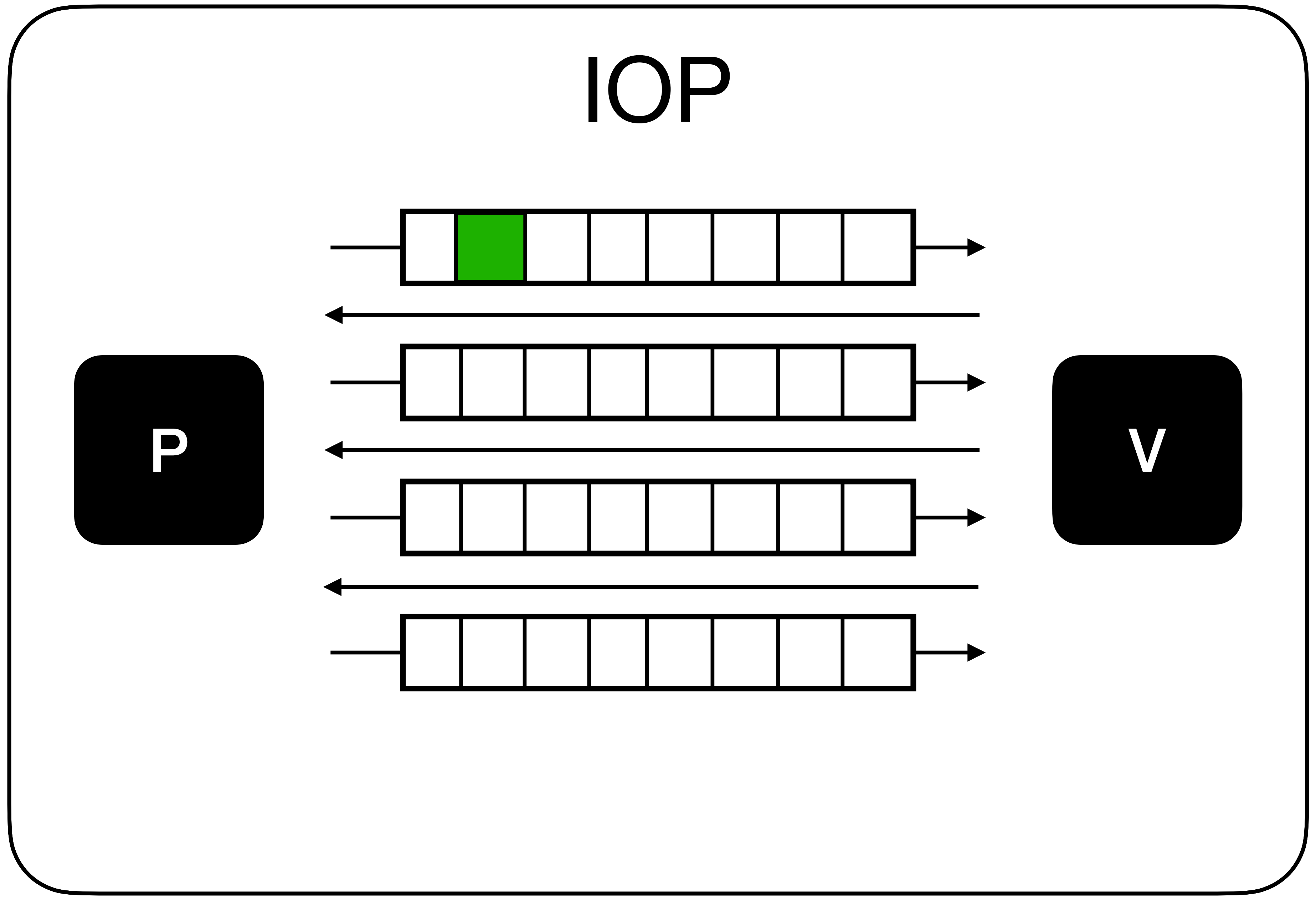
Constructing SNARKs

[BCS16] Construction



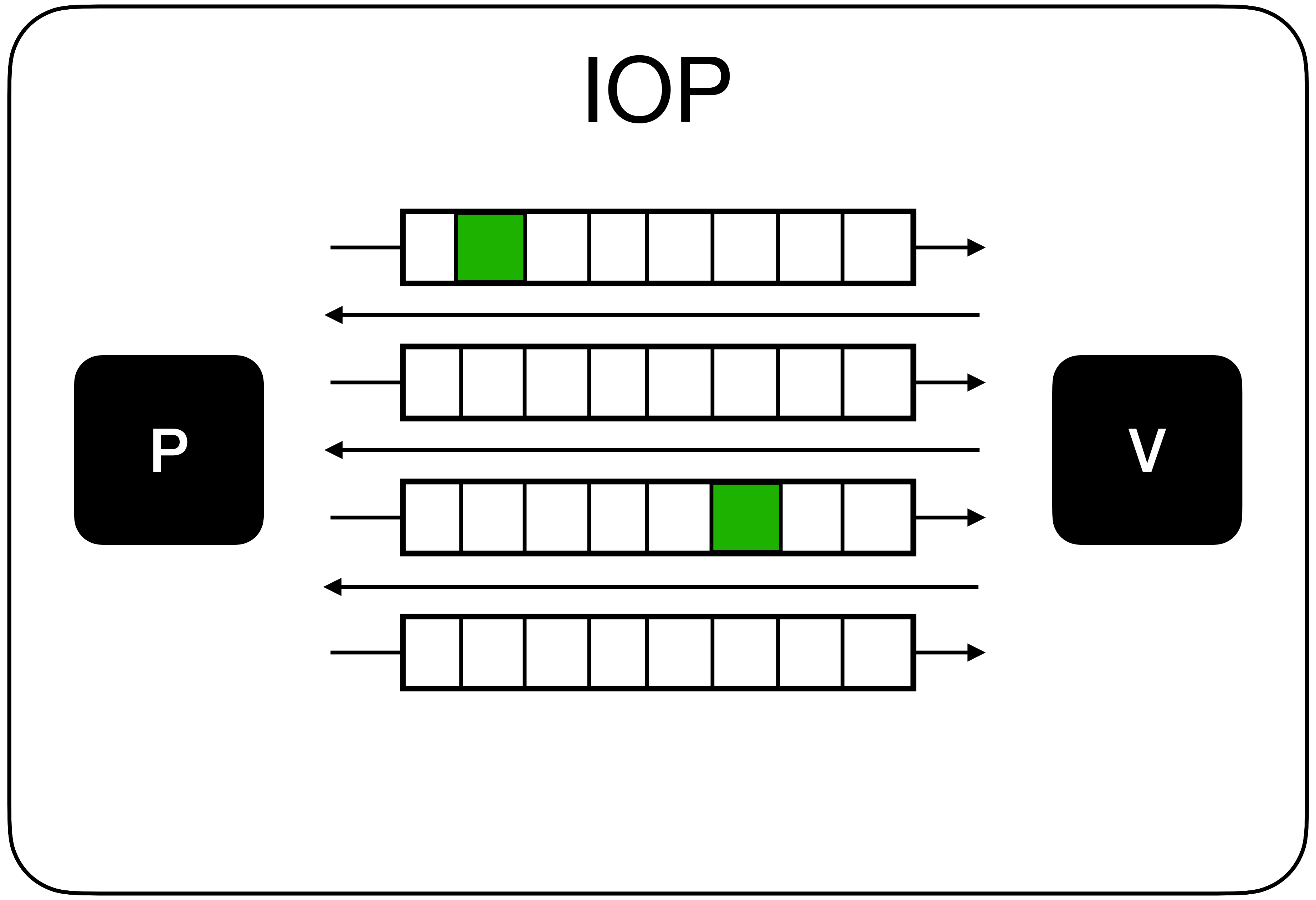
Constructing SNARKs

[BCS16] Construction



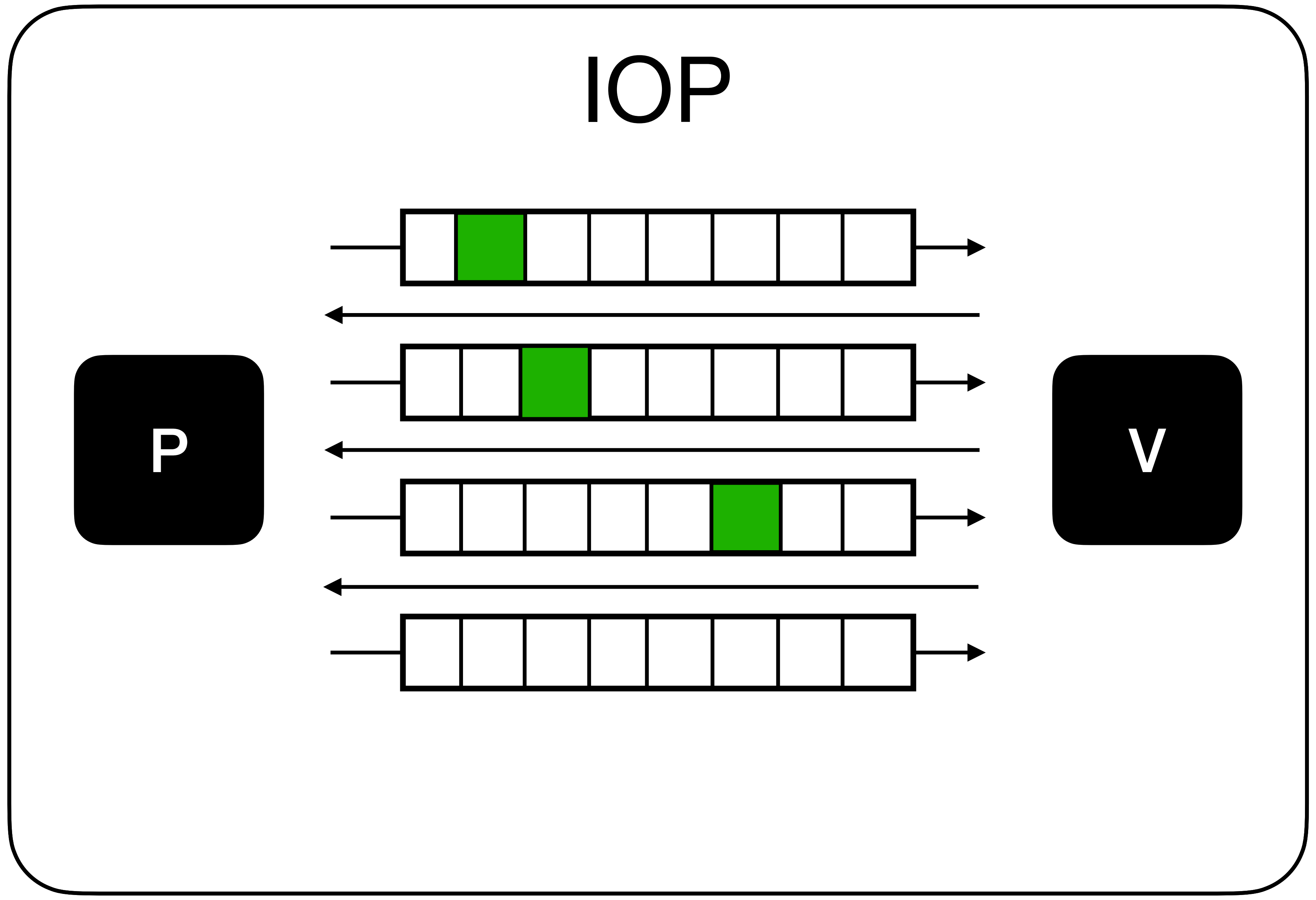
Constructing SNARKs

[BCS16] Construction



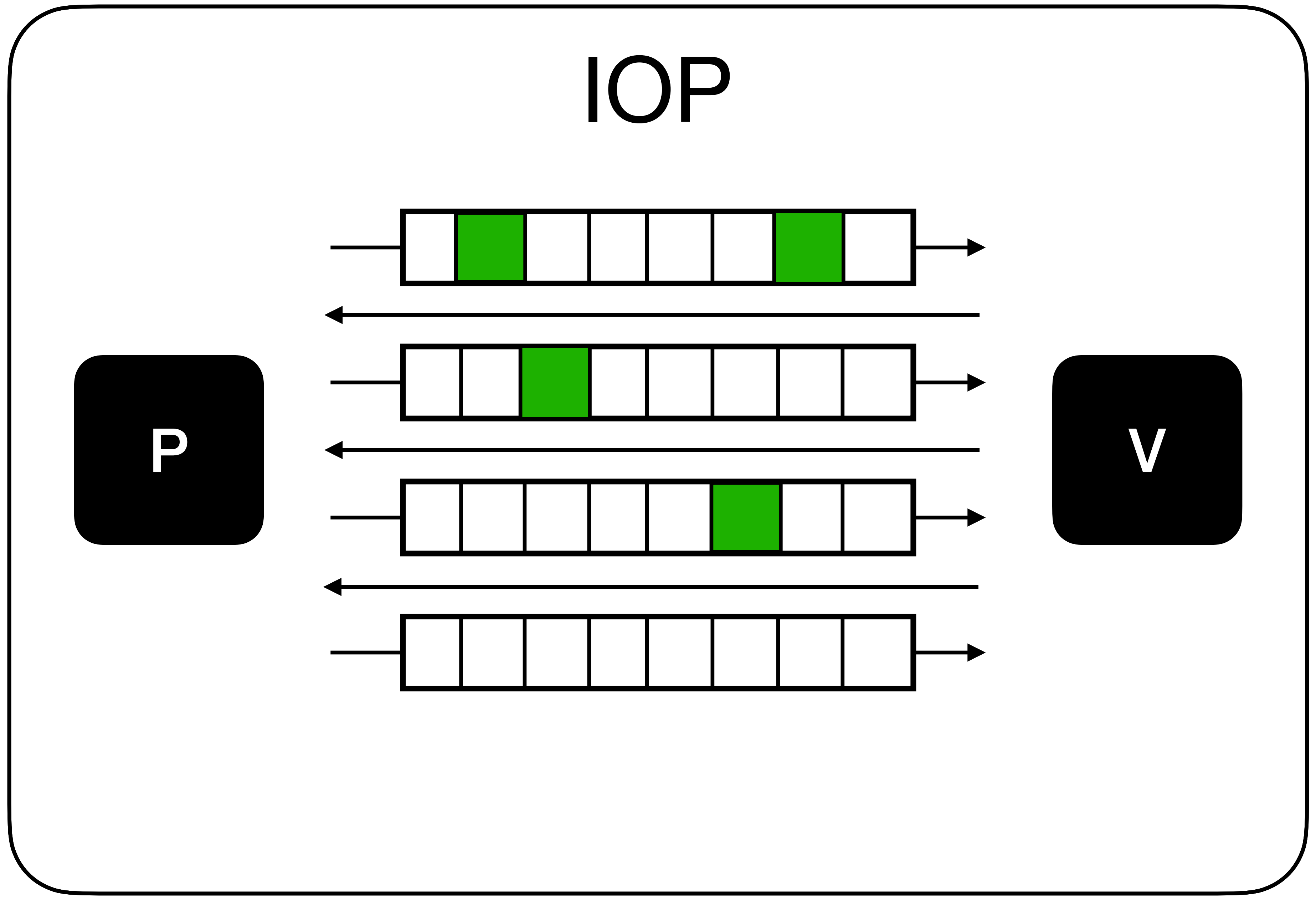
Constructing SNARKs

[BCS16] Construction



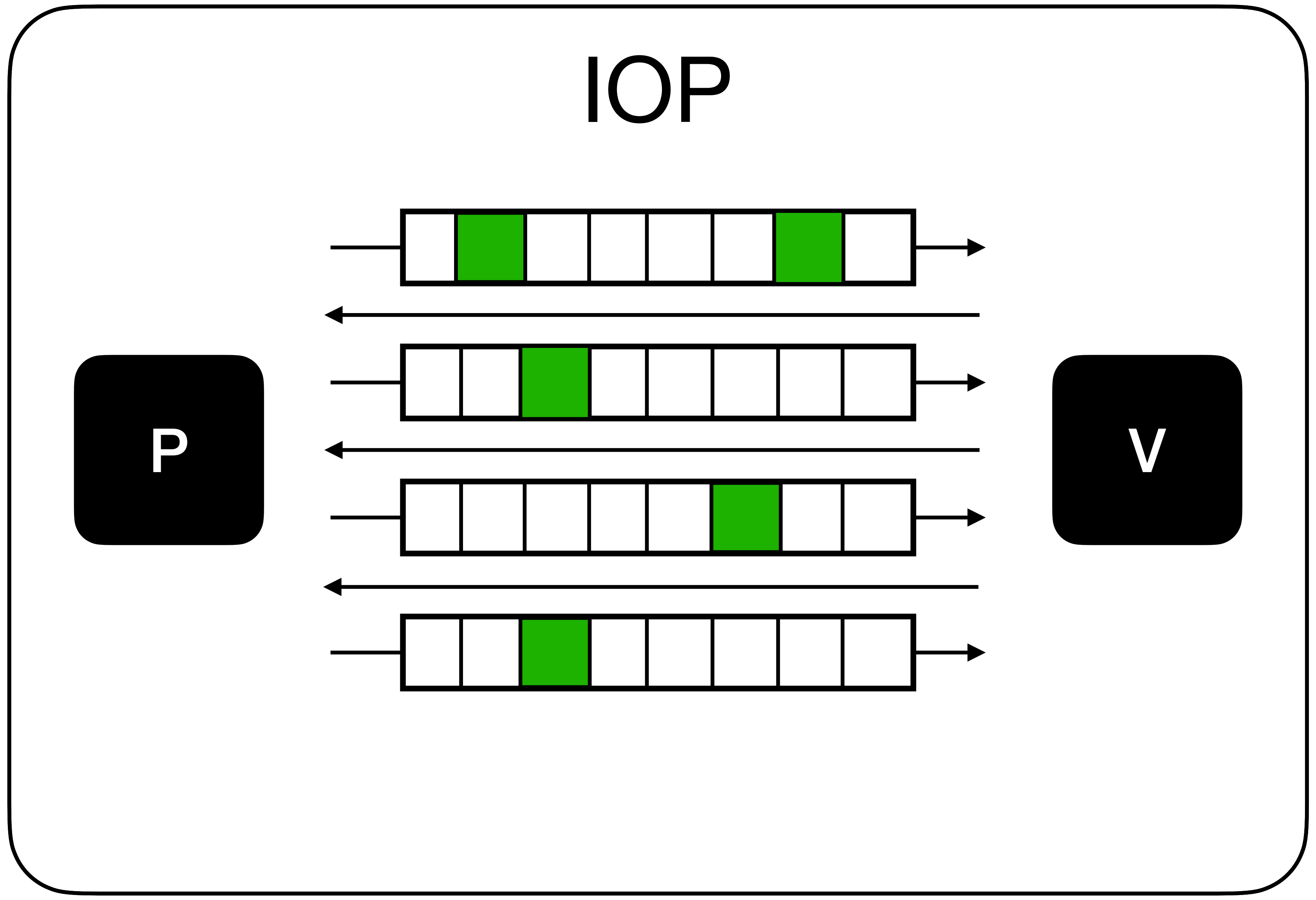
Constructing SNARKs

[BCS16] Construction



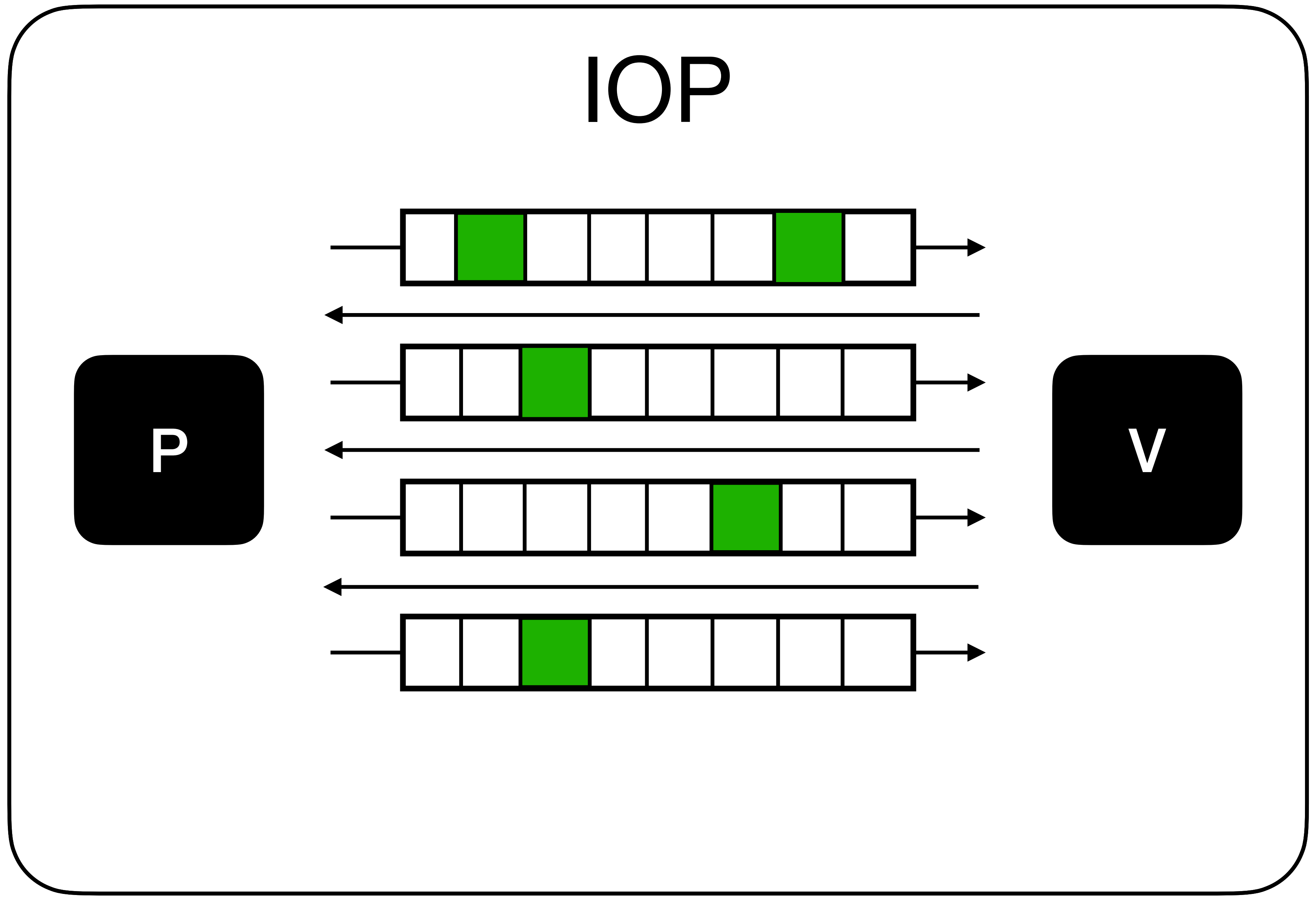
Constructing SNARKs

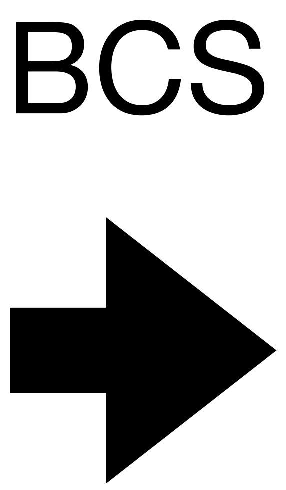
[BCS16] Construction



Constructing SNARKs

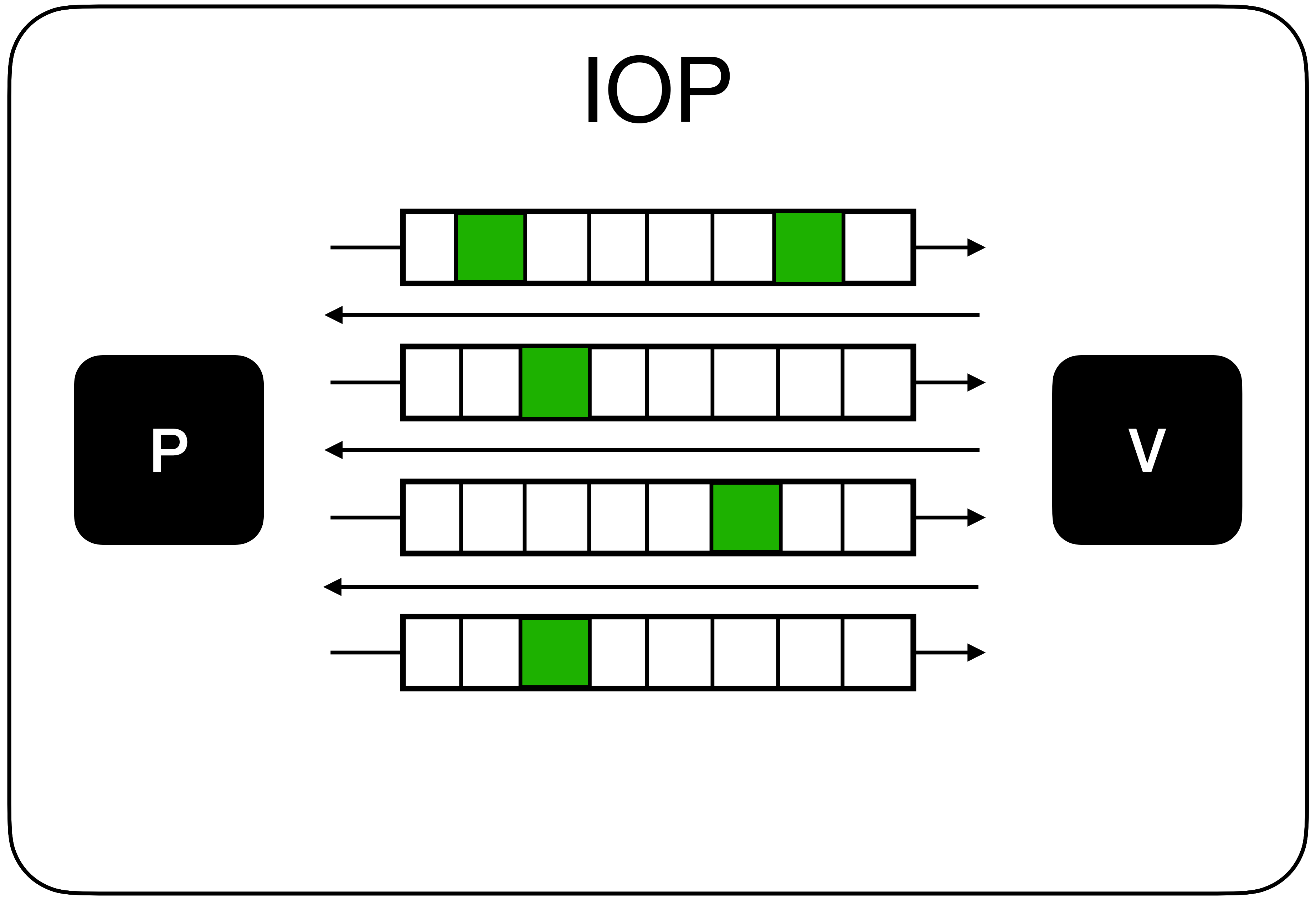
[BCS16] Construction

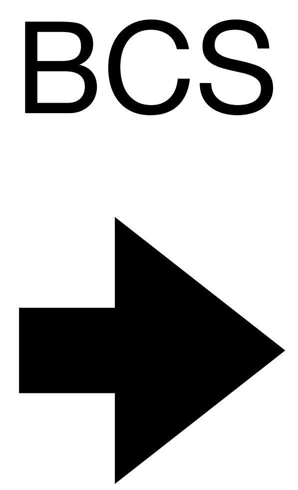


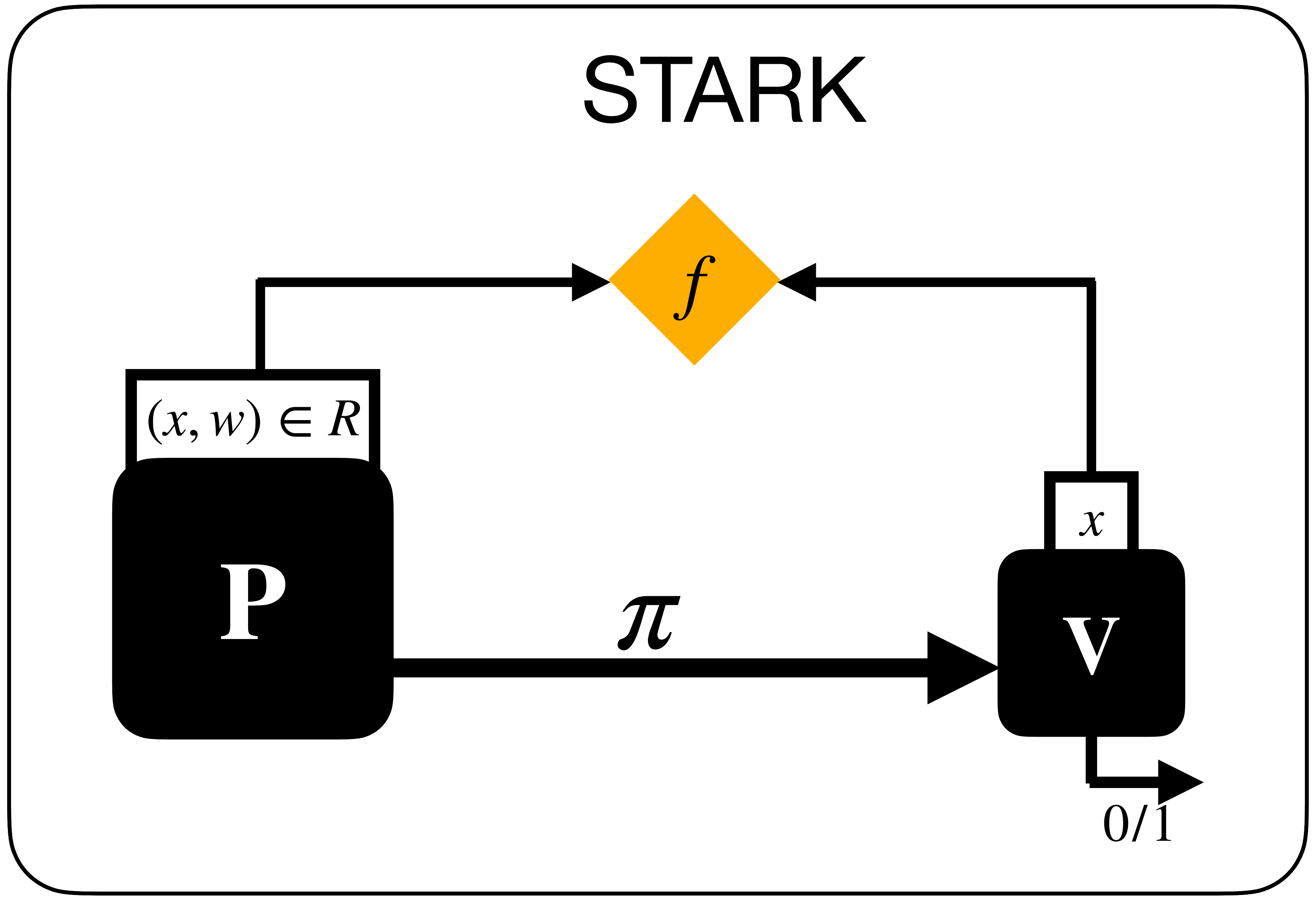


Constructing SNARKs

[BCS16] Construction

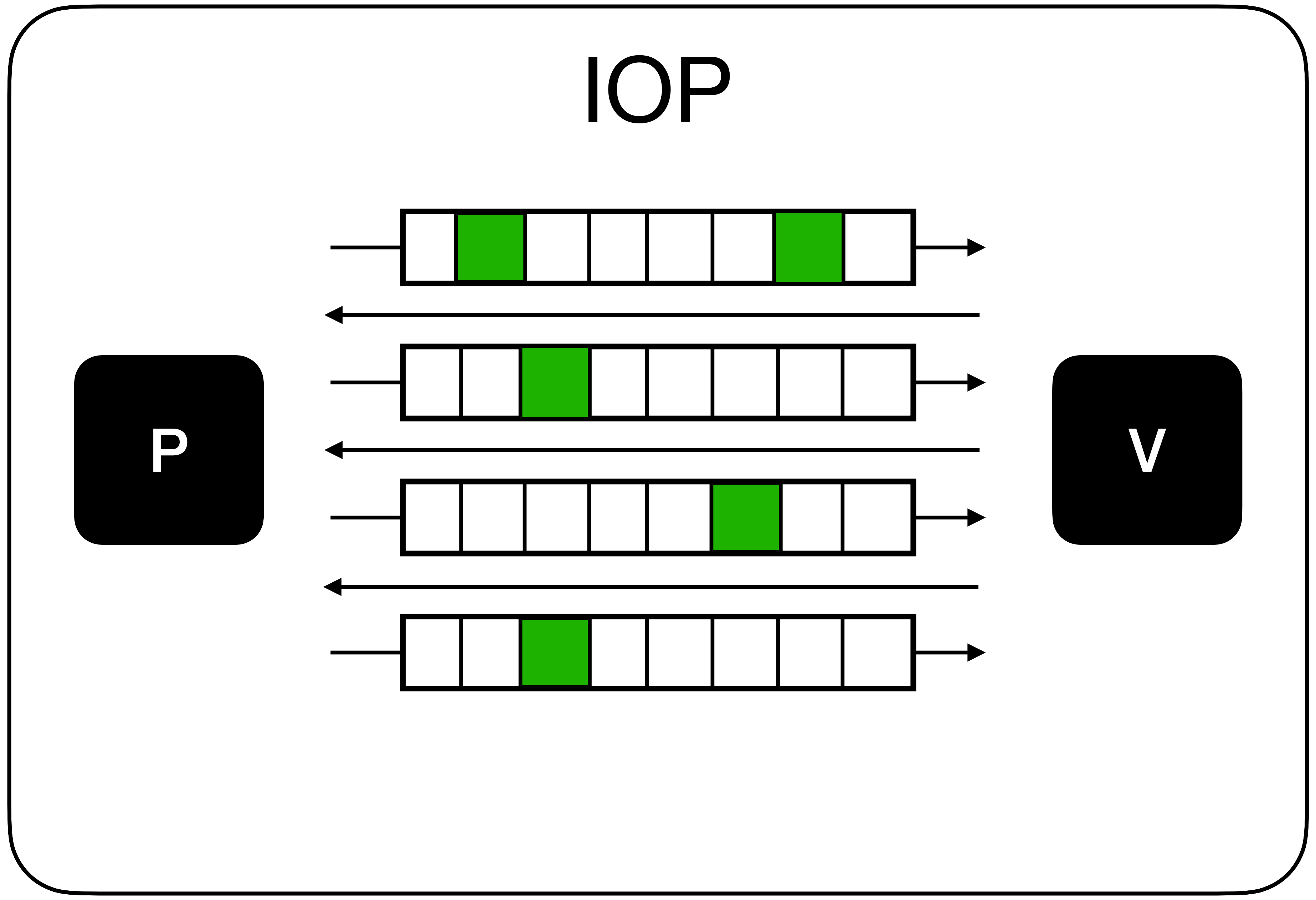


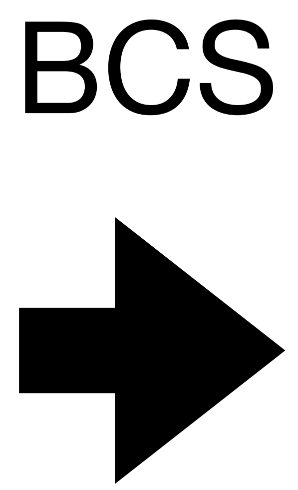


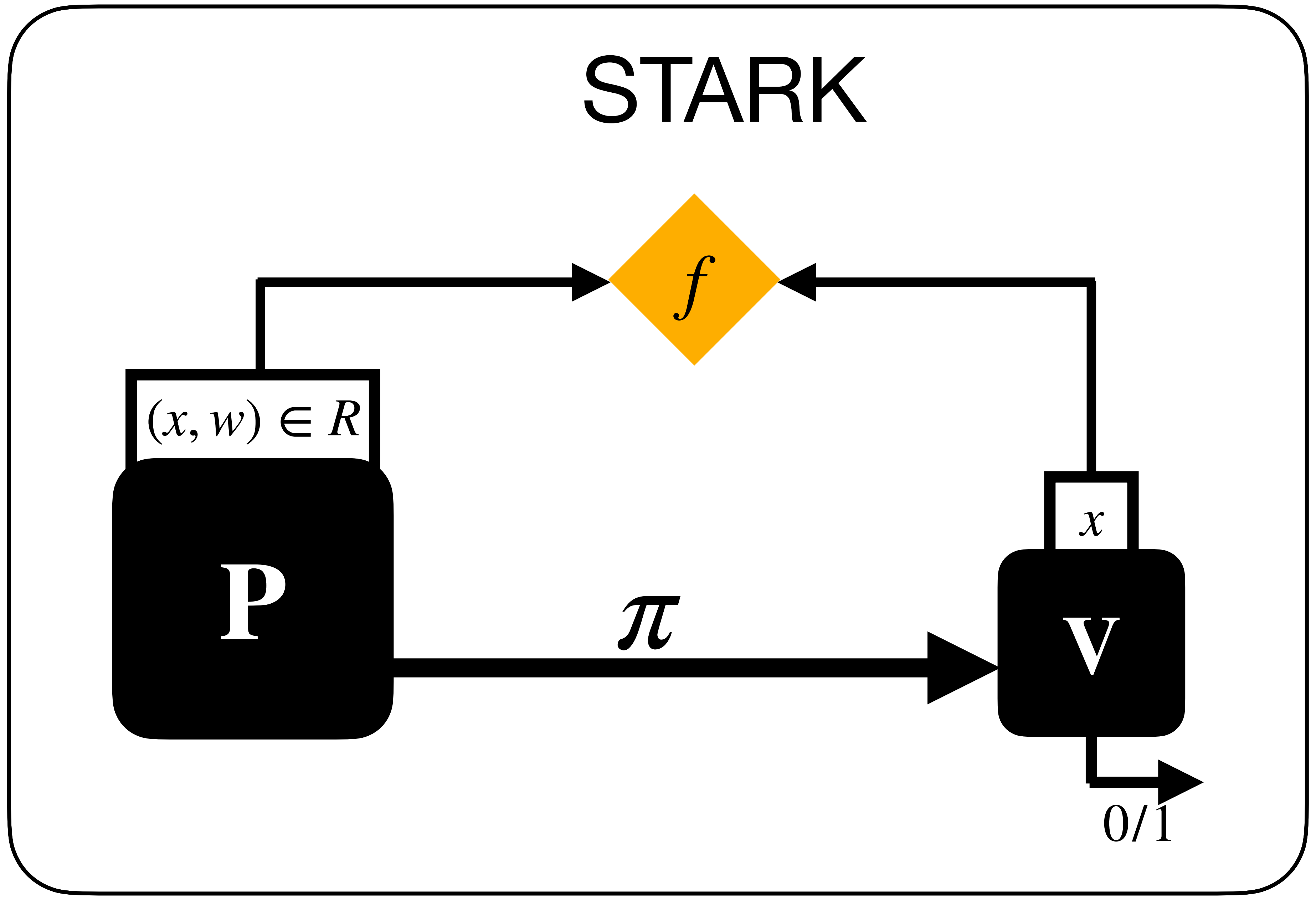


Constructing SNARKs

[BCS16] Construction





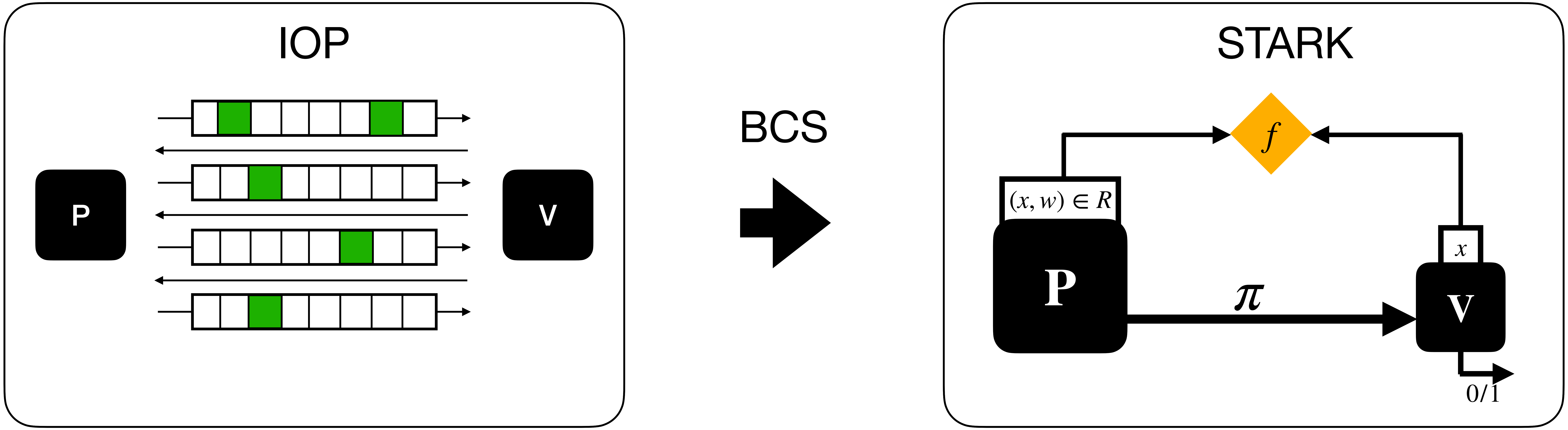


Proof length l $\approx O(n)$

Queries q $\approx O(\log n)$

Constructing SNARKs

[BCS16] Construction

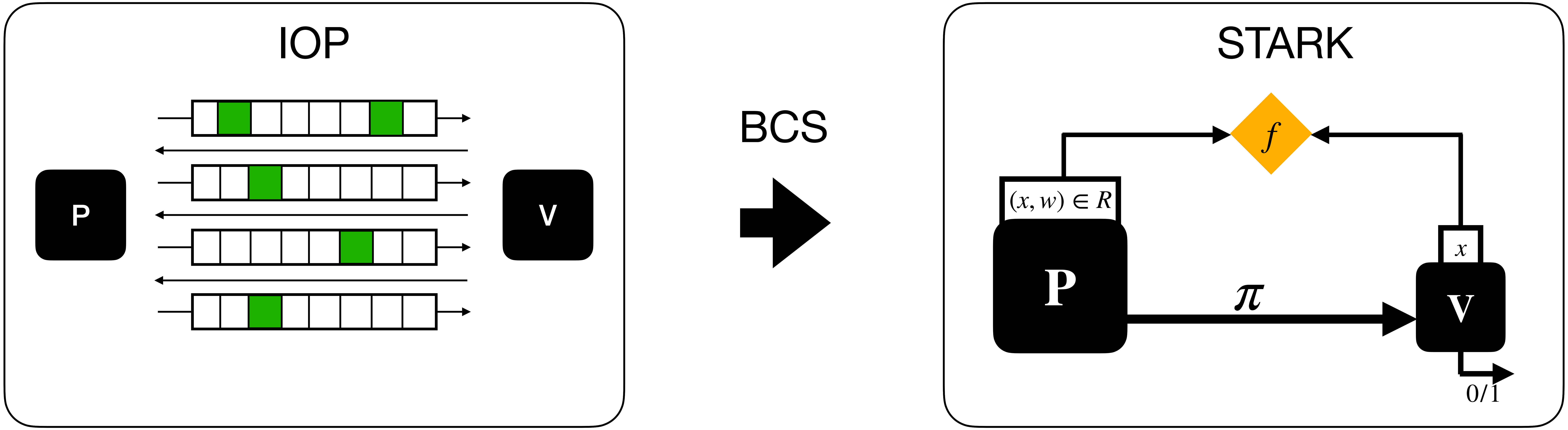


Proof length l $\approx O(n)$ — Large, think 2^{24}

Queries q $\approx O(\log n)$ — Small, think ~400

Constructing SNARKs

[BCS16] Construction



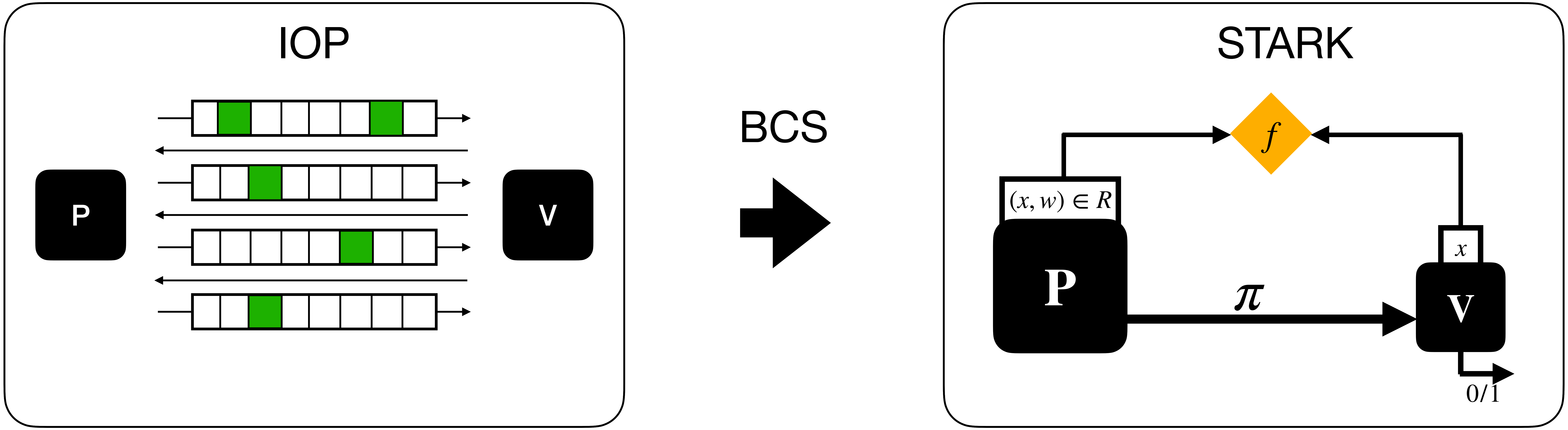
Proof length l $\approx O(n)$ — Large, think 2^{24}

Queries q $\approx O(\log n)$ — Small, think ~400

Argument size $O(\lambda \cdot \mathrm{q} \cdot \log \mathrm{l})$

Constructing SNARKs

[BCS16] Construction



Proof length $\mathsf{l} \approx O(n)$ — Large, think 2^{24}

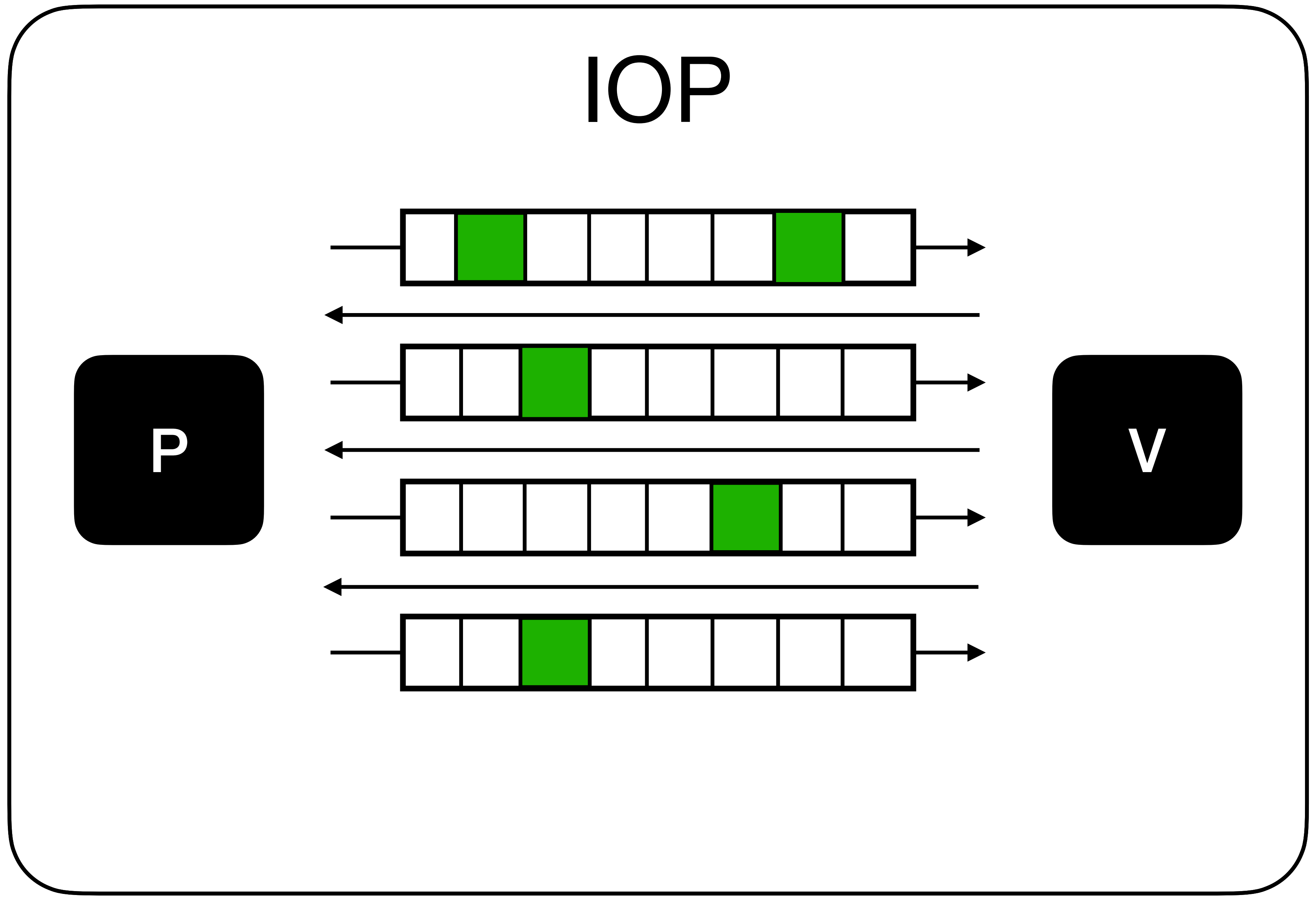
Queries $\mathsf{q} \approx O(\log n)$ — Small, think ~400

Argument size $O(\lambda \cdot \mathsf{q} \cdot \log \mathsf{l})$ — Small, tens of KiB

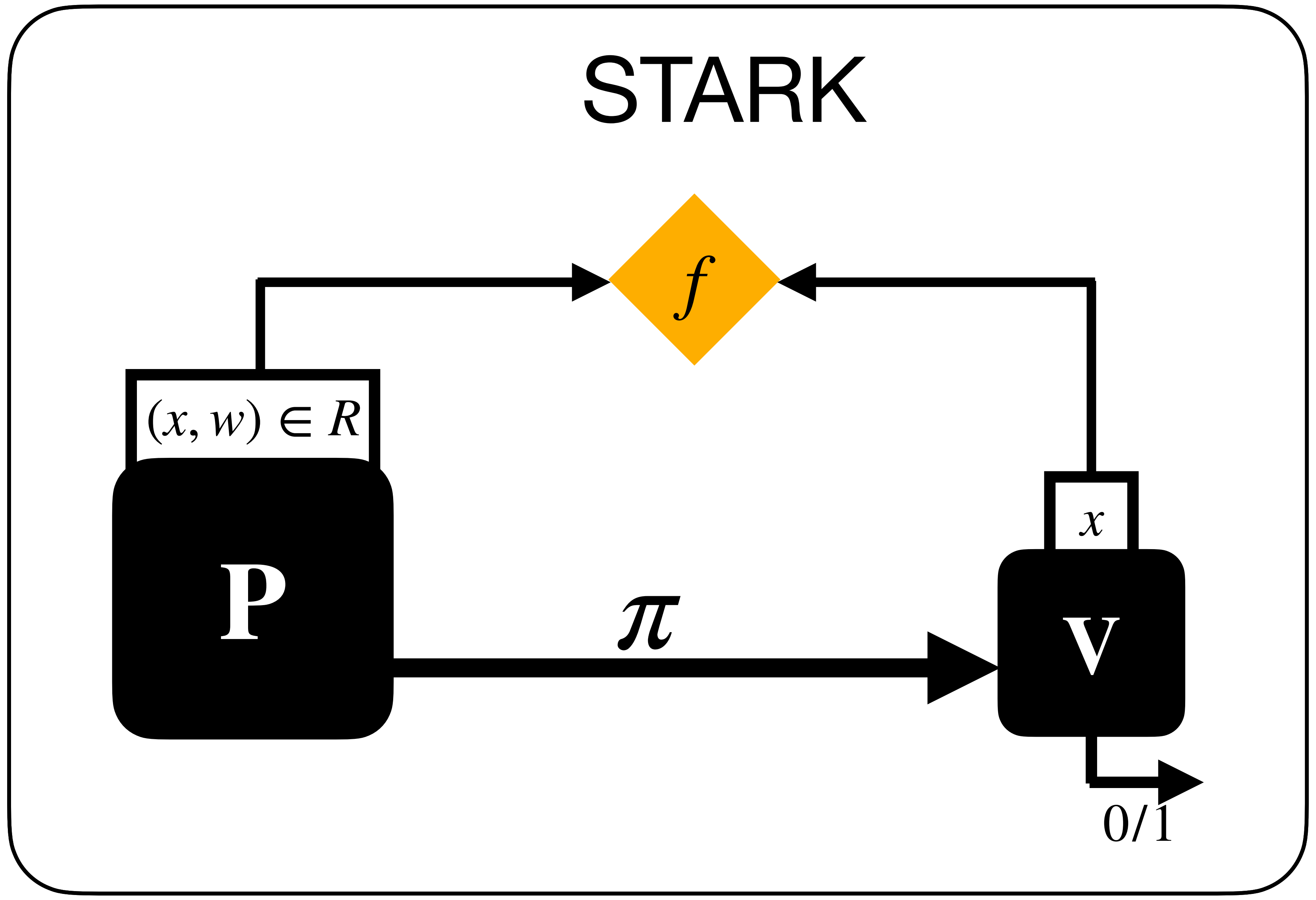
Constructing SNARKs

[BCS16] Construction

In this talk, we focus on the IOP!



BCS



Proof length l $\approx O(n)$ — Large, think 2^{24}

Queries q $\approx O(\log n)$ — Small, think ~400

Argument size $O(\lambda \cdot \mathsf{q} \cdot \log \mathsf{l})$ — Small, tens of KiB

Constructing IOPs

Traditionally

Constructing IOPs

Traditionally

PIOP

Constructing IOPs

Traditionally

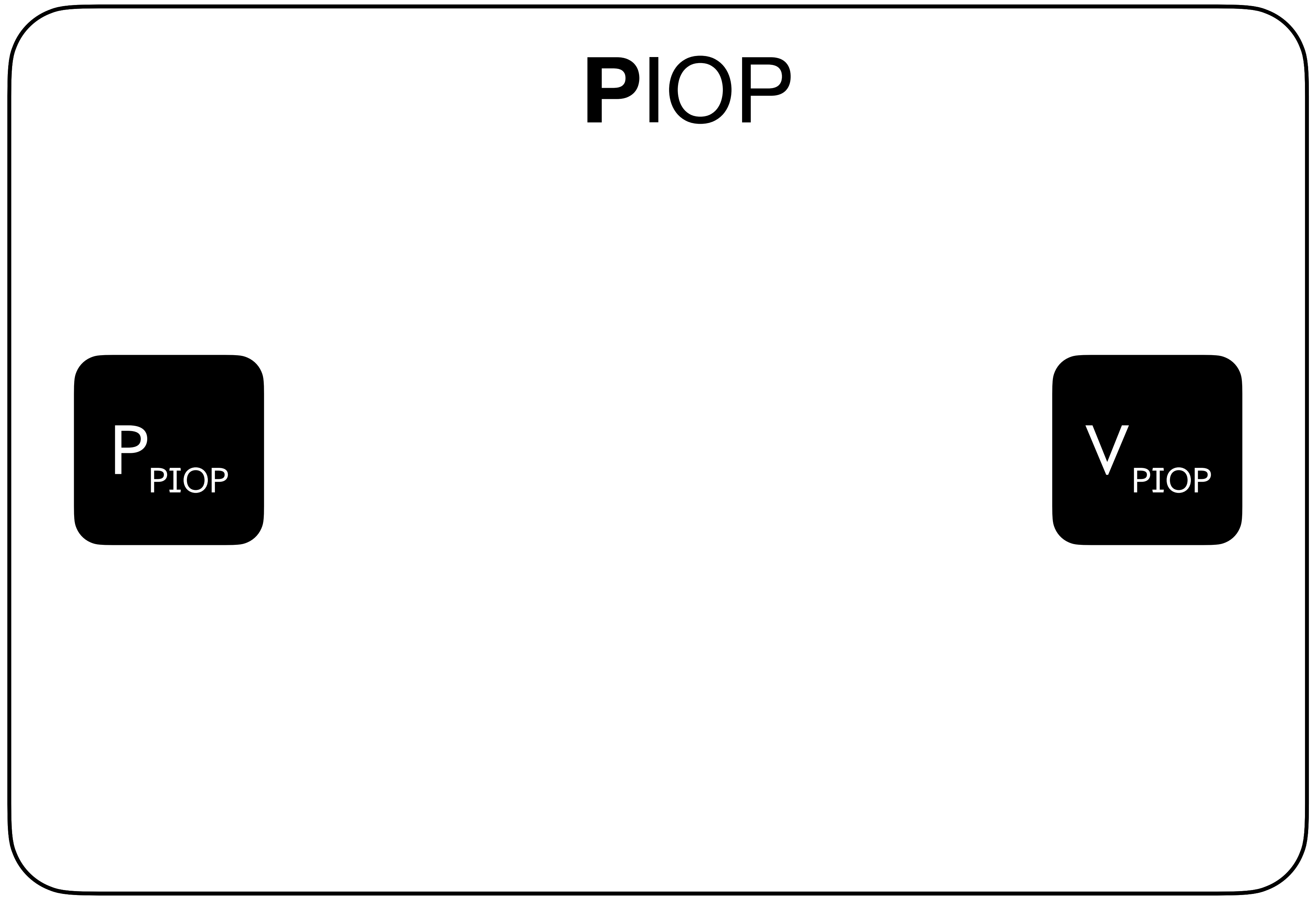
PIOP

Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

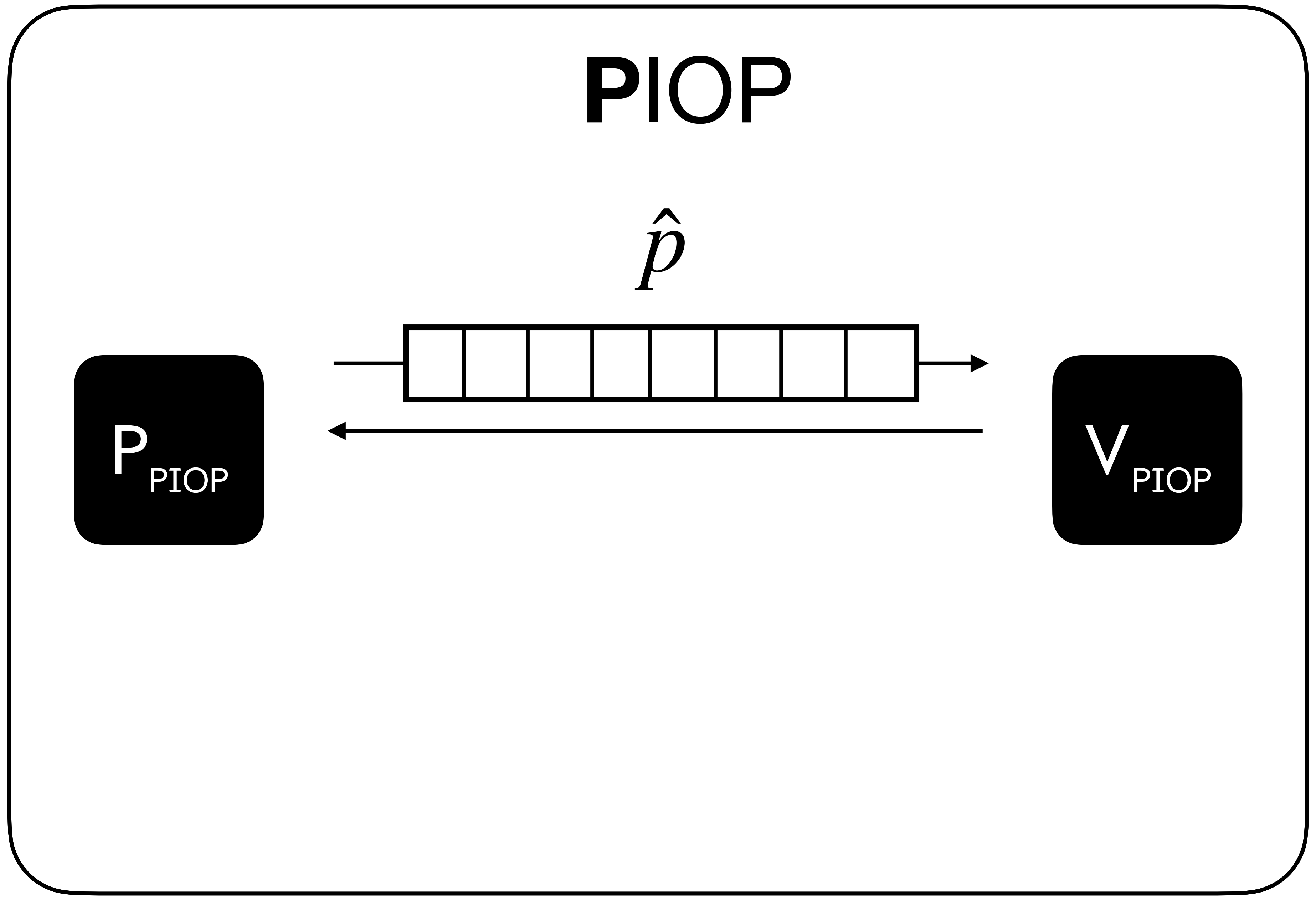


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

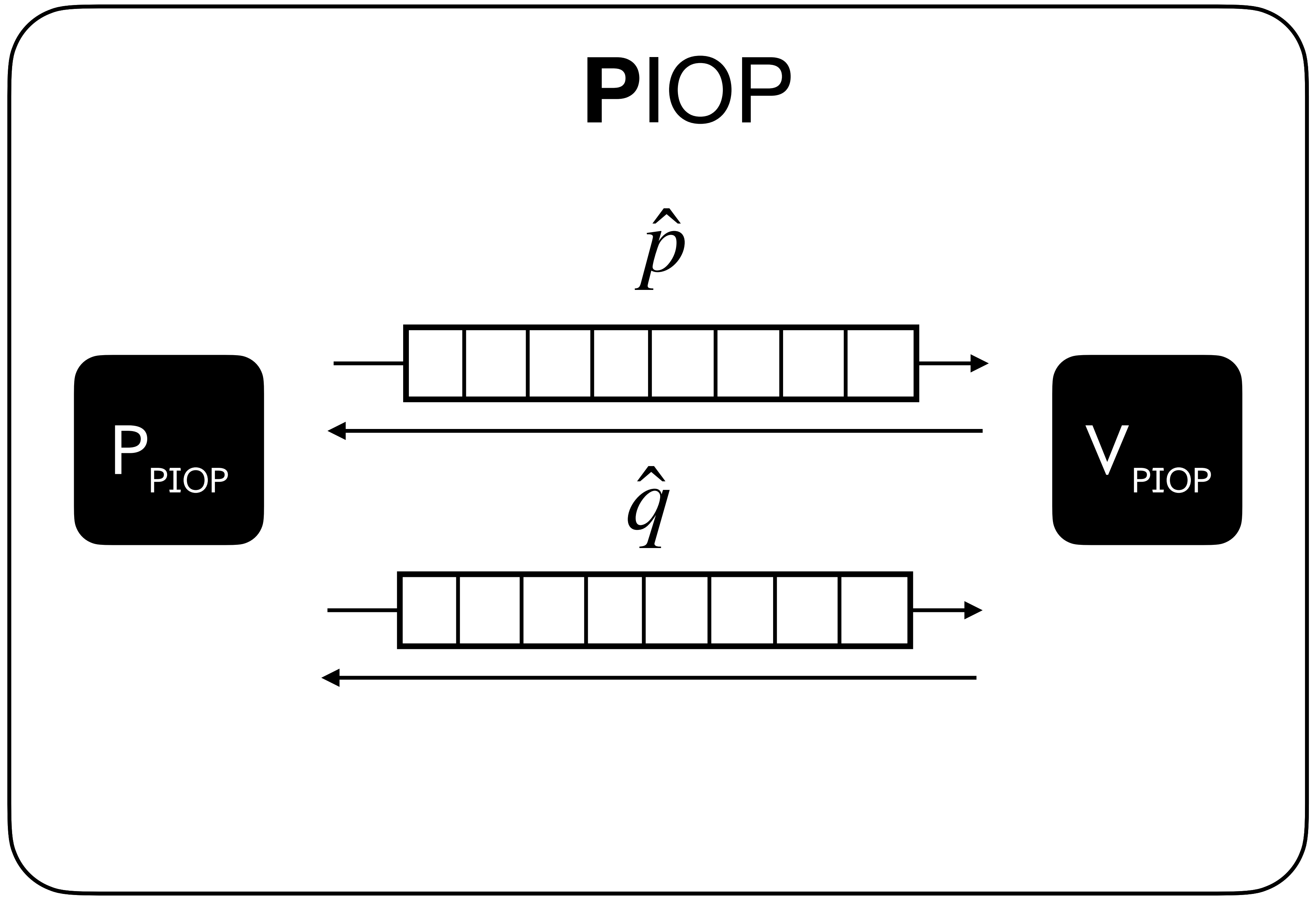


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

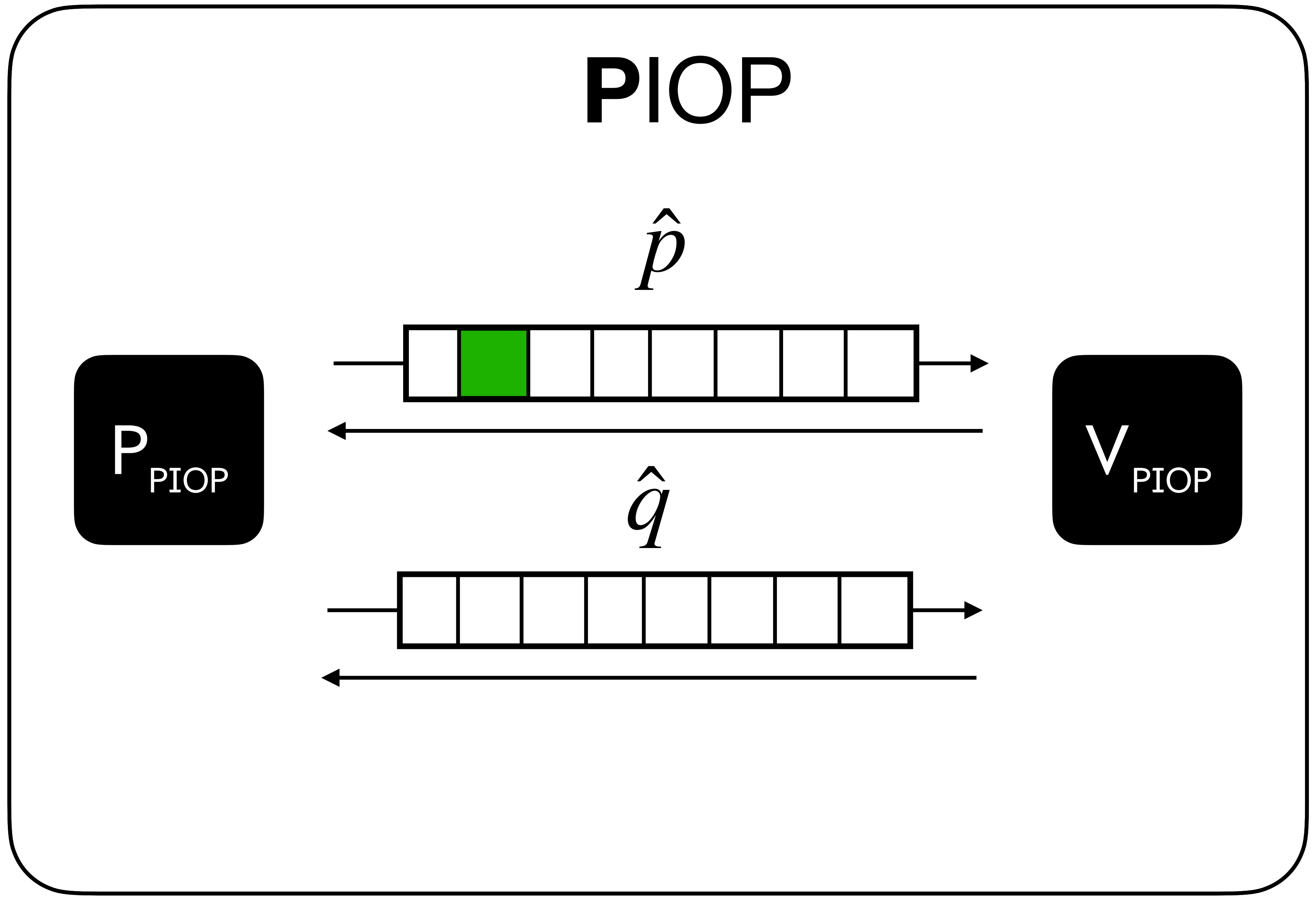


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

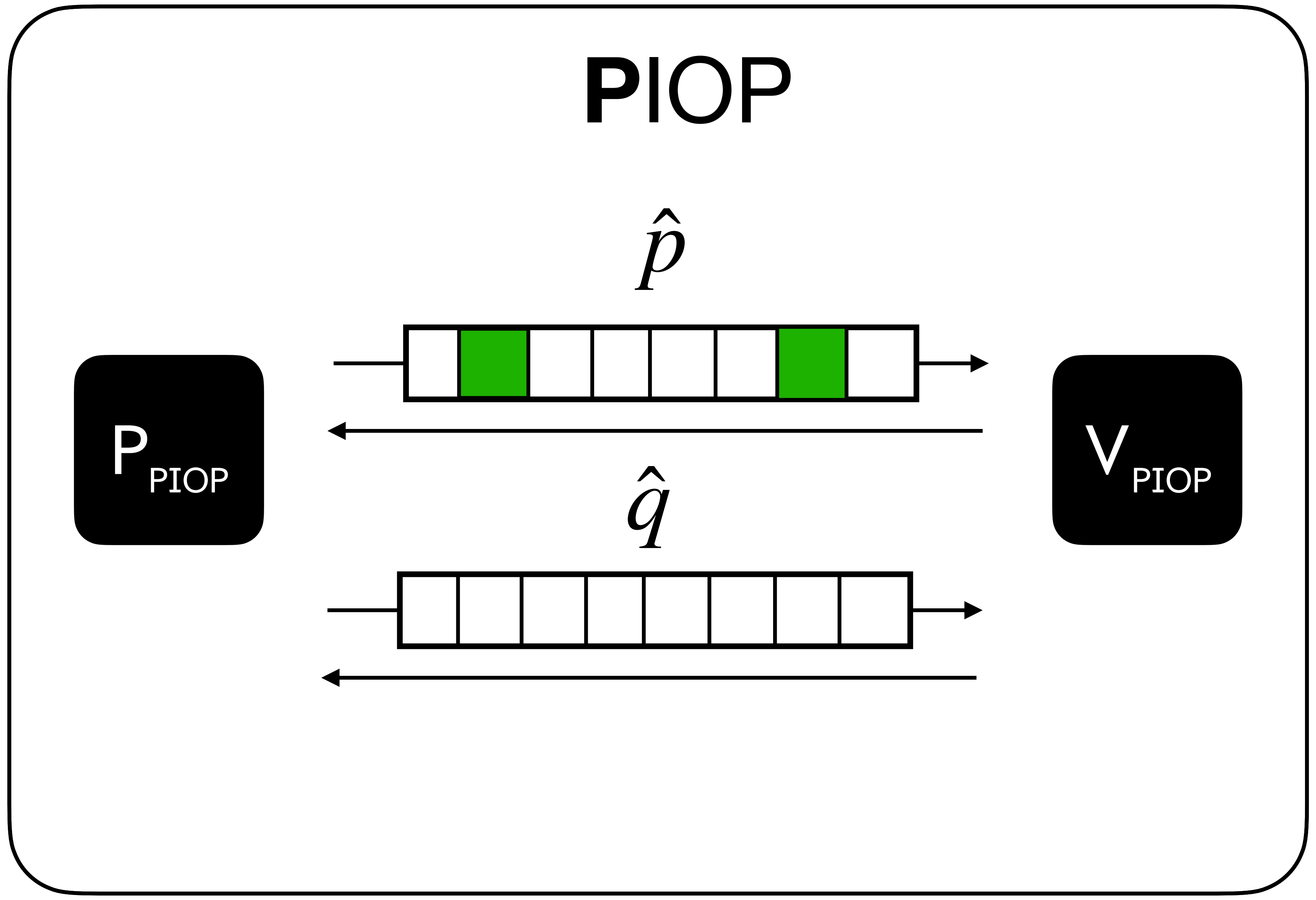


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

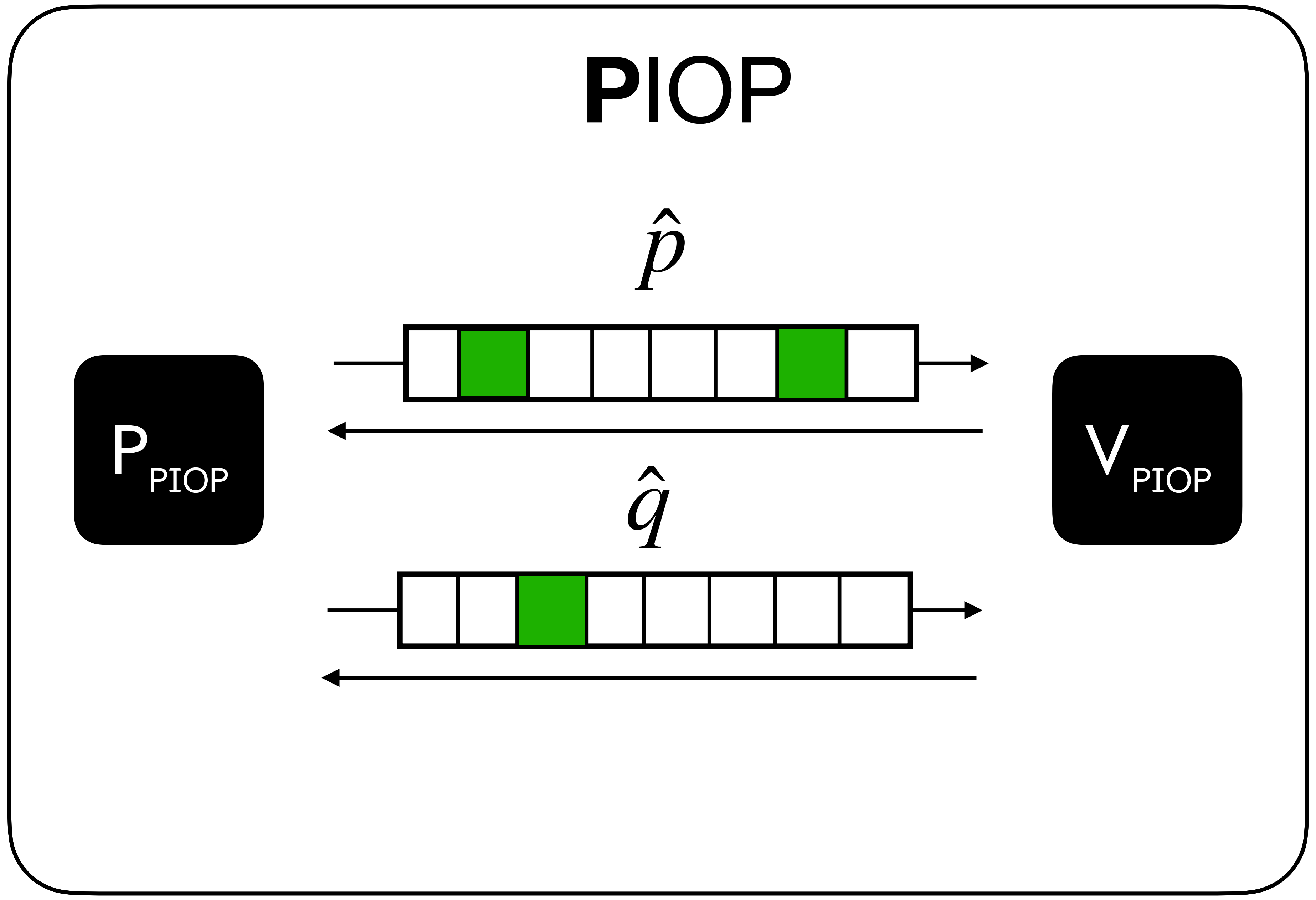


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

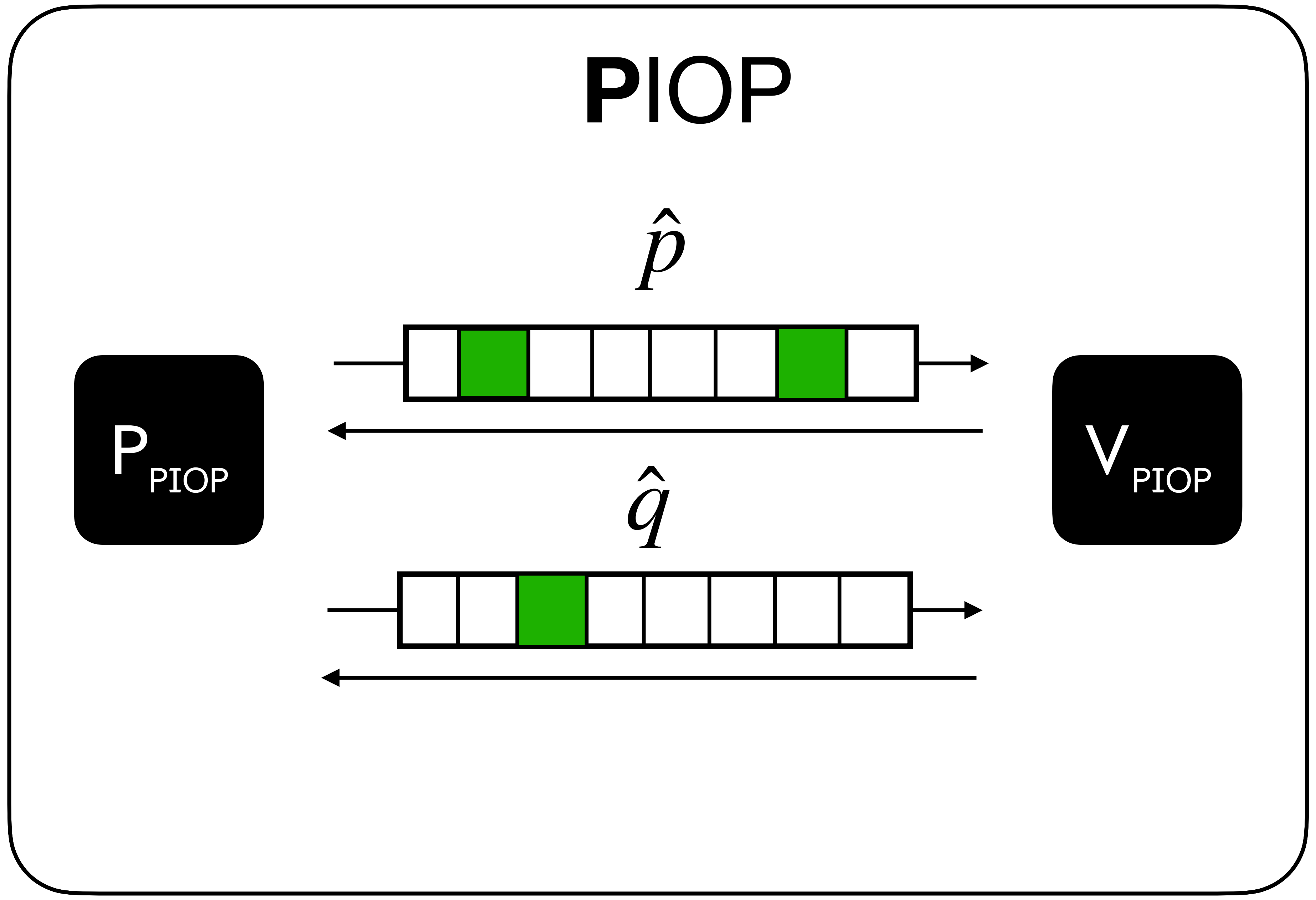


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally



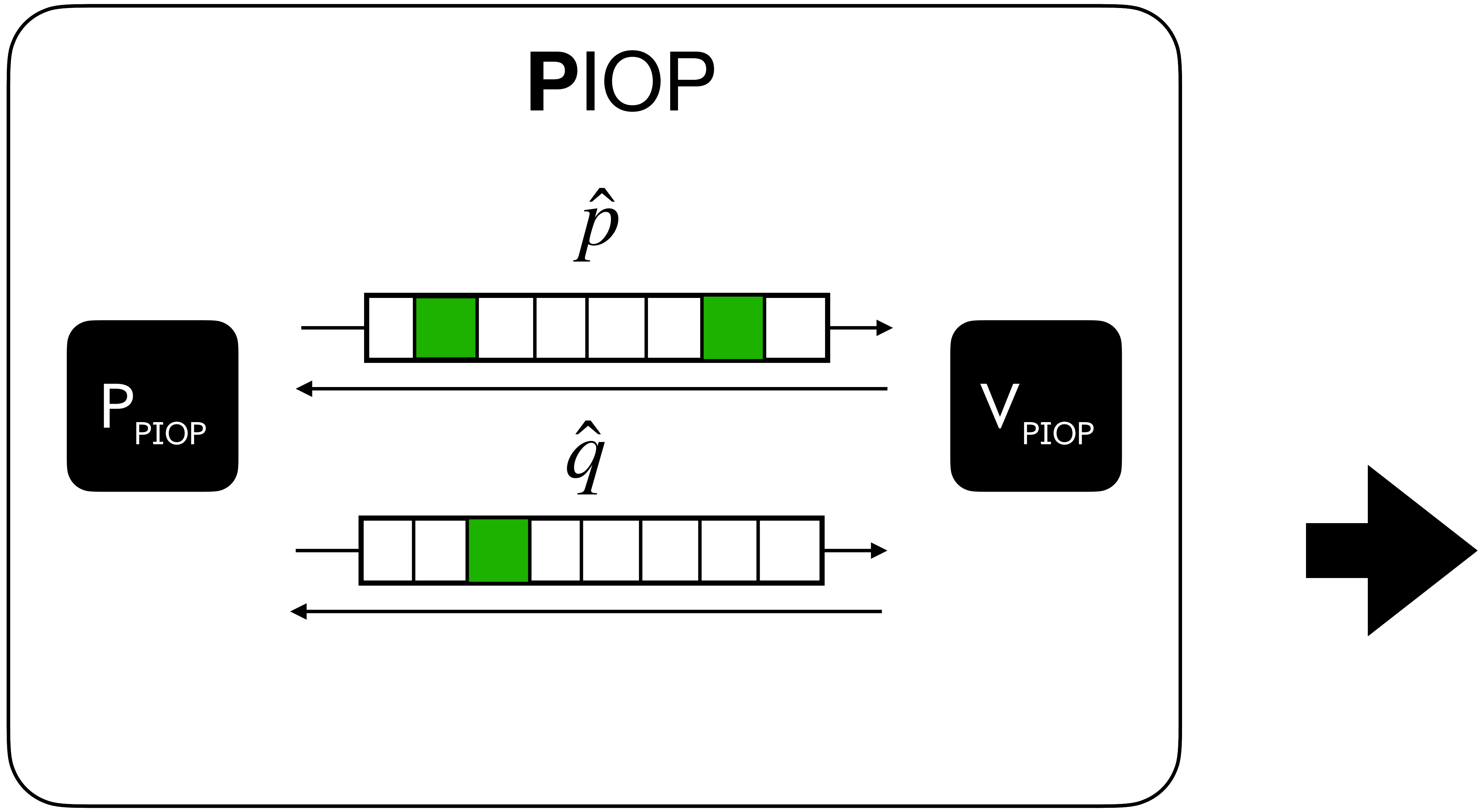
Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.

Constructing IOPs

Traditionally



Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

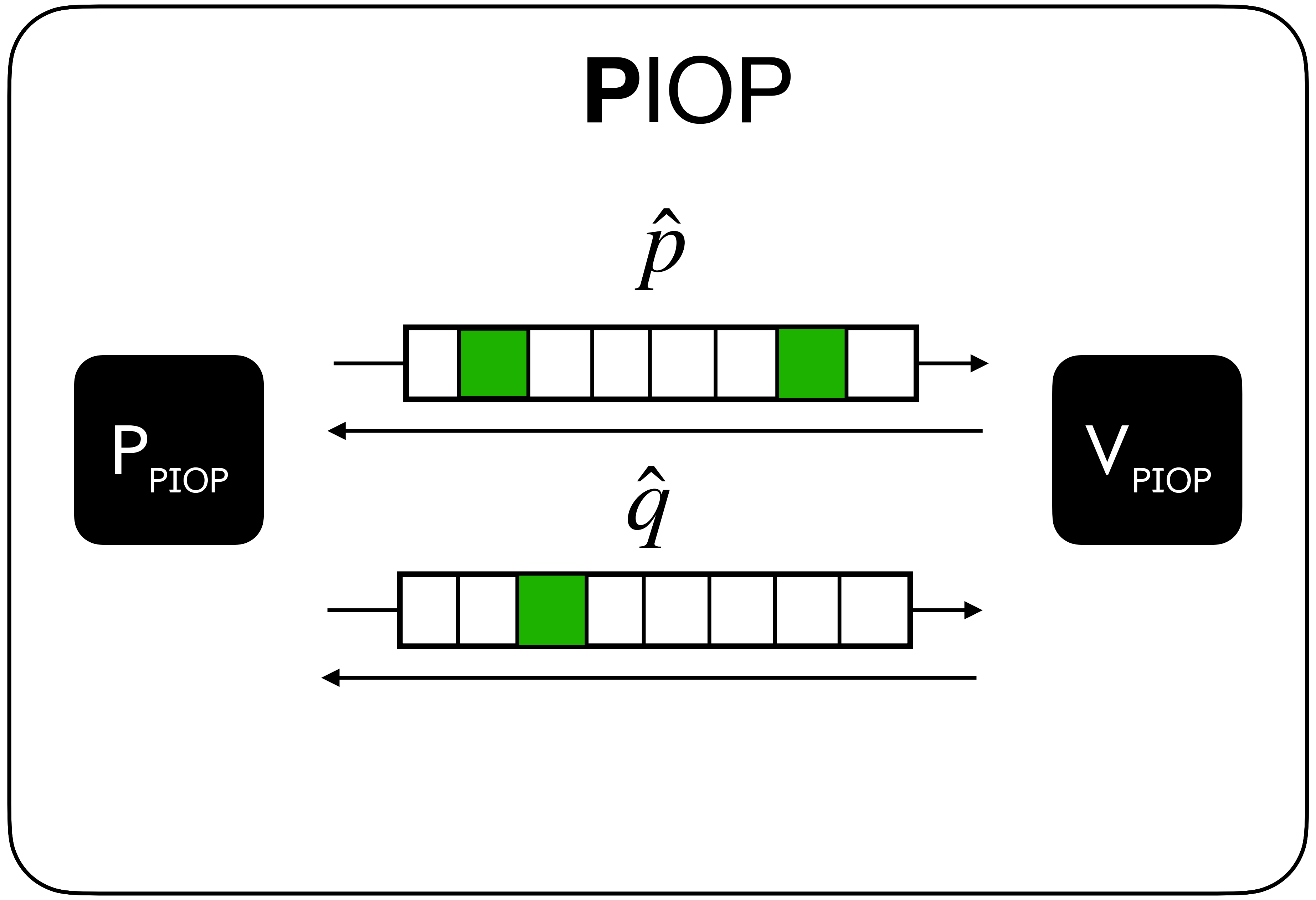
E.g. Aurora, STARK PIOP etc.

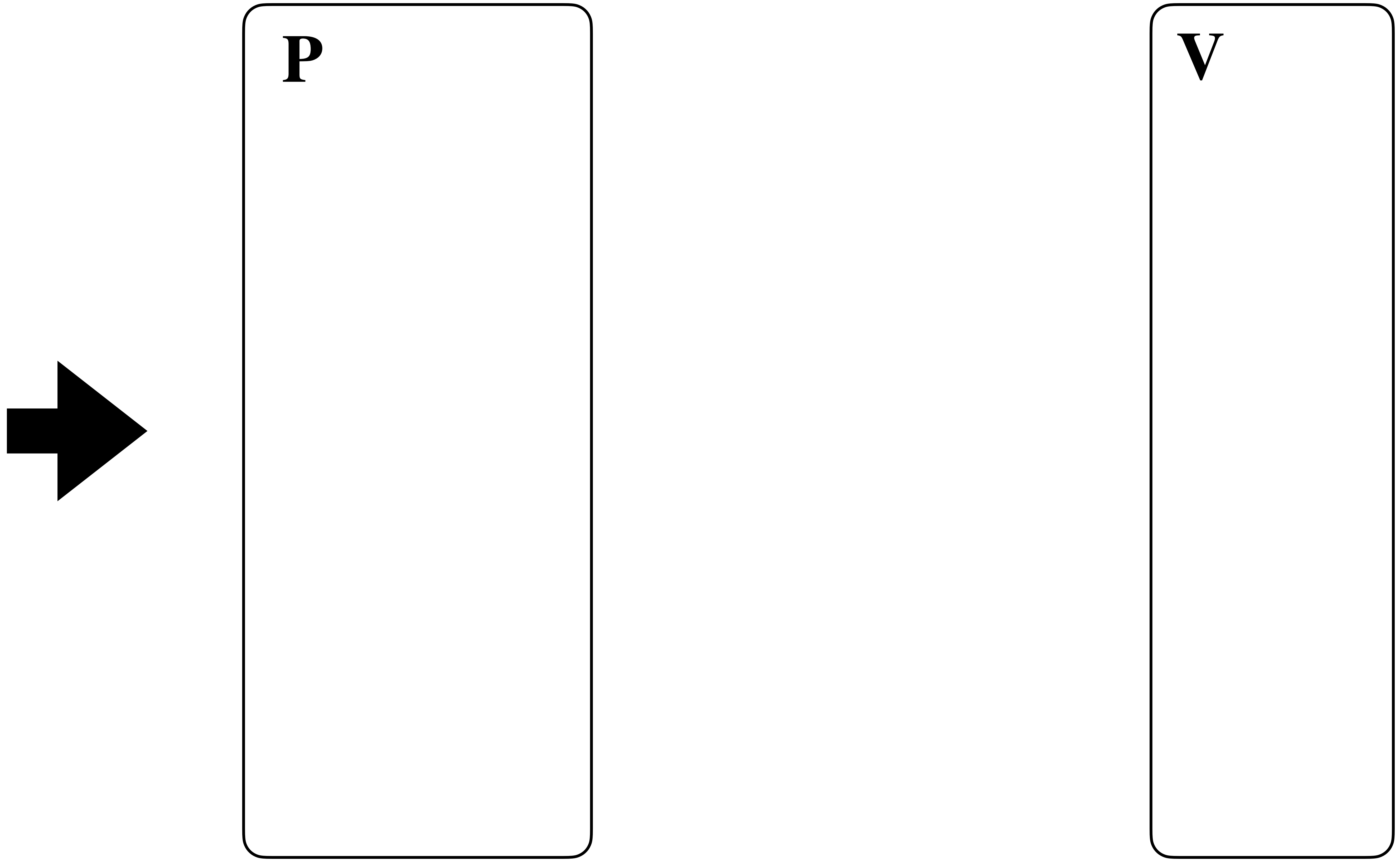
Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.

Constructing IOPs

Traditionally

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.





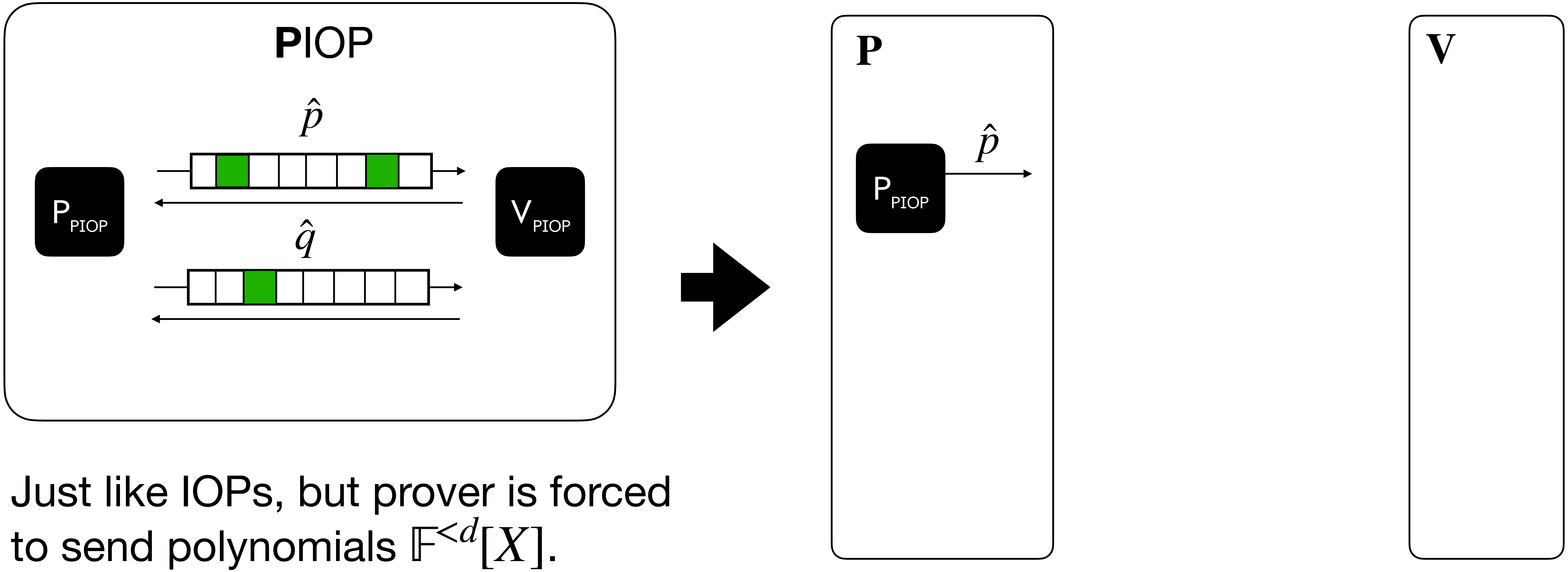
Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.



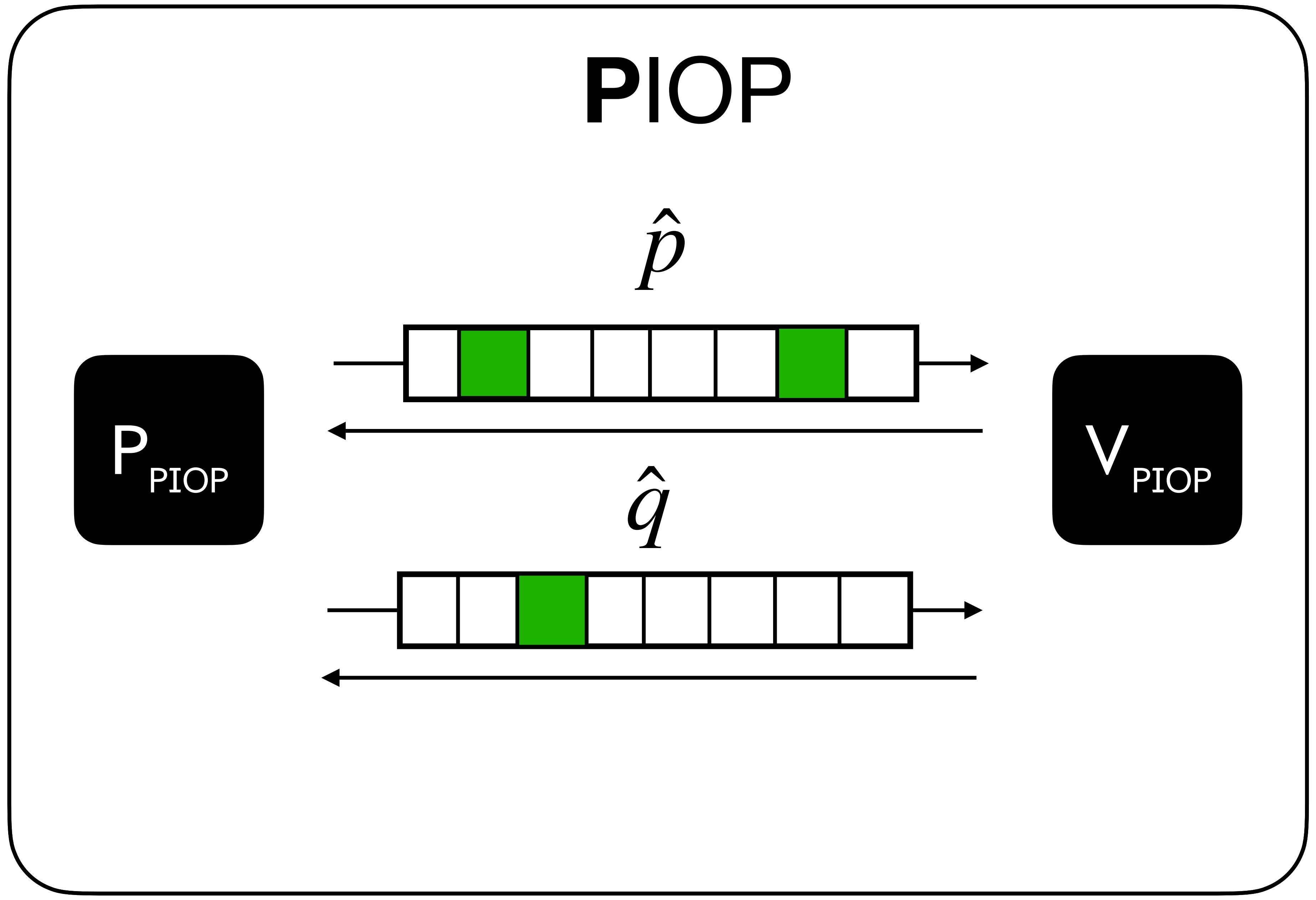
Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

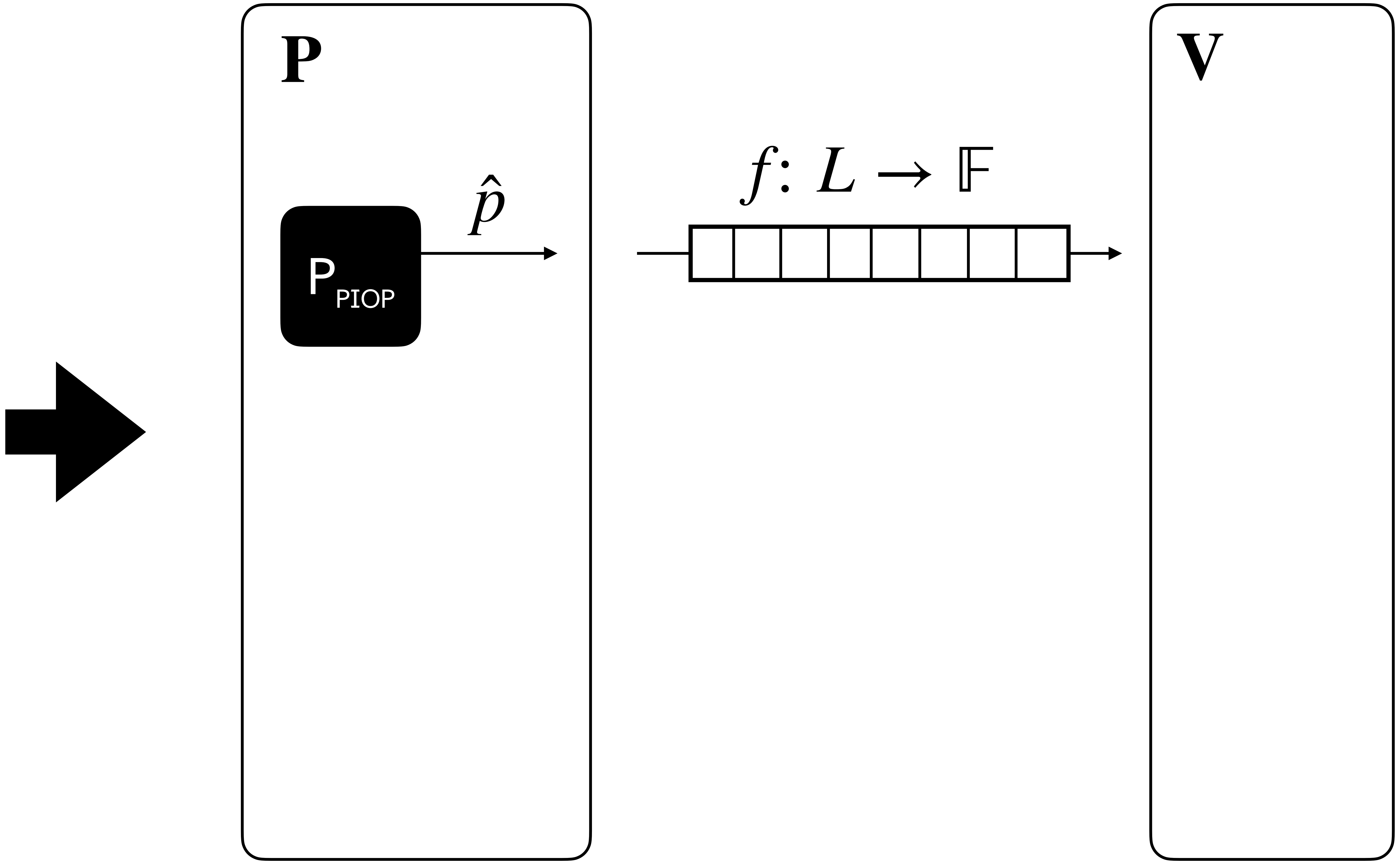
E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.



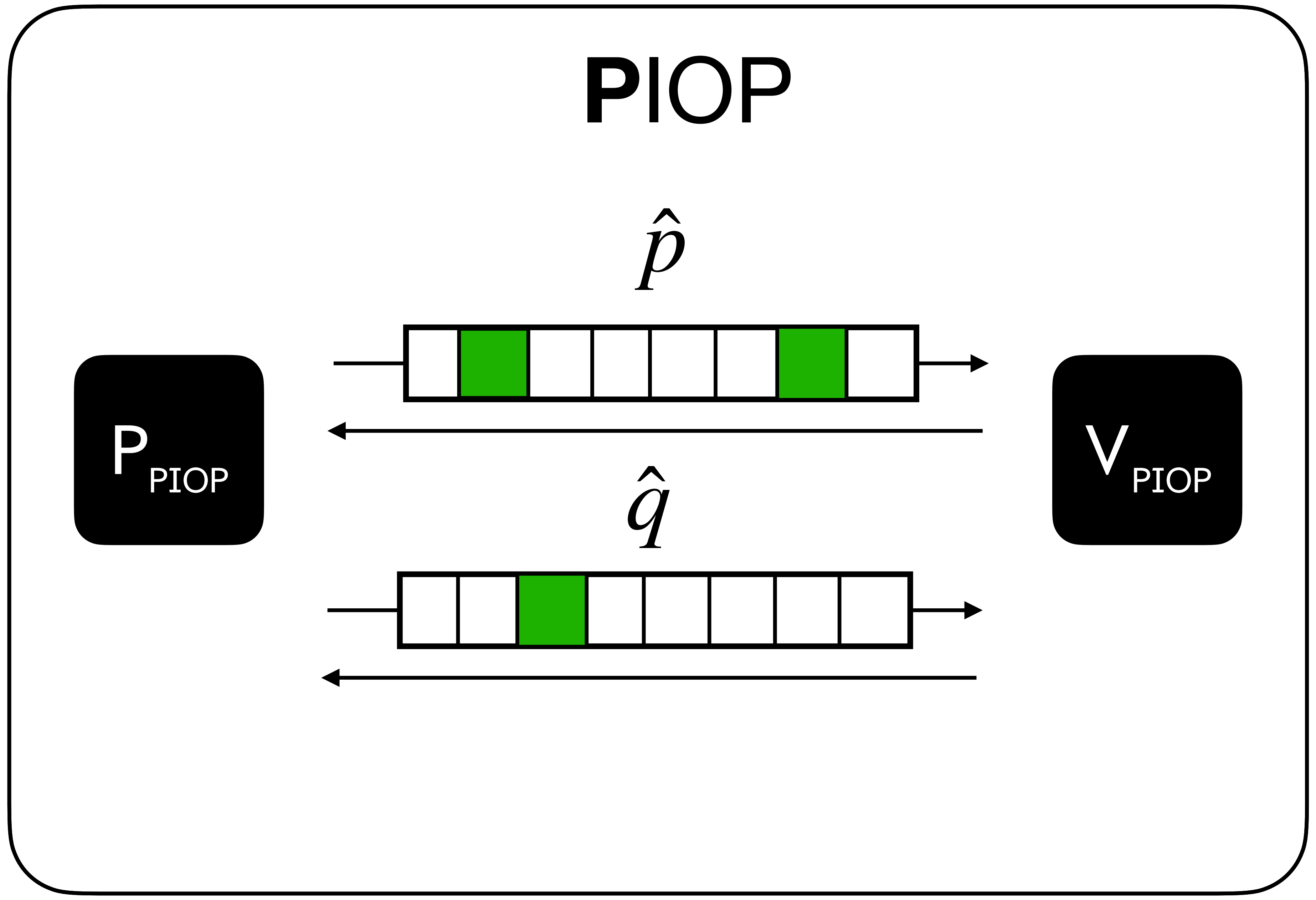


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

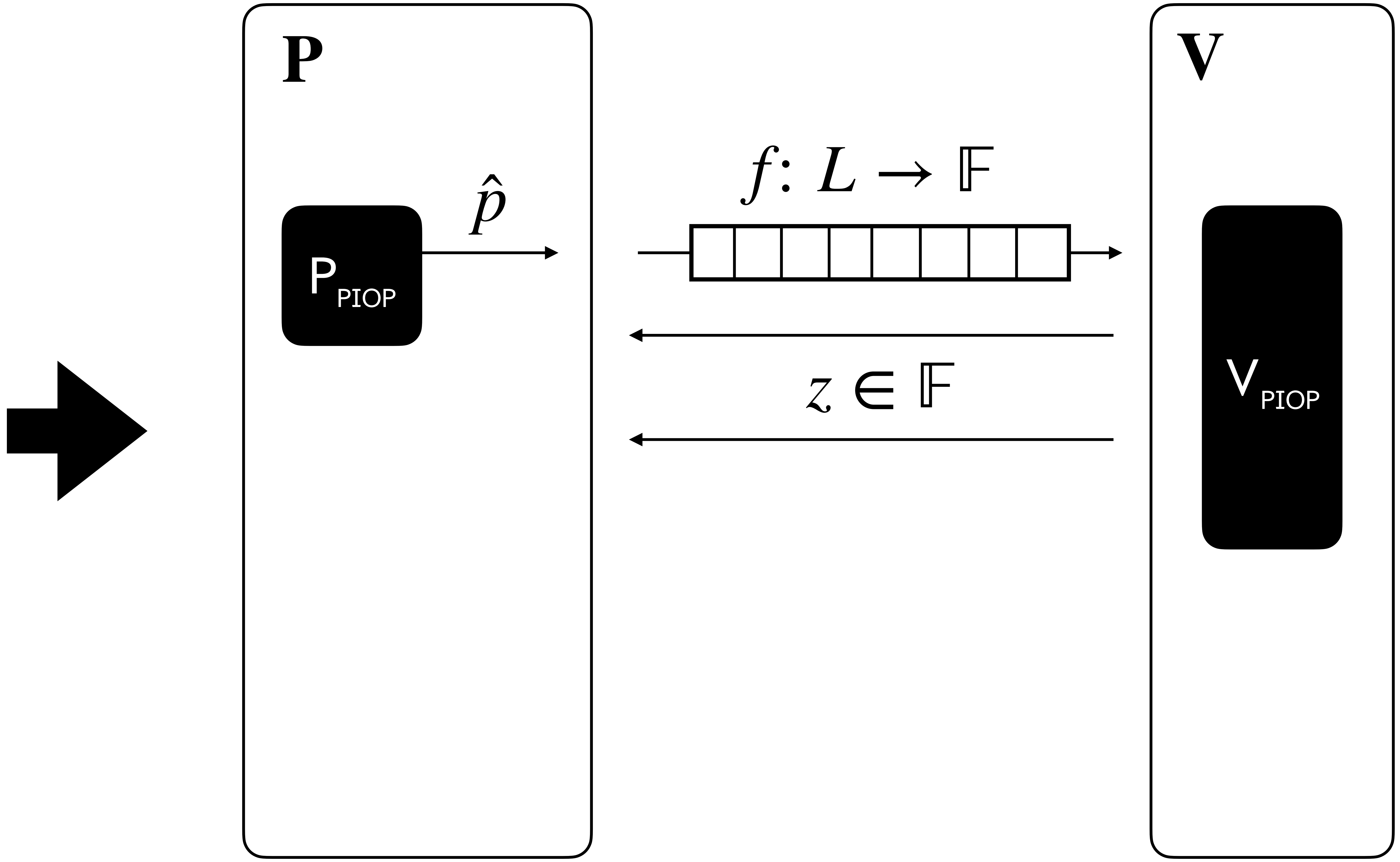
Traditionally



Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

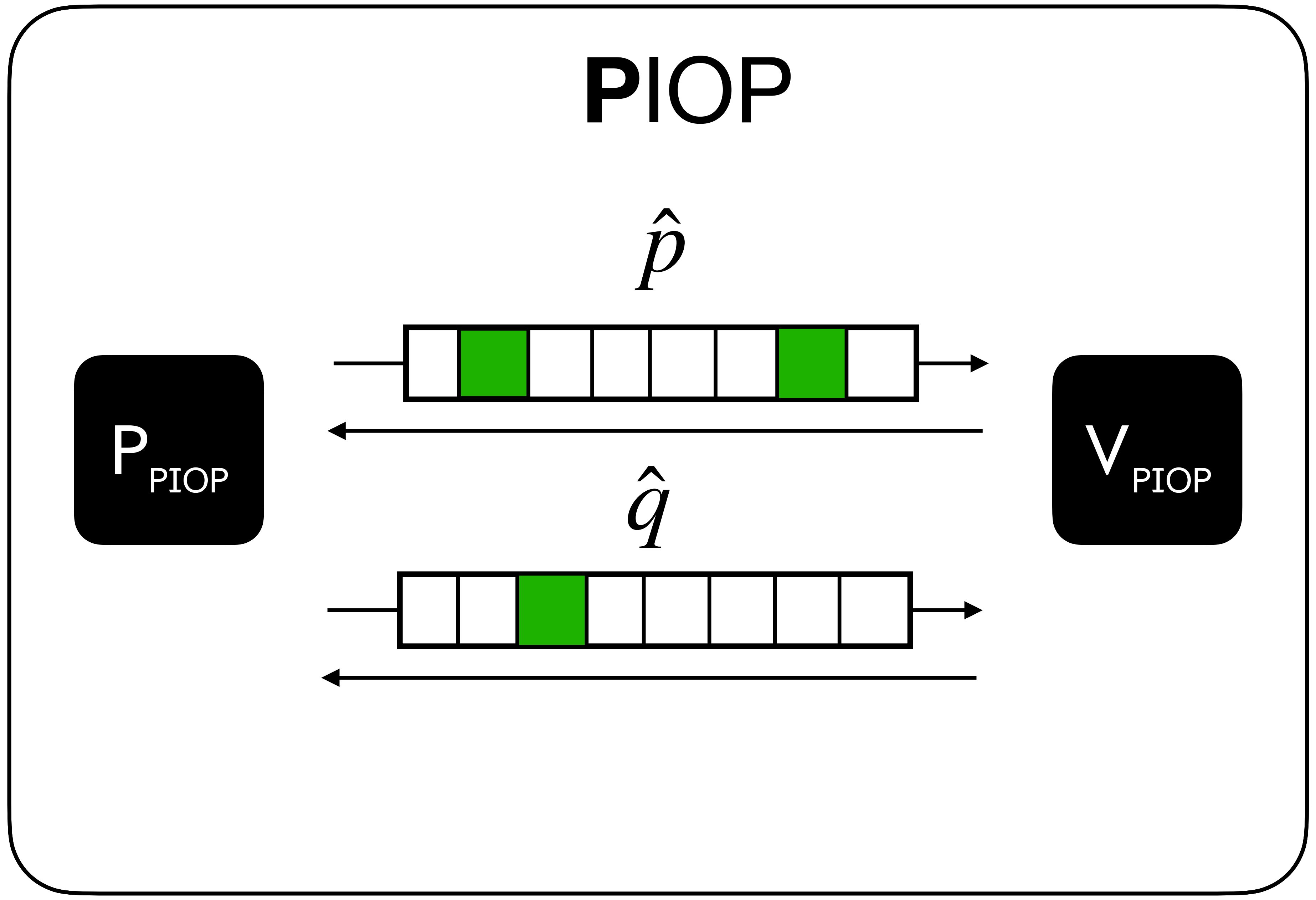
Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.

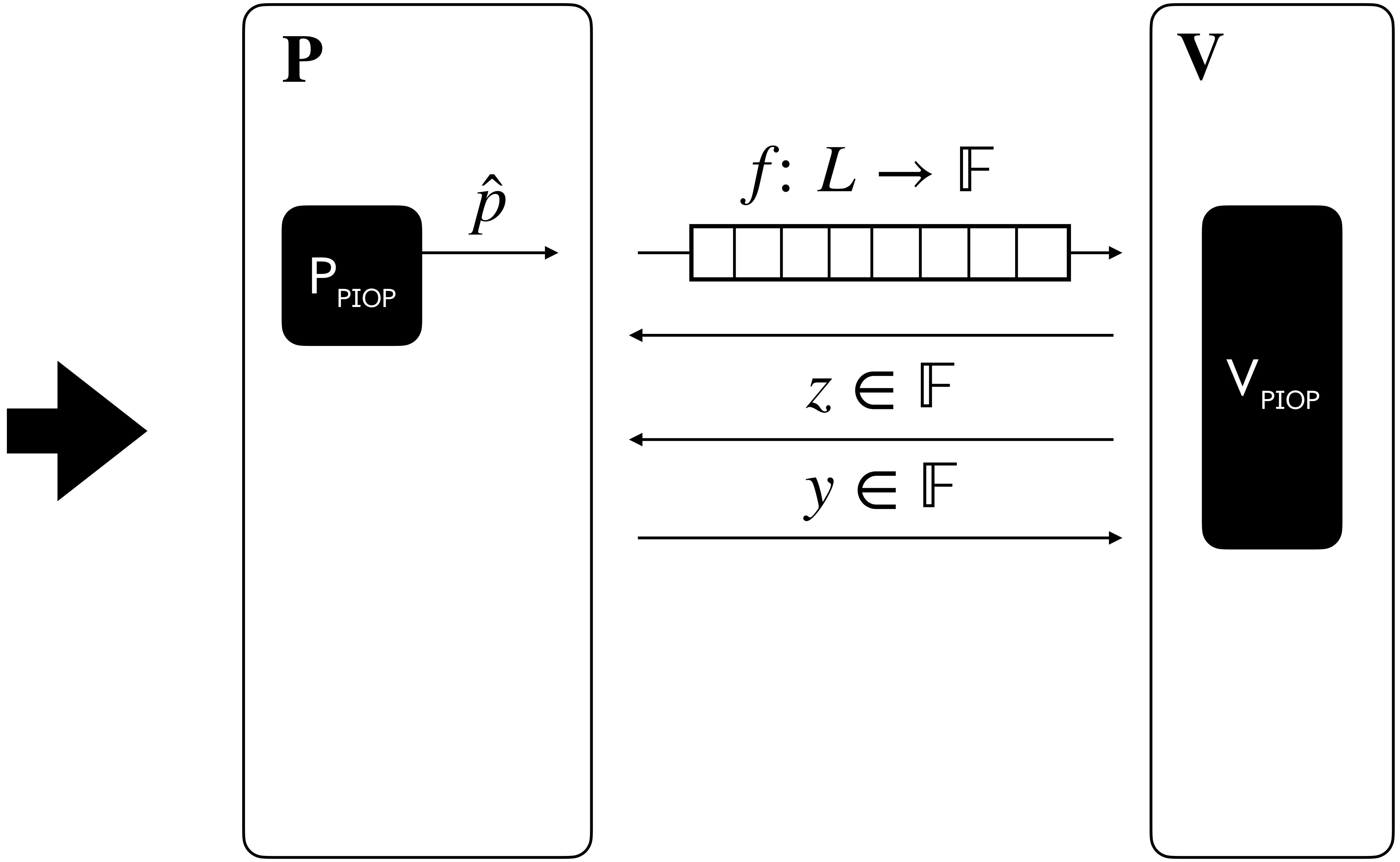


Constructing IOPs

Traditionally

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.





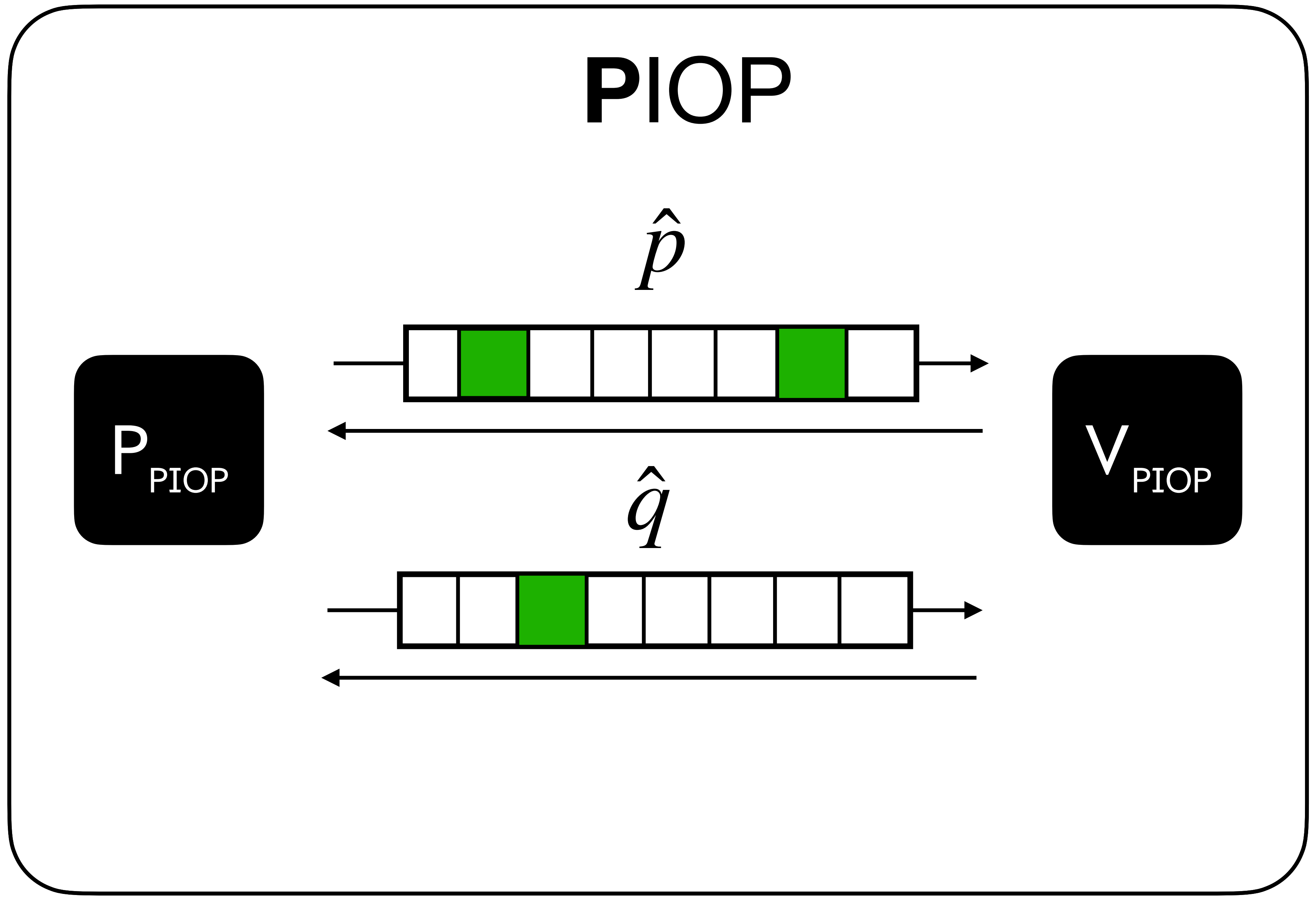
Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

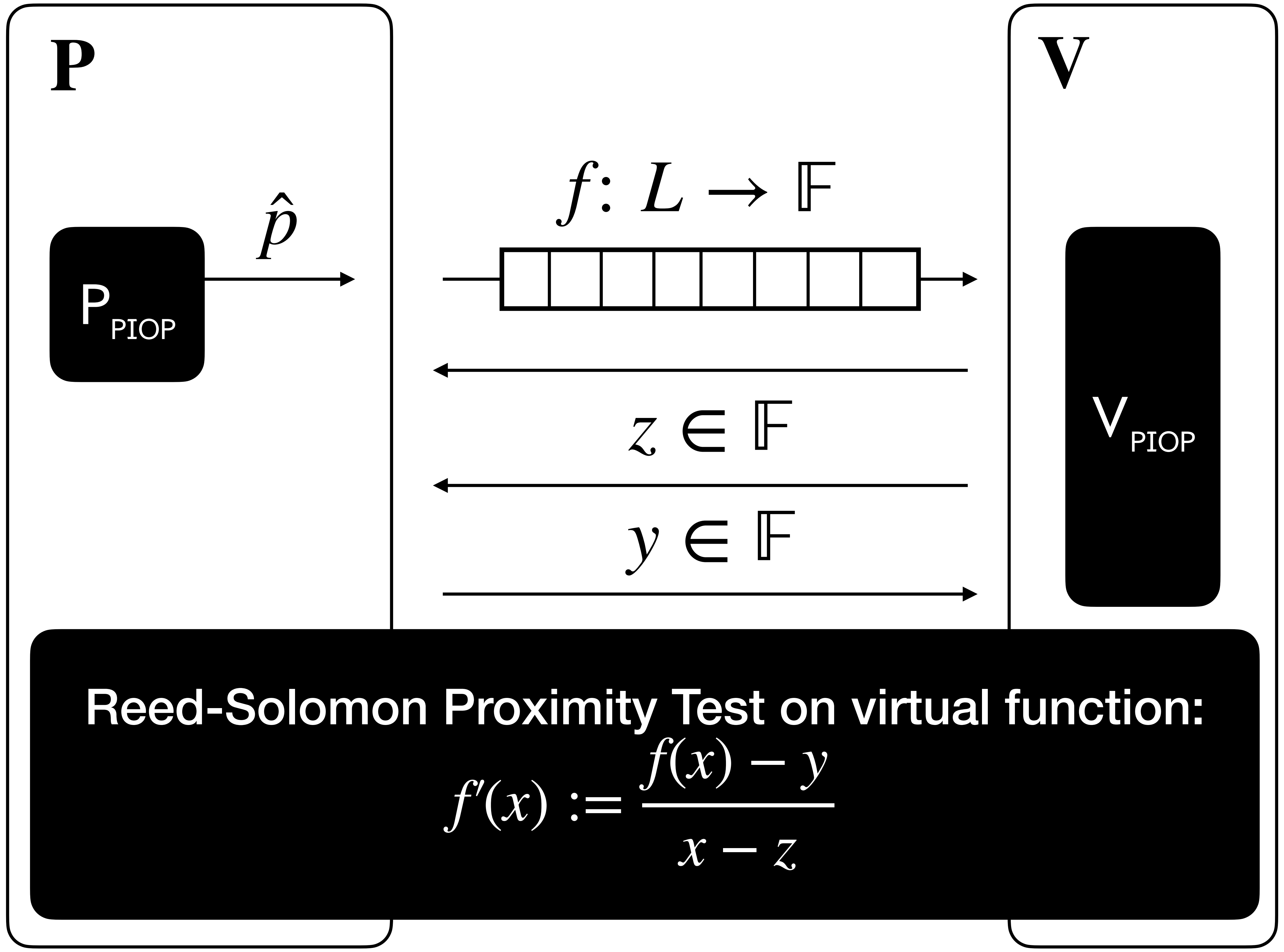
E.g. Aurora, STARK PIOP etc.

Constructing IOPs

Traditionally

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.



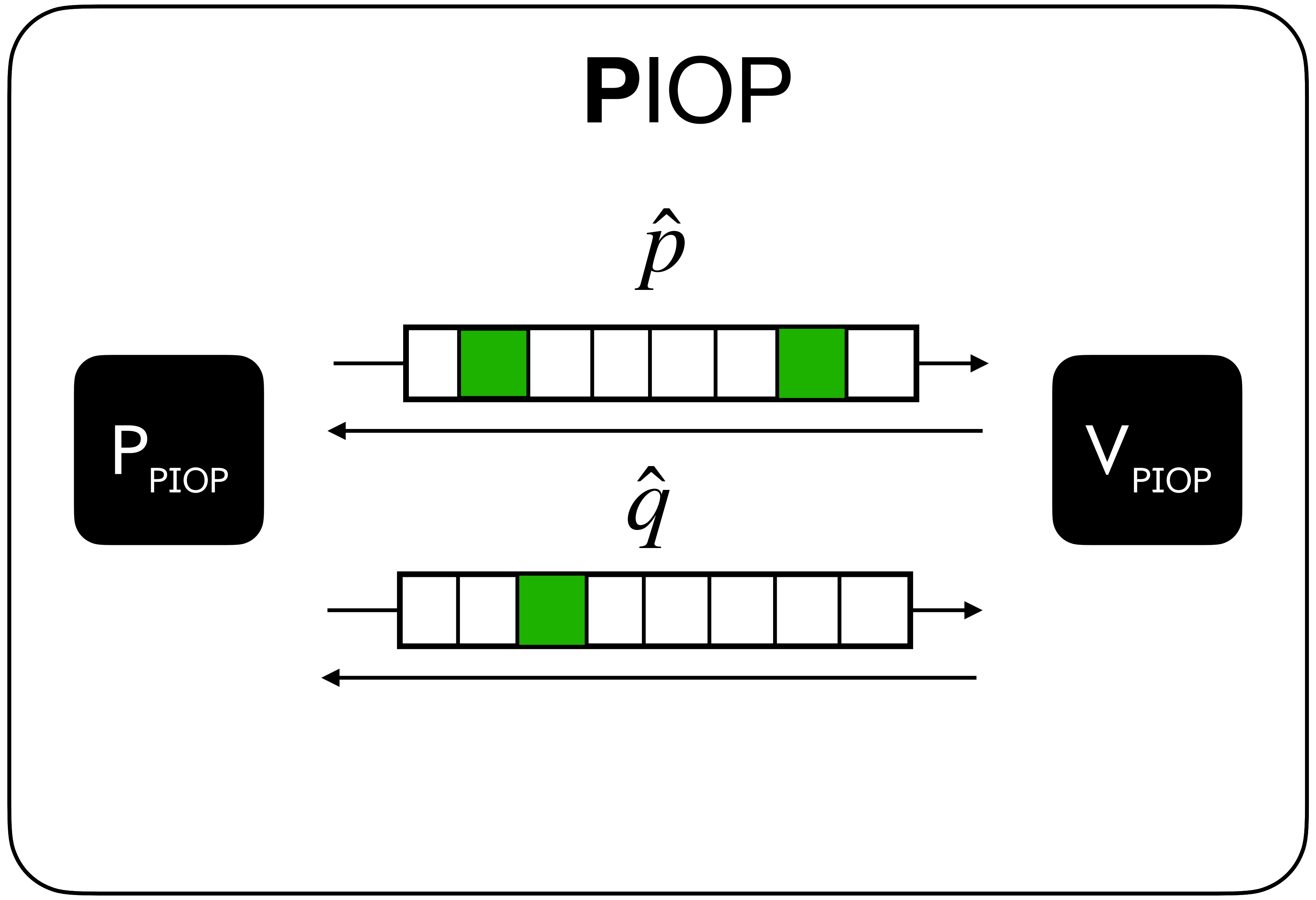


Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Constructing IOPs

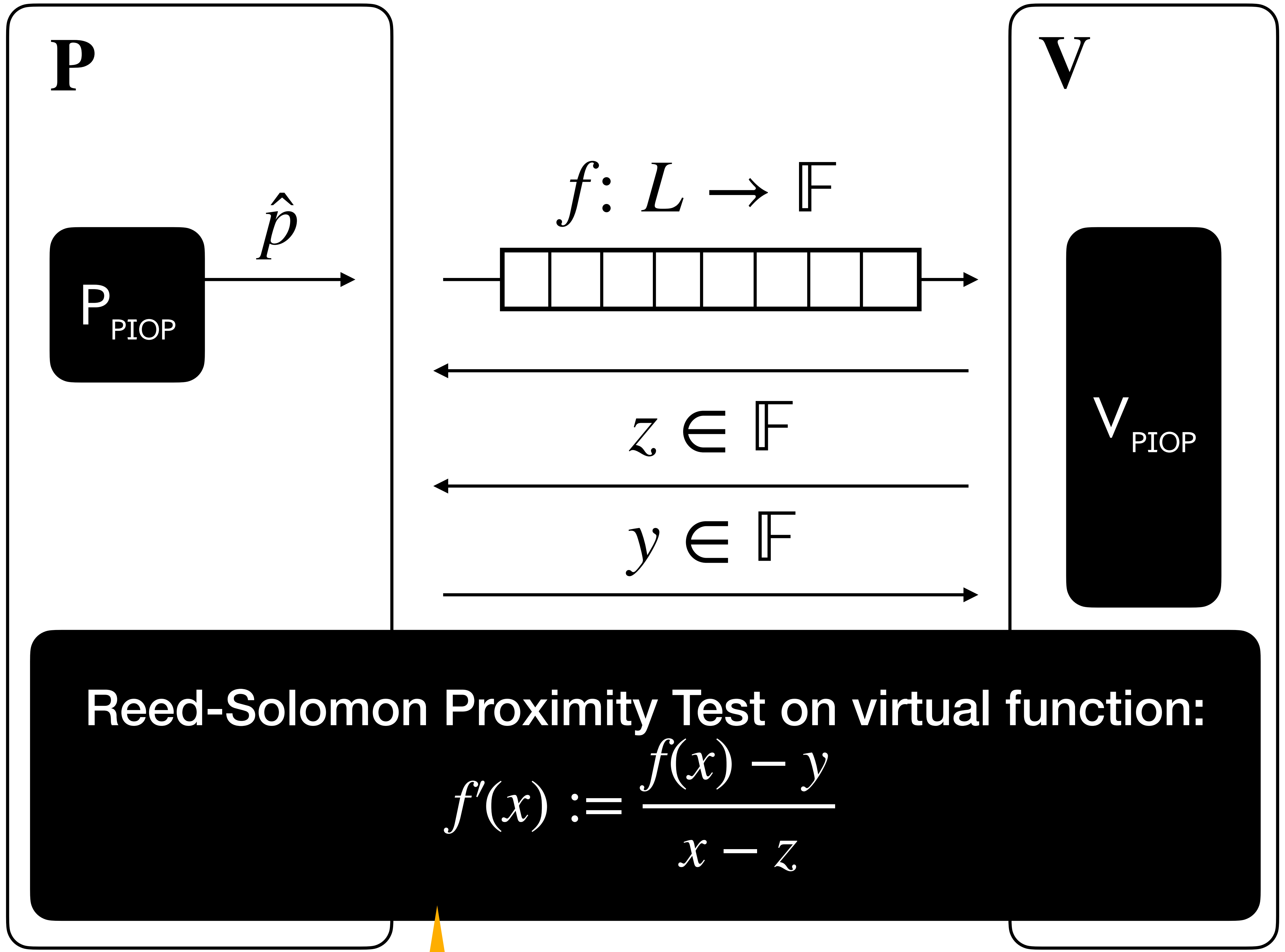
Traditionally



Just like IOPs, but prover is forced to send polynomials $\mathbb{F}^{<d}[X]$.

E.g. Aurora, STARK PIOP etc.

Strategy: use Reed-Solomon codes as "redundant" encoding. Use a proximity test to check claims on encoded oracles.



$> 80\,\%$ **of argument size from proximity test!**

IOP of Proximity to RS codes

IOP of Proximity to RS codes

$$\mathsf{RS}[n, m, \rho] :=$$

IOP of Proximity to RS codes

Convenience

$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

IOP of Proximity to RS codes

Convenience

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Rate of the code

IOP of Proximity to RS codes

Convenience

$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Rate of the code

IOPP for RS

IOP of Proximity to RS codes

Convenience

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Rate of the code

IOPP for RS

P

V

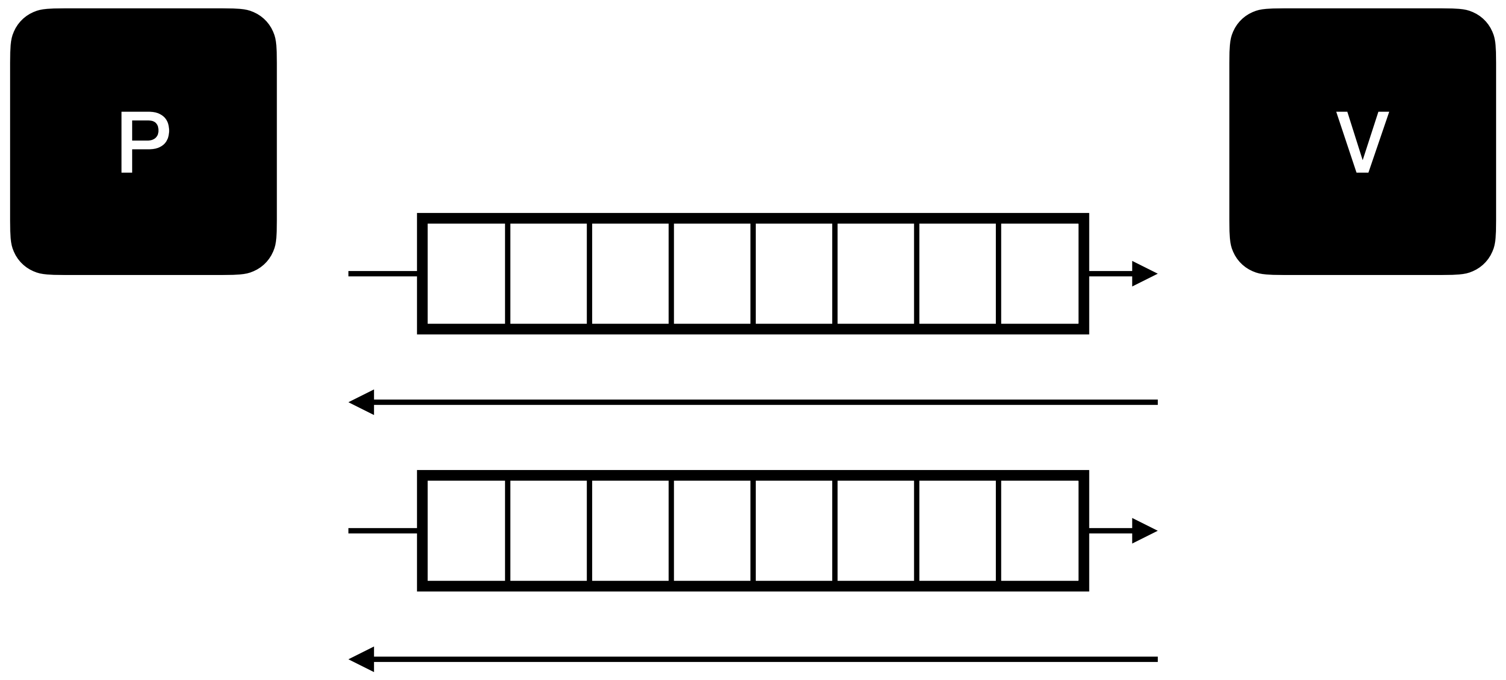
IOP of Proximity to RS codes

Convenience

$$\mathrm{RS}[n, m, \rho] := \left\{ \text{Evaluations of polynomials of degree } < 2^m \text{ on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \frac{2^m}{n} \right\}$$

Rate of the code

IOPP for RS



IOP of Proximity to RS codes

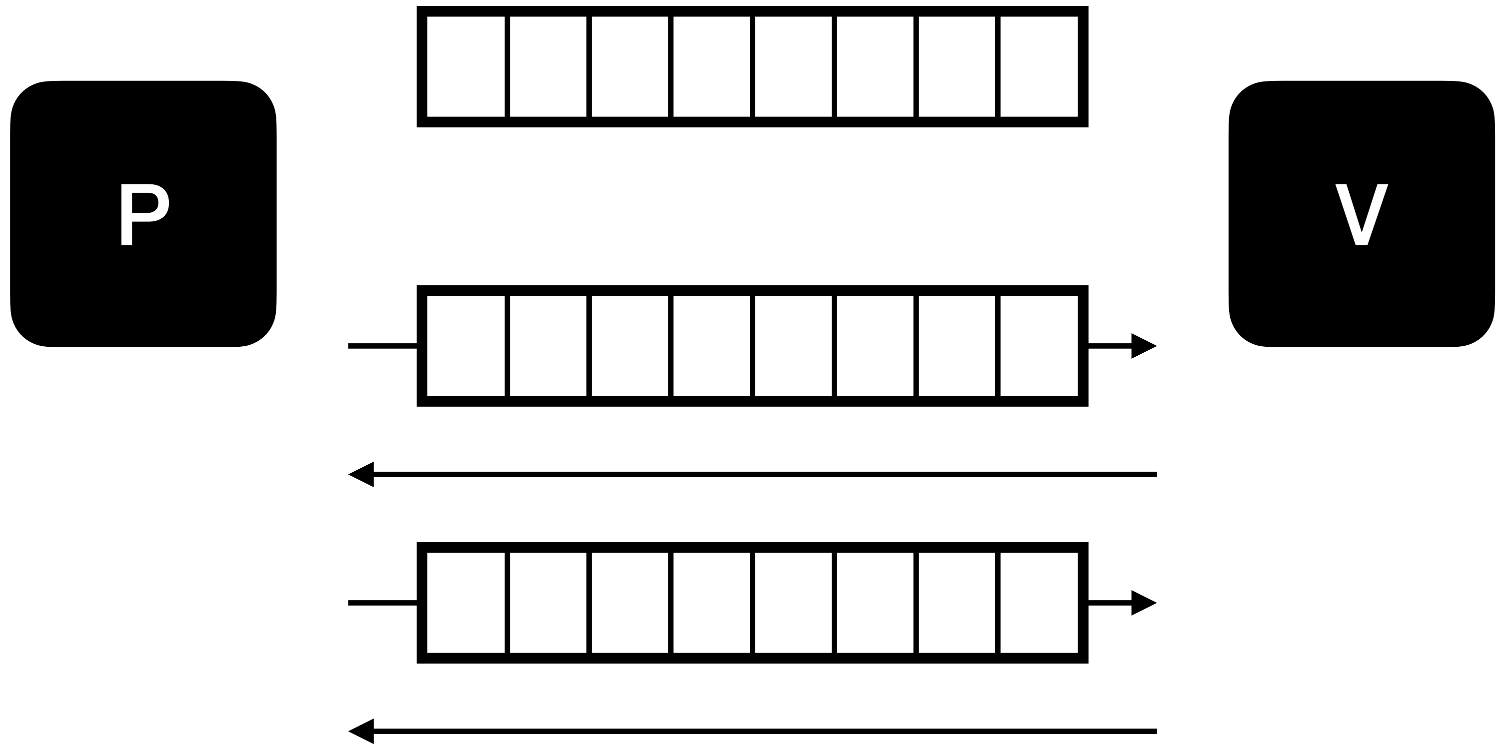
Convenience

$$\mathsf{RS}[n, m, \rho] := \left\{ \text{Evaluations of polynomials of degree } < 2^m \text{ on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \frac{2^m}{n} \right\}$$

Rate of the code

IOPP for RS

$f : L \to \mathbb{F}$



IOP of Proximity to RS codes

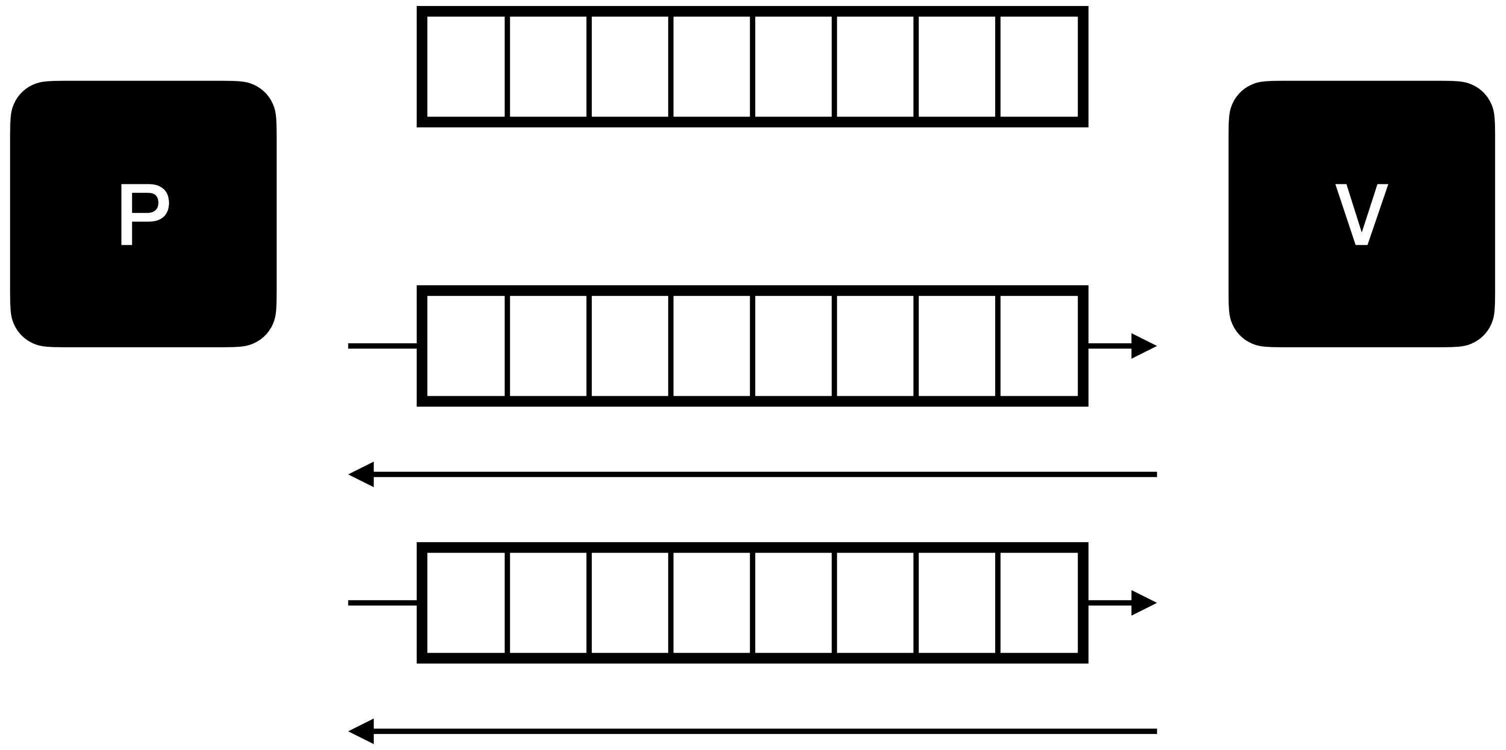
$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Convenience

Rate of the code

IOPP for RS

$f : L \to \mathbb{F}$



- If $f \in \mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts.
- If f is δ-far from $\mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts w.p. $\varepsilon_{\mathsf{RBR}} \leq 2^{-\lambda}$

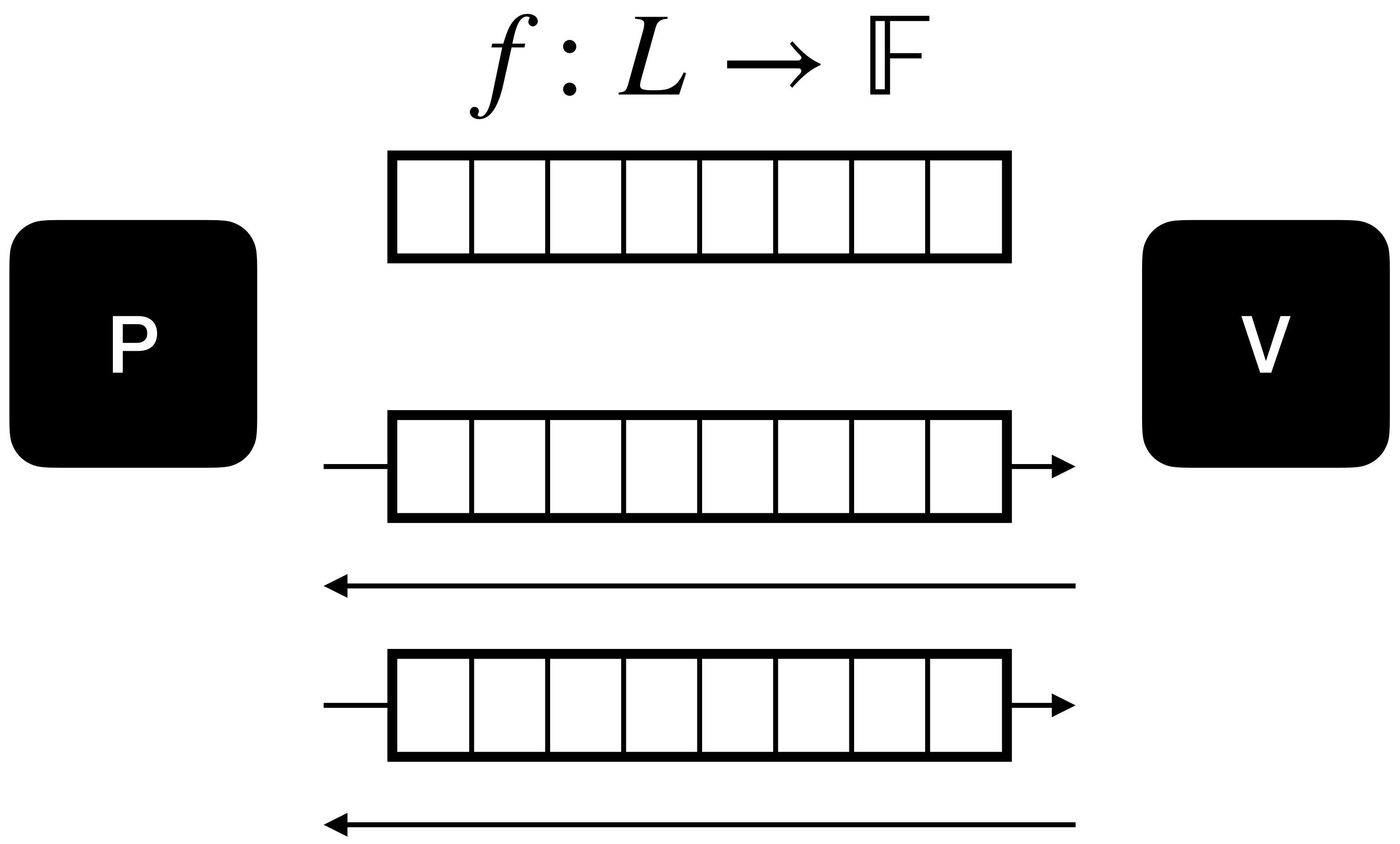
IOP of Proximity to RS codes

$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Convenience

Rate of the code

IOPP for RS



- If $f \in \mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts.
- If f is δ-far from $\mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts w.p. $\varepsilon_{\mathsf{RBR}} \leq 2^{-\lambda}$

Goal: minimize queries to f and other proof oracles.

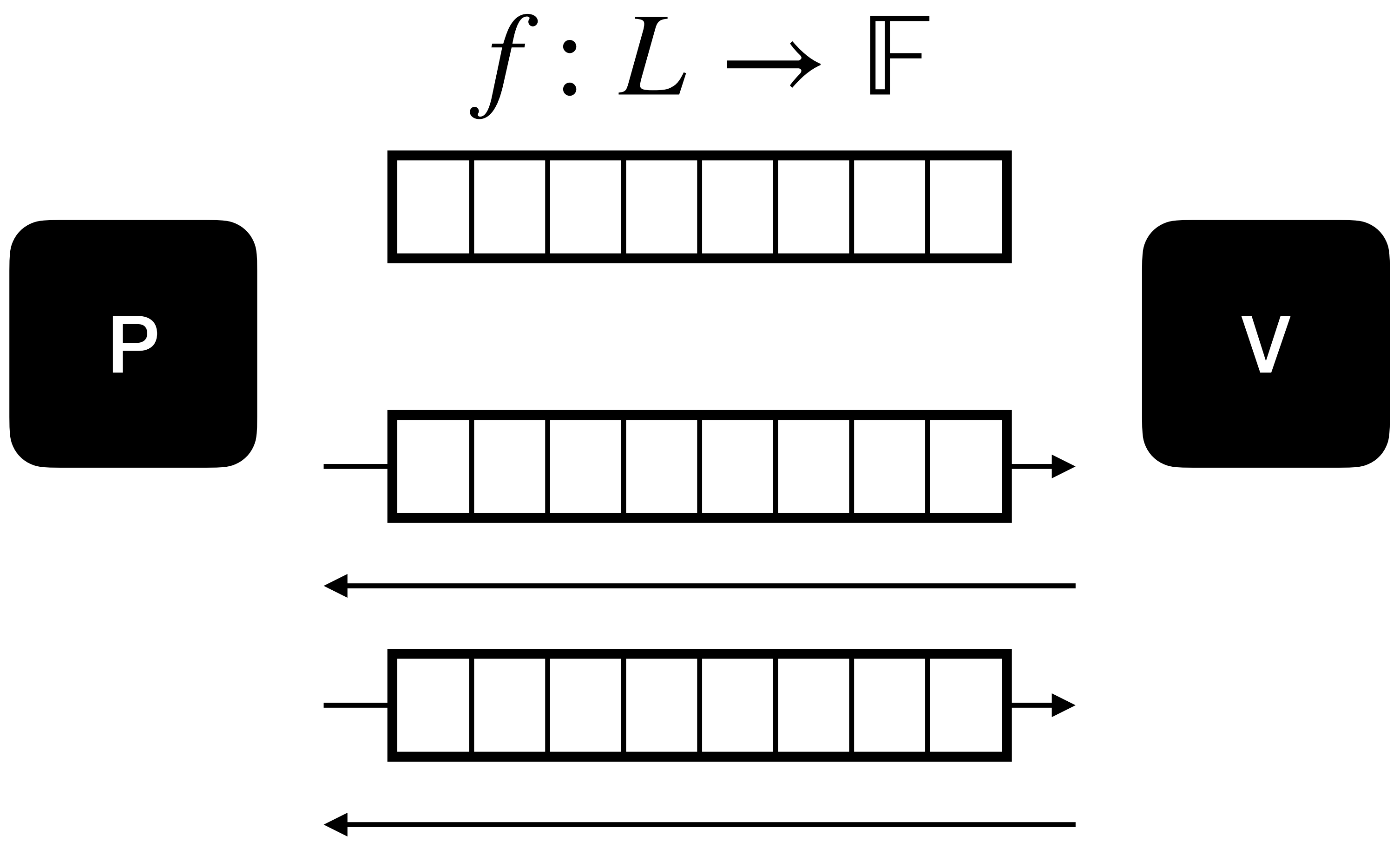
IOP of Proximity to RS codes

$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of polynomials of degree } < 2^m \\ \text{on a domain } L \subseteq \mathbb{F} \text{ of size } n.\ \rho := \dfrac{2^m}{n} \end{array} \right\}$$

Convenience

Rate of the code

IOPP for RS



- If $f \in \mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts.
- If f is δ-far from $\mathsf{RS}[n, m, \rho]$, $\mathbf{V}$ accepts w.p. $\varepsilon_{\mathsf{RBR}} \leq 2^{-\lambda}$

Round by round, required by BCS transform.

Goal: minimize queries to f and other proof oracles.

Constrained RS tests

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Break it down as:

Test for **constrained** encoding

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Break it down as:

Test for **constrained** encoding

Quotient $f'(x) := \dfrac{f(x) - y}{x - z}$

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Break it down as:

Test for **constrained** encoding

Quotient $f'(x) := \frac{f(x) - y}{x - z}$ **+** Reed—Solomon proximity test for f'

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Break it down as:

Test for **constrained** encoding

Quotient $f'(x) := \dfrac{f(x) - y}{x - z}$ **+** Reed—Solomon proximity test for f'

We are designing a proximity test **just** to check this constraint.

Constrained RS tests

What we are running:

Reed-Solomon Proximity Test on virtual function:

$$f'(x) := \frac{f(x) - y}{x - z}$$

What we really want to show:

I have a polynomial $\hat{f}$ and a commitment to (an encoding of it) f such that

$$\hat{f}(z) = y$$

Break it down as:

Test for **constrained** encoding

Quotient $f'(x) := \dfrac{f(x) - y}{x - z}$ + Reed—Solomon proximity test for f'

We are designing a proximity test **just** to check this constraint.

Can we move the constraint directly into the IOPP?

Constrained RS codes

Constrained RS codes

$$\mathsf{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

Constrained RS codes

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

Rewrite RS codes to be about multilinear polynomials:
$\mathrm{coeff}(\hat{p}) = \mathrm{coeff}(\hat{q})$
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

Constrained RS codes

$$\text{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Rewrite RS codes to be about multilinear polynomials:
$\text{coeff}(\hat{p}) = \text{coeff}(\hat{q})$
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

Constrained RS codes

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Rewrite RS codes to be about multilinear polynomials:
$\mathrm{coeff}(\hat{p}) = \mathrm{coeff}(\hat{q})$
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

$$\mathrm{CRS}[n, m, \rho, \hat{w}, \sigma] :=$$

Constrained RS codes

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Rewrite RS codes to be about multilinear polynomials:
$\mathrm{coeff}(\hat{p}) = \mathrm{coeff}(\hat{q})$
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

Constraint

$$\mathrm{CRS}[n, m, \rho, \hat{w}, \sigma] :=$$

Constrained RS codes

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Rewrite RS codes to be about multilinear polynomials:
$\mathrm{coeff}(\hat{p}) = \mathrm{coeff}(\hat{q})$
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

Constraint

Value of constraint

$$\mathrm{CRS}[n, m, \rho, \hat{w}, \sigma] :=$$

Constrained RS codes

Rewrite RS codes to be about multilinear polynomials:
coeff($\hat{p}$) = coeff($\hat{q}$)
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Constraint

Value of constraint

$$\mathrm{CRS}[n, m, \rho, \hat{w}, \sigma] := \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} : \sum_{b \in \{0,1\}^m} \hat{w}(\hat{f}(b), b) = \sigma \right\}$$

Constrained RS codes

Rewrite RS codes to be about multilinear polynomials:
coeff($\hat{p}$) = coeff($\hat{q}$)
implies that
$\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

$$\mathrm{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Constraint

Value of constraint

$$\mathrm{CRS}[n, m, \rho, \hat{w}, \sigma] := \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} : \sum_{b \in \{0,1\}^m} \hat{w}(\hat{f}(b), b) = \sigma \right\}$$

$\mathrm{RS}[n, m, \rho] = \mathrm{CRS}[n, m, \rho, 0, 0]$

Constrained RS codes

Rewrite RS codes to be about multilinear polynomials: $\text{coeff}(\hat{p}) = \text{coeff}(\hat{q})$ implies that $\hat{p}(z) = \hat{q}(z, z^2, \ldots, z^{2^{m-1}})$

$$\text{RS}[n, m, \rho] := \left\{ \begin{array}{l} \text{Evaluations of univariate} \\ \hat{f} \in \mathbb{F}^{<2^m}[X] \text{ on } L \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} \right\}$$

Constraint

Value of constraint

$$\text{CRS}[n, m, \rho, \hat{w}, \sigma] := \left\{ \begin{array}{l} \text{Evaluations of multilinear} \\ \hat{f} \in \mathbb{F}^{\leq 1}[X_1, \ldots, X_m] \text{ on } L \end{array} : \sum_{b \in \{0,1\}^m} \hat{w}(\hat{f}(b), b) = \sigma \right\}$$

$\text{RS}[n, m, \rho] = \text{CRS}[n, m, \rho, 0, 0]$

If $\hat{w} = Z \cdot \text{eq}(\mathbf{X}, \mathbf{r})$ we recover multilinear polynomial evaluation

Our results

WHIR 🌪

$k > 1$ is a folding parameter

A constrained Reed-Solomon proximity test

WHIR 🌪️

A constrained Reed-Solomon proximity test

$k > 1$ is a folding parameter

Rounds: $O\left(m\right)$

Alphabet: $\mathbb{F}^{2^k}$

Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\mathsf{WHIR}} \cdot (2^k + m)\right)$

WHIR 🌪️

A <u>constrained</u> Reed-Solomon proximity test

$k > 1$ is a folding parameter

Rounds: $O(m)$

Alphabet: $\mathbb{F}^{2^k}$

Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\text{WHIR}} \cdot (2^k + m)\right)$

Query complexity:

$$q_{\text{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right) = O(\lambda)$$

WHIR 🌪️

A constrained Reed-Solomon proximity test

$k > 1$ is a folding parameter

Rounds: $O(m)$

Alphabet: $\mathbb{F}^{2^k}$

Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\text{WHIR}} \cdot (2^k + m)\right)$

Query complexity:

$$q_{\text{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right) = O(\lambda)$$

$\lambda \gg m$

WHIR 🌪️

A constrained Reed-Solomon proximity test

$k > 1$ is a folding parameter

Rounds: $O(m)$

Alphabet: $\mathbb{F}^{2^k}$

Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\text{WHIR}} \cdot (2^k + m)\right)$

Query complexity:

$$q_{\text{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right) = O(\lambda)$$

$k \approx \log m$

$\lambda \gg m$

Comparison with prior work

	Queries	Verifier Time	Alphabet
BaseFold	$q_{\mathsf{BF}} = O(\lambda \cdot m)$	$O(q_{\mathsf{BF}})$	$\mathbb{F}^2$
FRI	$q_{\mathsf{FRI}} = O\left(\frac{\lambda}{k} \cdot m\right)$	$O(q_{\mathsf{FRI}} \cdot 2^k)$	$\mathbb{F}^{2^k}$
STIR	$q_{\mathsf{STIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right)$	$O(q_{\mathsf{STIR}} \cdot 2^k + \lambda^2 \cdot 2^k)$	$\mathbb{F}^{2^k}$
WHIR	$q_{\mathsf{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right)$	$O(q_{\mathsf{WHIR}} \cdot (2^k + m))$	$\mathbb{F}^{2^k}$

Comparison to STIR and FRI

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and similar prover time.

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and similar prover time.
- **Additionally, richer proximity tests means that:**

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k}\cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k}\cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and similar prover time.
- **Additionally, richer proximity tests means that:**
  - Can be used as a **multilinear** PCS (instead of BaseFold, FRI-Binius, etc)

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathrm{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and similar prover time.
- **Additionally, richer proximity tests means that:**
  - Can be used as a **multilinear** PCS (instead of BaseFold, FRI-Binius, etc)
  - Can be used in compiler for Σ-IOP (extra slides)

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & **WHIR** $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and similar prover time.
- **Additionally, richer proximity tests means that:**
    - Can be used as a **multilinear** PCS (instead of BaseFold, FRI-Binius, etc)
    - Can be used in compiler for Σ-IOP (extra slides)
- **Further,** super-fast verification (next)

Implementation

```
========================================
Whir (PCS) 🌪️
Field: Goldilocks2 and MT: Blake3
Number of variables: 20, folding factor: 4
Security level: 100 bits using ConjectureList security and 19 bits of PoW
initial_folding_pow_bits: 0
Num_queries: 41, rate: 2^-2, pow_bits: 18, ood_samples: 2, folding_pow: 0
Num_queries: 17, rate: 2^-5, pow_bits: 15, ood_samples: 2, folding_pow: 2
Num_queries: 11, rate: 2^-8, pow_bits: 12, ood_samples: 2, folding_pow: 4
Num_queries: 8, rate: 2^-11, pow_bits: 12, ood_samples: 2, folding_pow: 6
final_queries: 6, final_rate: 2^-14, final_pow_bits: 16, final_folding_pow_bits: 0
------------------------------------
Round by round soundness analysis:
------------------------------------
167.0 bits -- OOD commitment
102.0 bits -- (x4) prox gaps: 103.0, sumcheck: 102.0, pow: 0.0
171.0 bits -- OOD sample
100.0 bits -- query error: 82.0, combination: 94.6, pow: 18.0
100.0 bits -- (x4) prox gaps: 101.0, sumcheck: 100.0, pow: 0.0
175.0 bits -- OOD sample
100.0 bits -- query error: 85.0, combination: 93.8, pow: 15.0
100.0 bits -- (x4) prox gaps: 99.0, sumcheck: 98.0, pow: 2.0
179.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 92.3, pow: 12.0
100.0 bits -- (x4) prox gaps: 97.0, sumcheck: 96.0, pow: 4.0
183.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 90.7, pow: 12.0
100.0 bits -- (x4) prox gaps: 95.0, sumcheck: 94.0, pow: 6.0
100.0 bits -- query error: 84.0, pow: 16.0

Prover time: 356.9ms
Proof size: 58.7 KiB
Verifier time: 342.8µs
Average hashes: 1.1k
```

Implementation

- Rust 🦀 implementation, available at WizardOfMenlo/whir

```
========================================
Whir (PCS) 🌪
Field: Goldilocks2 and MT: Blake3
Number of variables: 20, folding factor: 4
Security level: 100 bits using ConjectureList security and 19 bits of PoW
initial_folding_pow_bits: 0
Num_queries: 41, rate: 2^-2, pow_bits: 18, ood_samples: 2, folding_pow: 0
Num_queries: 17, rate: 2^-5, pow_bits: 15, ood_samples: 2, folding_pow: 2
Num_queries: 11, rate: 2^-8, pow_bits: 12, ood_samples: 2, folding_pow: 4
Num_queries: 8, rate: 2^-11, pow_bits: 12, ood_samples: 2, folding_pow: 6
final_queries: 6, final_rate: 2^-14, final_pow_bits: 16, final_folding_pow_bits: 0
------------------------------------
Round by round soundness analysis:
------------------------------------
167.0 bits -- OOD commitment
102.0 bits -- (x4) prox gaps: 103.0, sumcheck: 102.0, pow: 0.0
171.0 bits -- OOD sample
100.0 bits -- query error: 82.0, combination: 94.6, pow: 18.0
100.0 bits -- (x4) prox gaps: 101.0, sumcheck: 100.0, pow: 0.0
175.0 bits -- OOD sample
100.0 bits -- query error: 85.0, combination: 93.8, pow: 15.0
100.0 bits -- (x4) prox gaps: 99.0, sumcheck: 98.0, pow: 2.0
179.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 92.3, pow: 12.0
100.0 bits -- (x4) prox gaps: 97.0, sumcheck: 96.0, pow: 4.0
183.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 90.7, pow: 12.0
100.0 bits -- (x4) prox gaps: 95.0, sumcheck: 94.0, pow: 6.0
100.0 bits -- query error: 84.0, pow: 16.0

Prover time: 356.9ms
Proof size: 58.7 KiB
Verifier time: 342.8μs
Average hashes: 1.1k
```

Implementation

- Rust 🦀 implementation, available at WizardOfMenlo/whir
- Arkworks as backend, (extension of) Goldilocks for benchmarks

```
========================================
Whir (PCS) 🌪️
Field: Goldilocks2 and MT: Blake3
Number of variables: 20, folding factor: 4
Security level: 100 bits using ConjectureList security and 19 bits of PoW
initial_folding_pow_bits: 0
Num_queries: 41, rate: 2^-2, pow_bits: 18, ood_samples: 2, folding_pow: 0
Num_queries: 17, rate: 2^-5, pow_bits: 15, ood_samples: 2, folding_pow: 2
Num_queries: 11, rate: 2^-8, pow_bits: 12, ood_samples: 2, folding_pow: 4
Num_queries: 8, rate: 2^-11, pow_bits: 12, ood_samples: 2, folding_pow: 6
final_queries: 6, final_rate: 2^-14, final_pow_bits: 16, final_folding_pow_bits: 0
------------------------------------
Round by round soundness analysis:
------------------------------------
167.0 bits -- OOD commitment
102.0 bits -- (x4) prox gaps: 103.0, sumcheck: 102.0, pow: 0.0
171.0 bits -- OOD sample
100.0 bits -- query error: 82.0, combination: 94.6, pow: 18.0
100.0 bits -- (x4) prox gaps: 101.0, sumcheck: 100.0, pow: 0.0
175.0 bits -- OOD sample
100.0 bits -- query error: 85.0, combination: 93.8, pow: 15.0
100.0 bits -- (x4) prox gaps: 99.0, sumcheck: 98.0, pow: 2.0
179.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 92.3, pow: 12.0
100.0 bits -- (x4) prox gaps: 97.0, sumcheck: 96.0, pow: 4.0
183.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 90.7, pow: 12.0
100.0 bits -- (x4) prox gaps: 95.0, sumcheck: 94.0, pow: 6.0
100.0 bits -- query error: 84.0, pow: 16.0

Prover time: 356.9ms
Proof size: 58.7 KiB
Verifier time: 342.8µs
Average hashes: 1.1k
```

Implementation

- Rust 🦀 implementation, available at WizardOfMenlo/whir
- Arkworks as backend, (extension of) Goldilocks for benchmarks
  - Huge thanks to Remco Bloemen!!!

```
========================================
Whir (PCS) 🌪️
Field: Goldilocks2 and MT: Blake3
Number of variables: 20, folding factor: 4
Security level: 100 bits using ConjectureList security and 19 bits of PoW
initial_folding_pow_bits: 0
Num_queries: 41, rate: 2^-2, pow_bits: 18, ood_samples: 2, folding_pow: 0
Num_queries: 17, rate: 2^-5, pow_bits: 15, ood_samples: 2, folding_pow: 2
Num_queries: 11, rate: 2^-8, pow_bits: 12, ood_samples: 2, folding_pow: 4
Num_queries: 8, rate: 2^-11, pow_bits: 12, ood_samples: 2, folding_pow: 6
final_queries: 6, final_rate: 2^-14, final_pow_bits: 16, final_folding_pow_bits: 0
------------------------------------
Round by round soundness analysis:
------------------------------------
167.0 bits -- OOD commitment
102.0 bits -- (x4) prox gaps: 103.0, sumcheck: 102.0, pow: 0.0
171.0 bits -- OOD sample
100.0 bits -- query error: 82.0, combination: 94.6, pow: 18.0
100.0 bits -- (x4) prox gaps: 101.0, sumcheck: 100.0, pow: 0.0
175.0 bits -- OOD sample
100.0 bits -- query error: 85.0, combination: 93.8, pow: 15.0
100.0 bits -- (x4) prox gaps: 99.0, sumcheck: 98.0, pow: 2.0
179.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 92.3, pow: 12.0
100.0 bits -- (x4) prox gaps: 97.0, sumcheck: 96.0, pow: 4.0
183.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 90.7, pow: 12.0
100.0 bits -- (x4) prox gaps: 95.0, sumcheck: 94.0, pow: 6.0
100.0 bits -- query error: 84.0, pow: 16.0

Prover time: 356.9ms
Proof size: 58.7 KiB
Verifier time: 342.8μs
Average hashes: 1.1k
```

Implementation

- Rust 🦀 implementation, available at WizardOfMenlo/whir
- Arkworks as backend, (extension of) Goldilocks for benchmarks
  - Huge thanks to Remco Bloemen!!!
- We compared to FRI, STIR and BaseFold.

```
========================================
Whir (PCS) 🌪
Field: Goldilocks2 and MT: Blake3
Number of variables: 20, folding factor: 4
Security level: 100 bits using ConjectureList security and 19 bits of PoW
initial_folding_pow_bits: 0
Num_queries: 41, rate: 2^-2, pow_bits: 18, ood_samples: 2, folding_pow: 0
Num_queries: 17, rate: 2^-5, pow_bits: 15, ood_samples: 2, folding_pow: 2
Num_queries: 11, rate: 2^-8, pow_bits: 12, ood_samples: 2, folding_pow: 4
Num_queries: 8, rate: 2^-11, pow_bits: 12, ood_samples: 2, folding_pow: 6
final_queries: 6, final_rate: 2^-14, final_pow_bits: 16, final_folding_pow_bits: 0
------------------------------------
Round by round soundness analysis:
------------------------------------
167.0 bits -- OOD commitment
102.0 bits -- (x4) prox gaps: 103.0, sumcheck: 102.0, pow: 0.0
171.0 bits -- OOD sample
100.0 bits -- query error: 82.0, combination: 94.6, pow: 18.0
100.0 bits -- (x4) prox gaps: 101.0, sumcheck: 100.0, pow: 0.0
175.0 bits -- OOD sample
100.0 bits -- query error: 85.0, combination: 93.8, pow: 15.0
100.0 bits -- (x4) prox gaps: 99.0, sumcheck: 98.0, pow: 2.0
179.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 92.3, pow: 12.0
100.0 bits -- (x4) prox gaps: 97.0, sumcheck: 96.0, pow: 4.0
183.0 bits -- OOD sample
100.0 bits -- query error: 88.0, combination: 90.7, pow: 12.0
100.0 bits -- (x4) prox gaps: 95.0, sumcheck: 94.0, pow: 6.0
100.0 bits -- query error: 84.0, pow: 16.0

Prover time: 356.9ms
Proof size: 58.7 KiB
Verifier time: 342.8µs
Average hashes: 1.1k
```

Super fast verifier

Super fast verifier

- The WHIR verifier typically runs in a few hundred **micro**seconds.

Super fast verifier

- The WHIR verifier typically runs in a few hundred **micro**seconds.
- Other verifiers require several **milli**seconds (and more).

Super fast verifier

- The WHIR verifier typically runs in a few hundred **micro**seconds.
- Other verifiers require several **milli**seconds (and more).
- Without compromising prover time & argument size

Super fast verifier

- The WHIR verifier typically runs in a few hundred **micro**seconds.
- Other verifiers require several **milli**seconds (and more).
- Without compromising prover time & argument size
- As a PCS for degree 2^{22}, 100 bits of security:

Super fast verifier

- The WHIR verifier typically runs in a few hundred **micro**seconds.
- Other verifiers require several **milli**seconds (and more).
- Without compromising prover time & argument size
- As a PCS for degree 2^{22}, 100 bits of security:

```
Prover time: ~1s (MacBook Air)
Commit & open: 63 KiB
Verifier time: 270 µs (0.27 ms)
```

Super fast verifier

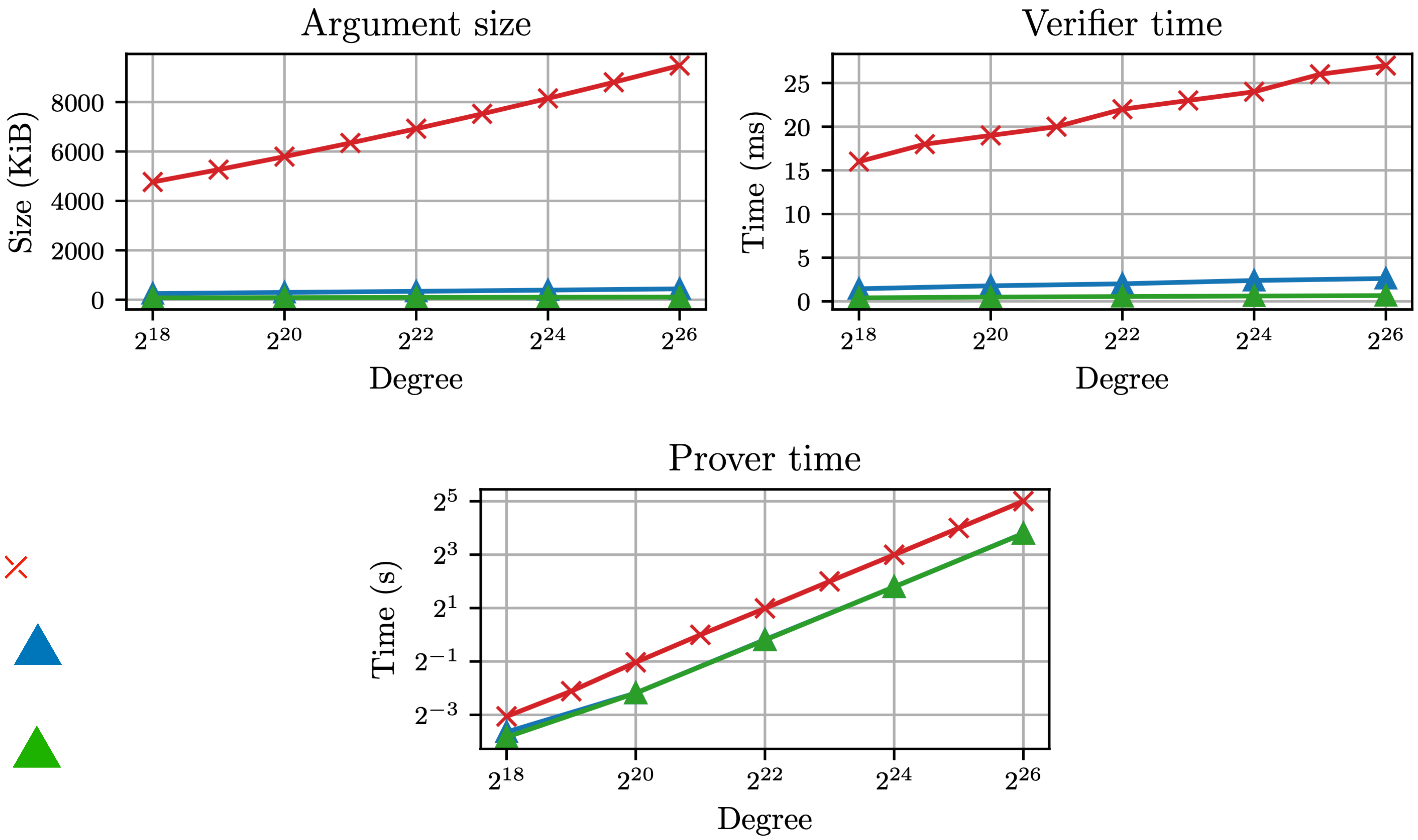
- The WHIR verifier typically runs in a few hundred **micro**seconds.
- Other verifiers require several **milli**seconds (and more).
- Without compromising prover time & argument size
- As a PCS for degree 2^{24}:

Schemes with trusted setup using pairings!

Verifier time (ms)	Brakedown	Ligero	Greyhound	Hyrax	PST	KZG	WHIR-1/2	WHIR-1/16
$\lambda = 100$	3500	733	-	100	7.81	2.42	0.61	0.29
$\lambda = 128$	3680	750	130	151	9.92	3.66	1.4	0.6

Table 4: Comparison of WHIR-CB's verifier time versus other polynomial commitment schemes, on 24 variables. For the KZG degree 2^{24} is used instead.

Comparison with BaseFold

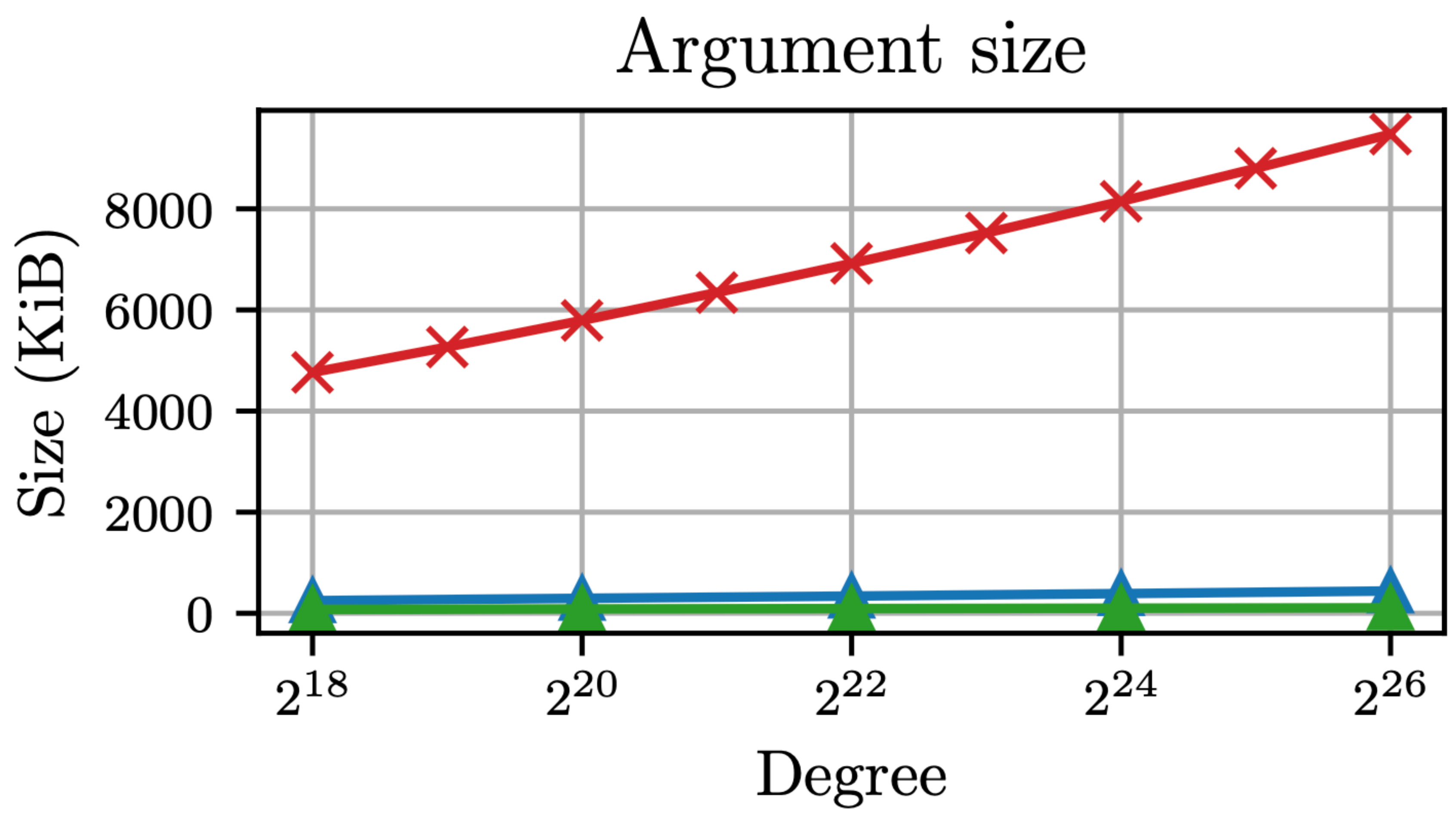


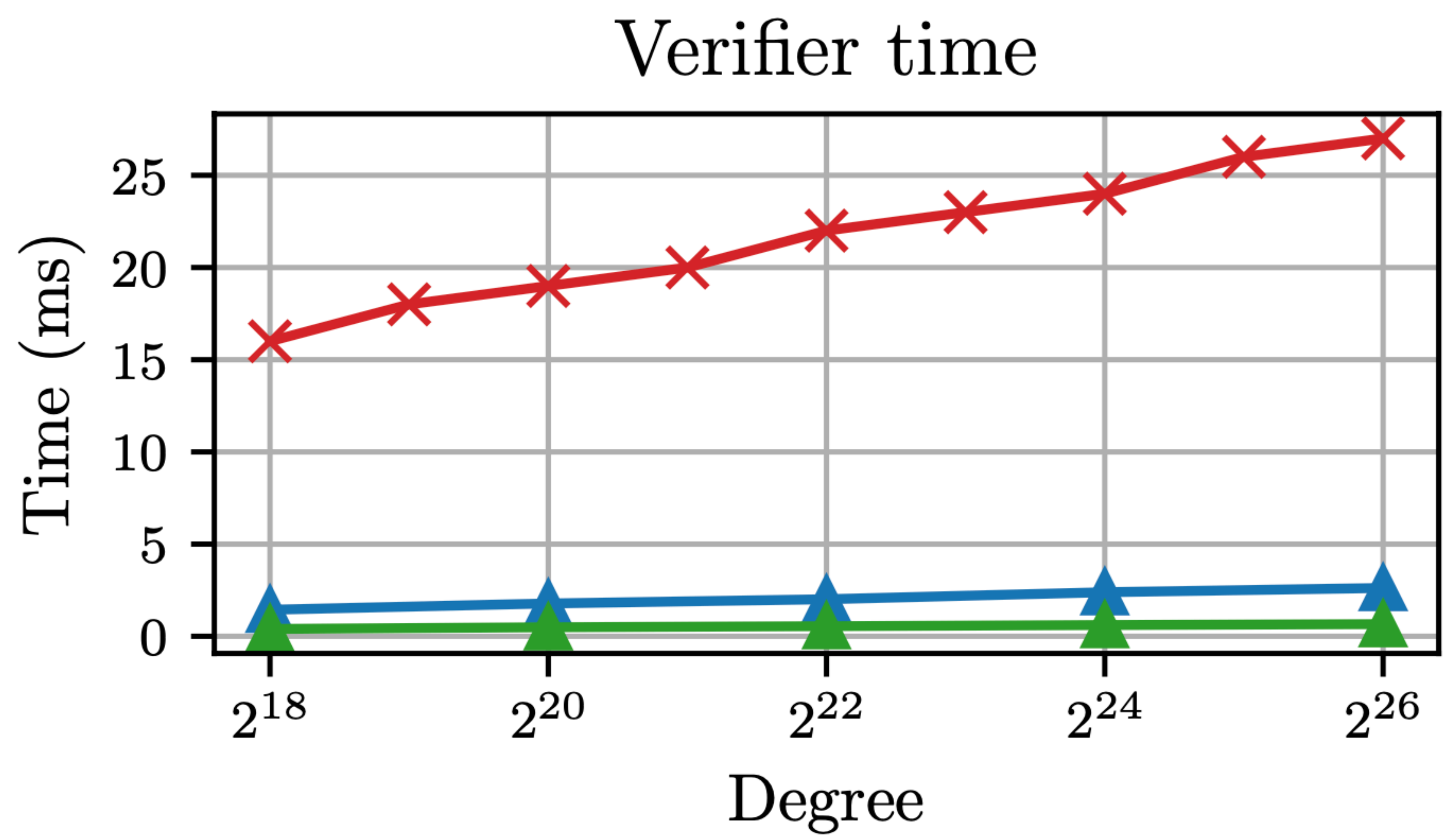
BaseFold: ×

WHIR-UD: ▲

WHIR-CB: ▲

Comparison with BaseFold

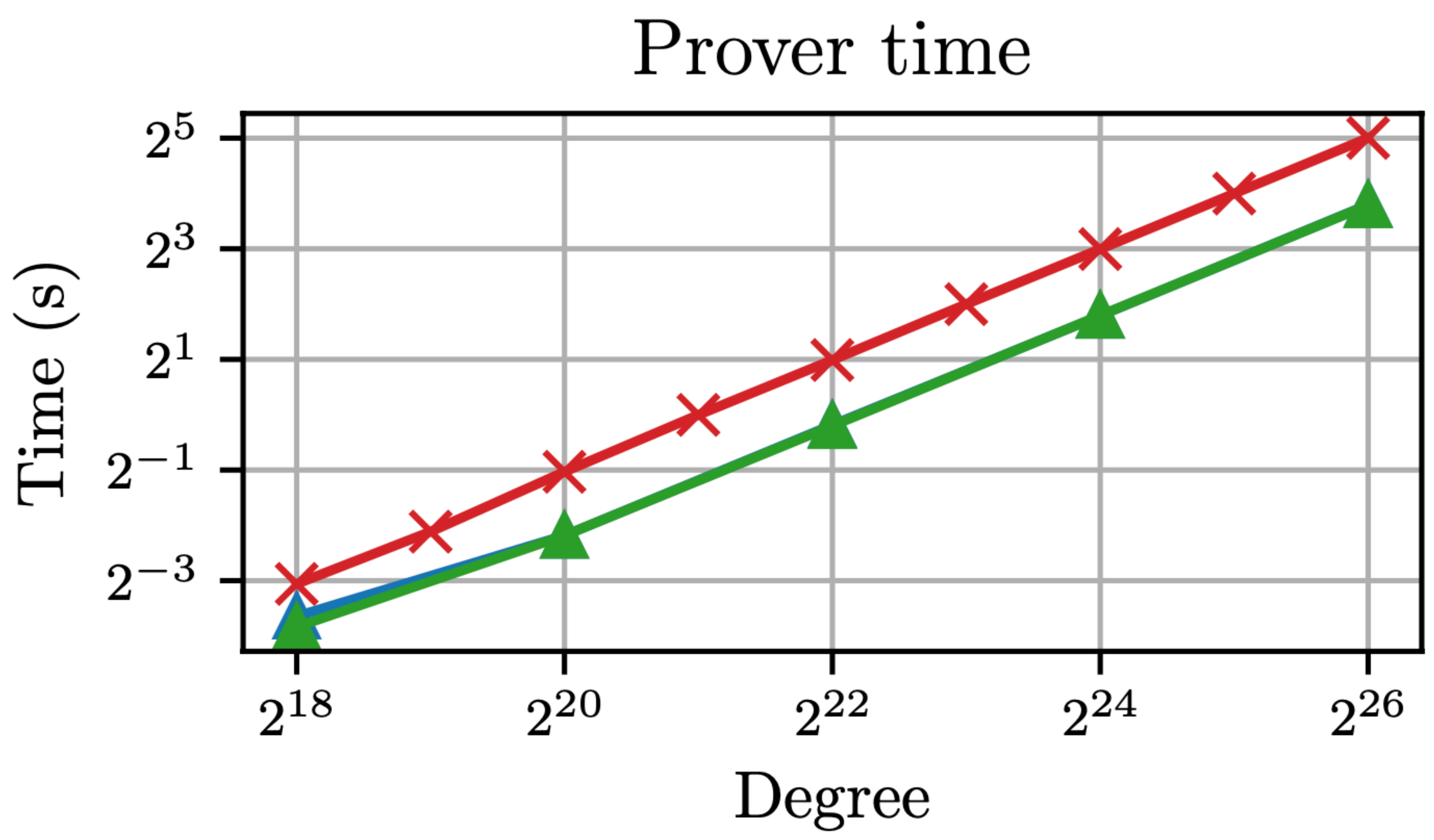




BaseFold: ×

WHIR-UD: ▲

WHIR-CB: ▲



Remark: BaseFold implementation is not fully optimised

Comparison with FRI (and STIR)

128-bits security level.

$\lambda = 106 + 22$ bits of PoW + “list-decoding” assumptions.

Comparison with FRI (and STIR)

128-bits security level.

$\lambda = 106 + 22$ bits of PoW + “list-decoding” assumptions.

$m = 24, \rho = 1/4$	**FRI**	**WHIR**
Size (KiB)	177	106
Verifier time	2.4ms	700μs

Comparison with FRI (and STIR)

128-bits security level.

$\lambda = 106 + 22$ bits of PoW + “list-decoding” assumptions.

$m = 24, \rho = 1/4$	FRI	WHIR
Size (KiB)	177	106
Verifier time	2.4ms	700μs

$d = 30, \rho = 1/2$	FRI	WHIR
Size (KiB)	494	187
Verifier time	4.4ms	1.3ms

Comparison with FRI (and STIR)

128-bits security level.

$\lambda = 106 + 22$ bits of PoW + "list-decoding" assumptions.

$\rho = 1/2$

$m = 24, \rho = 1/4$	FRI	WHIR
Size (KiB)	177	106
Verifier time	2.4ms	700μs

$d = 30, \rho = 1/2$	FRI	WHIR
Size (KiB)	494	187
Verifier time	4.4ms	1.3ms

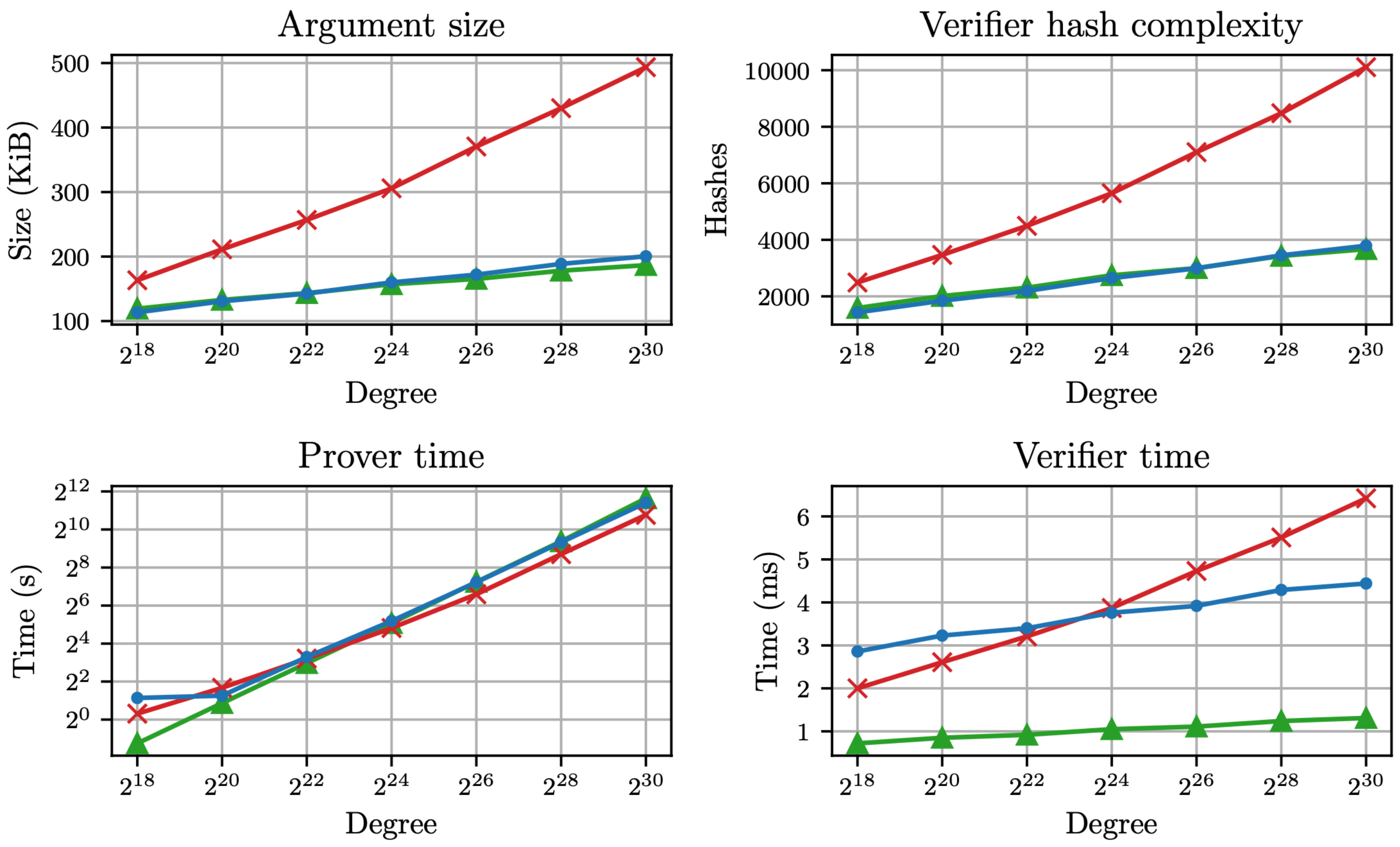


Figure 2: Comparison of FRI, STIR and WHIR for $\rho = 1/2$. FRI: ×, STIR: •, WHIR-CB: ▲. Pro time is displayed with logarithmic scaling.

Conclusion

Summary

Summary

WHIR 🌪️: a new IOPP for CRS codes.

Summary

WHIR 🌪: a new IOPP for CRS codes.

Query complexity:

$$O\left(\frac{\lambda}{k} \cdot \log m\right)$$

Verifier complexity:

$$O(q_{\mathsf{WHIR}} \cdot (2^k + m))$$

Summary

WHIR 🌪️: a new IOPP for CRS codes.

- **State-of-the-art** argument size and hash complexity

Query complexity:

$$O\left(\frac{\lambda}{k} \cdot \log m\right)$$

Verifier complexity:

$$O(q_{\mathsf{WHIR}} \cdot (2^k + m))$$

Summary

WHIR : a new IOPP for CRS codes.

- **State-of-the-art** argument size and hash complexity
- **Fastest** verification of any PCS (including trusted setups!)

Query complexity:

$$O\left(\frac{\lambda}{k} \cdot \log m\right)$$

Verifier complexity:

$$O(q_{\mathsf{WHIR}} \cdot (2^k + m))$$

Summary

WHIR 🌪️: a new IOPP for CRS codes.

Query complexity:

$$O\left(\frac{\lambda}{k} \cdot \log m\right)$$

Verifier complexity:

$$O(q_{\mathsf{WHIR}} \cdot (2^k + m))$$

- **State-of-the-art** argument size and hash complexity
- **Fastest** verification of any PCS (including trusted setups!)
- Enables high-soundness compilation for **Σ-IOP**

Σ-IOP + CRS IOPP (WHIR 🌪️) = IOP

Extra slides

Techniques

FRI & STIR Folding

Reduce $\mathsf{RS}[n, m, \rho]$ **to** $\mathsf{RS}[n/2^k, m - k, \rho]$

(Think $k = 4$)

FRI & STIR Folding

Reduce $\mathrm{RS}[n, m, \rho]$ to $\mathrm{RS}[n/2^k, m-k, \rho]$

(Think $k = 4$)

Unchanged!

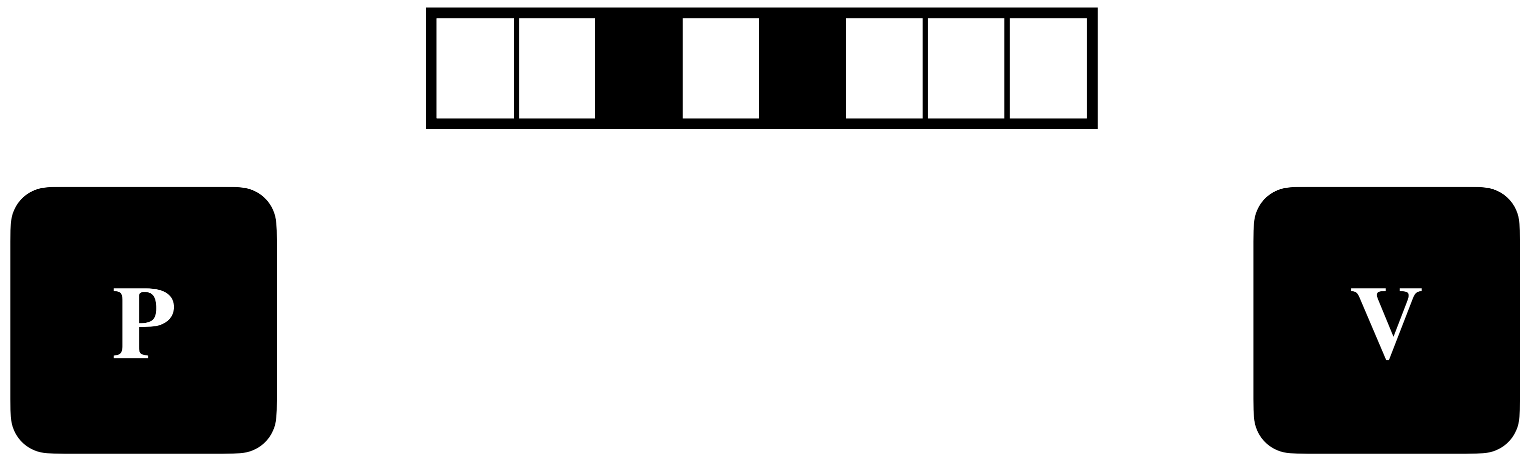
FRI & STIR Folding

Reduce $\mathrm{RS}[n, m, \rho]$ **to** $\mathrm{RS}[n/2^k, m-k, \rho]$

Unchanged!

(Think $k = 4$)

$$f : L \to \mathbb{F}$$

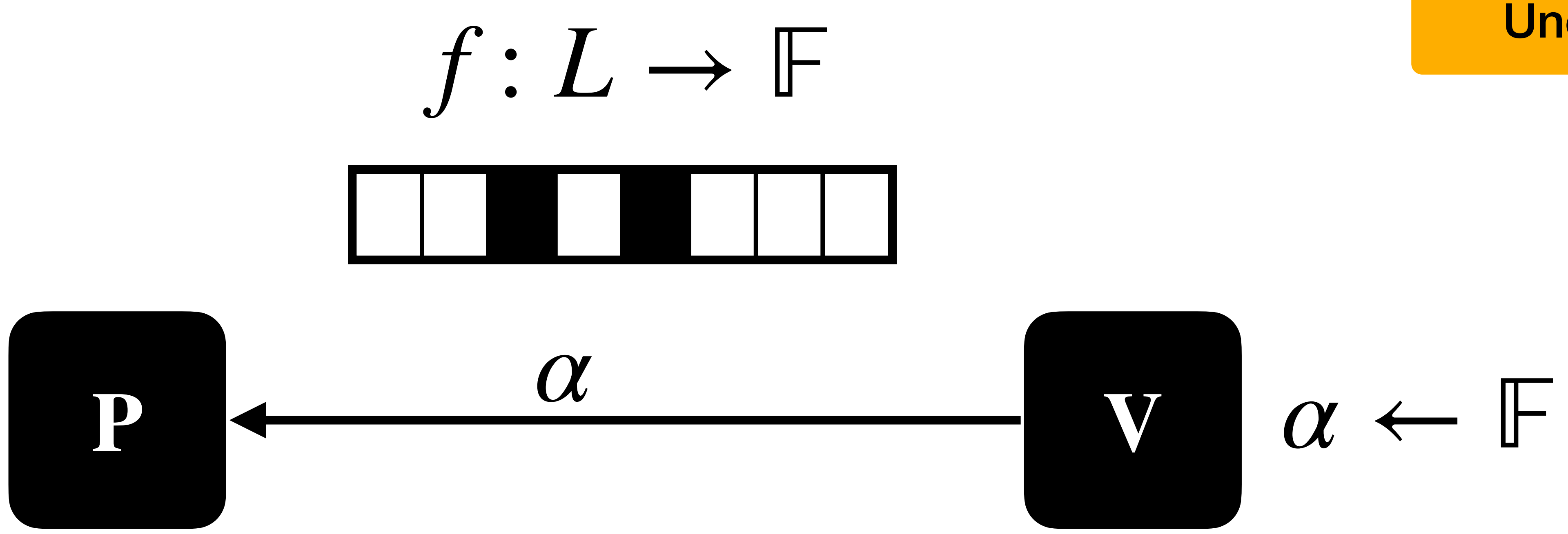


FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m - k, \rho]$

Unchanged!

(Think $k = 4$)

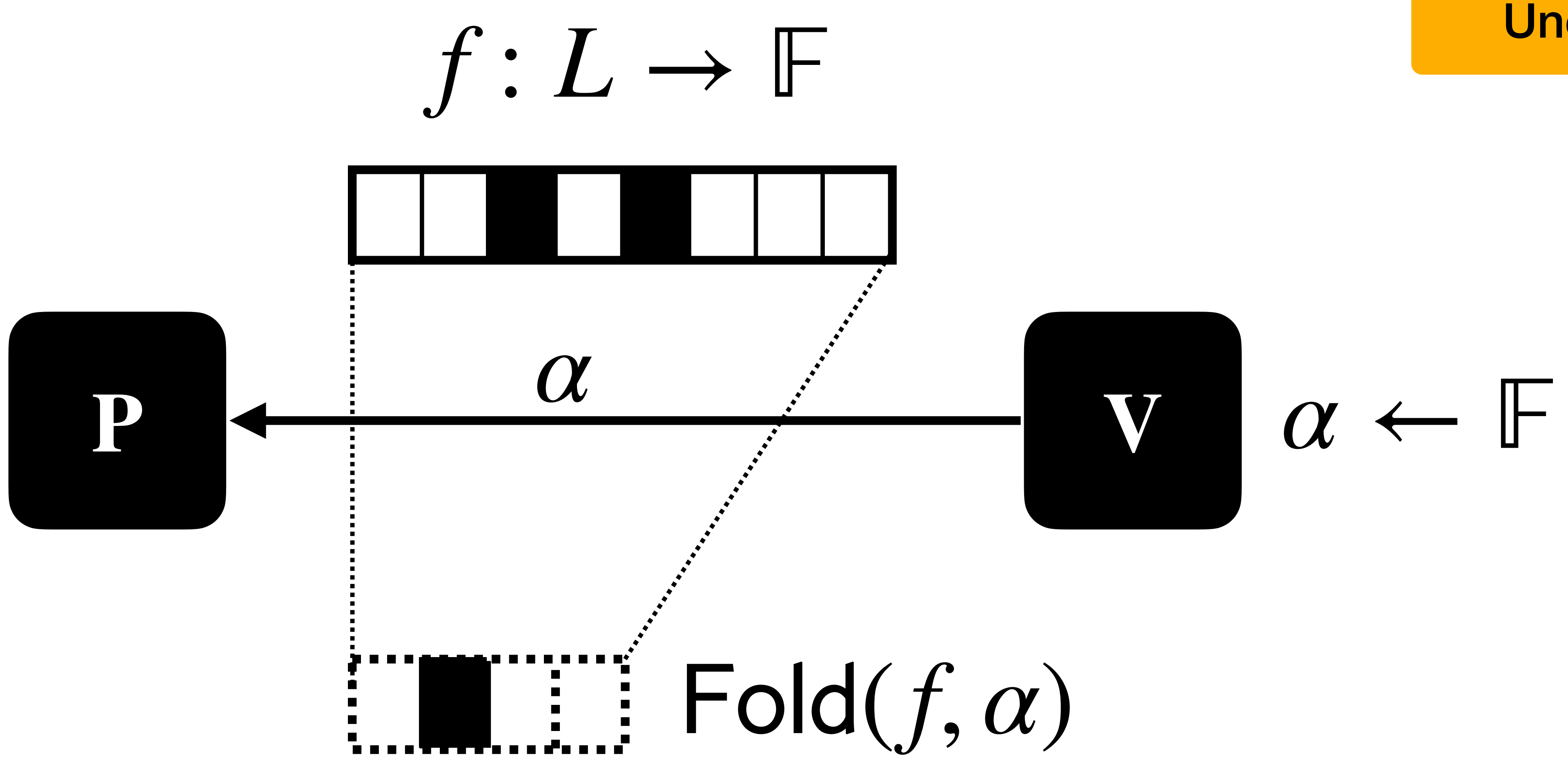


FRI & STIR Folding

Reduce $\mathsf{RS}[n, m, \rho]$ **to** $\mathsf{RS}[n/2^k, m - k, \rho]$

Unchanged!

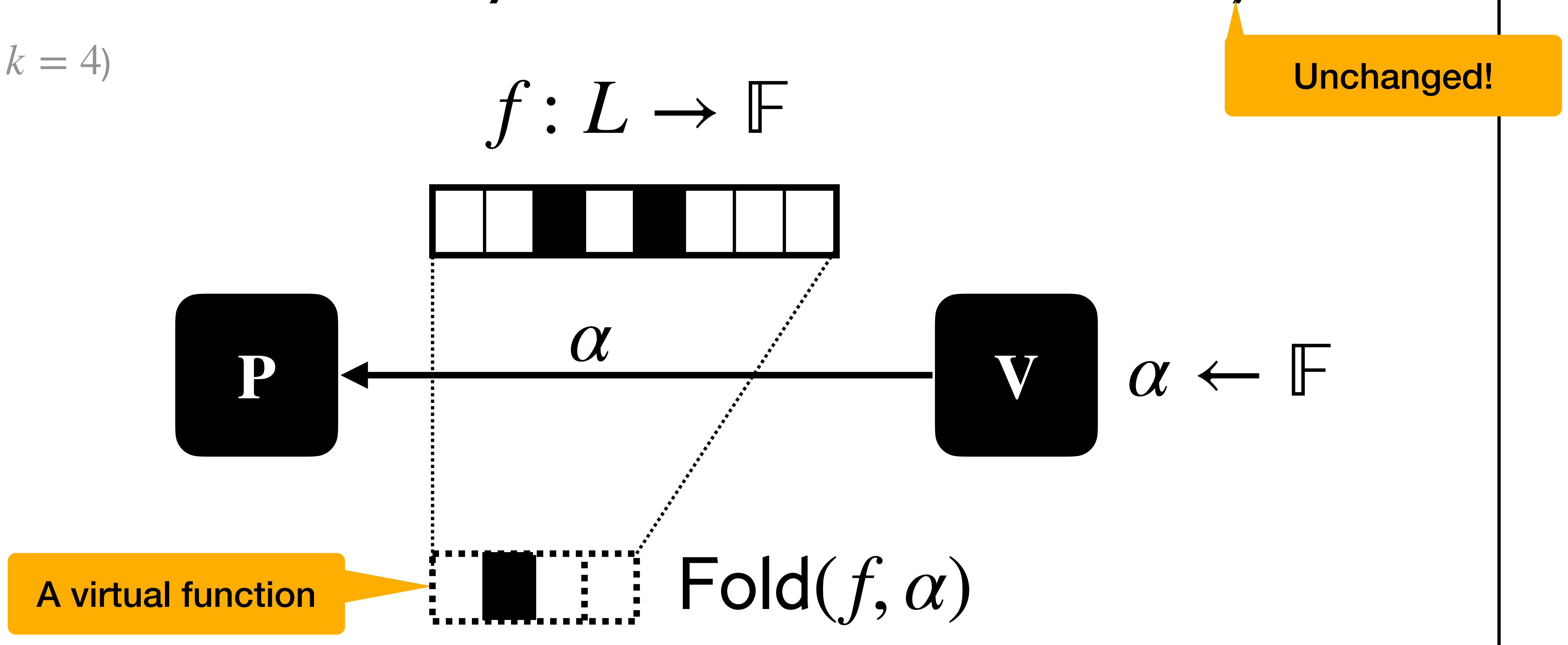
(Think $k = 4$)



FRI & STIR Folding

Reduce $\mathsf{RS}[n, m, \rho]$ **to** $\mathsf{RS}[n/2^k, m - k, \rho]$

(Think $k = 4$)

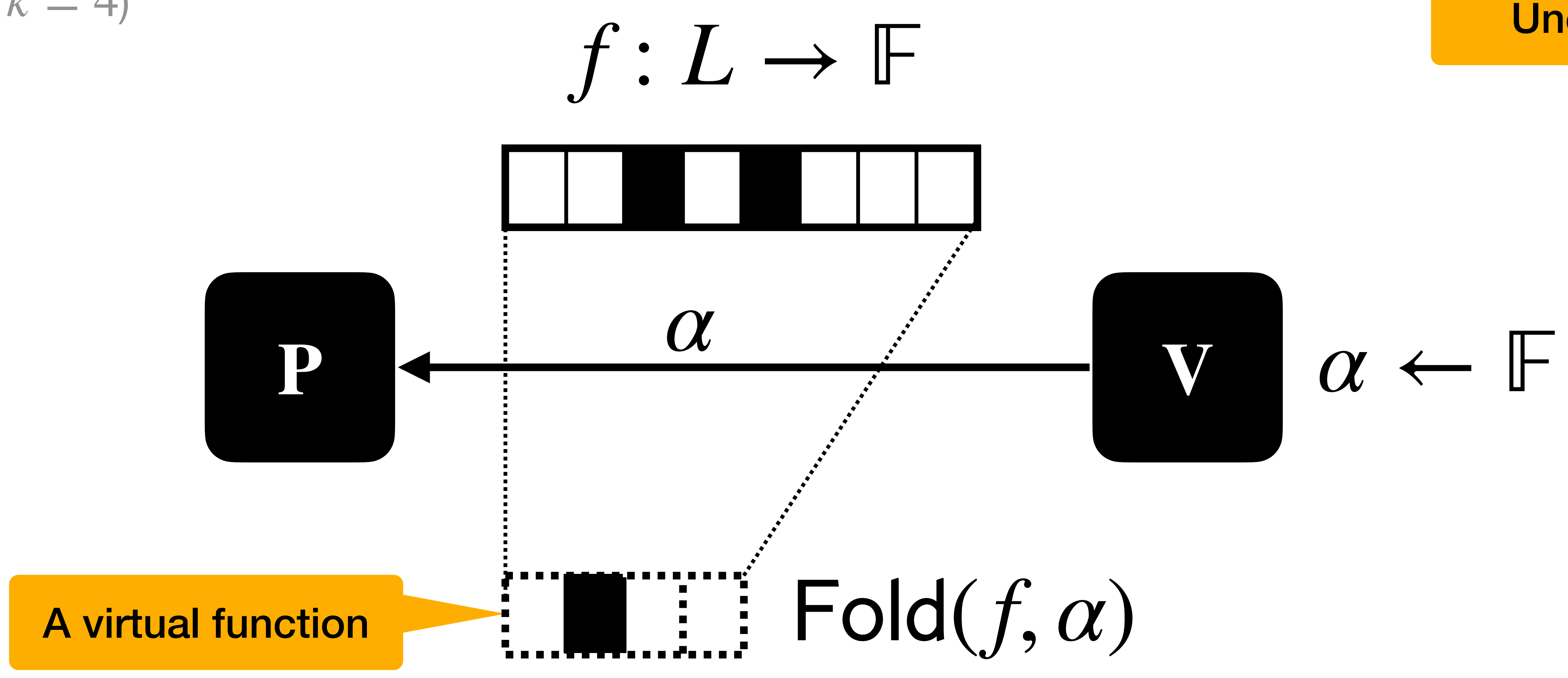


FRI & STIR Folding

Reduce $\mathsf{RS}[n, m, \rho]$ **to** $\mathsf{RS}[n/2^k, m-k, \rho]$

Unchanged!

(Think $k = 4$)



How? Inspiration from FFTs, for $k = 1$:

$$\mathsf{Fold}(f, \alpha) := f_{\mathsf{odd}} + \alpha \cdot f_{\mathsf{even}}$$

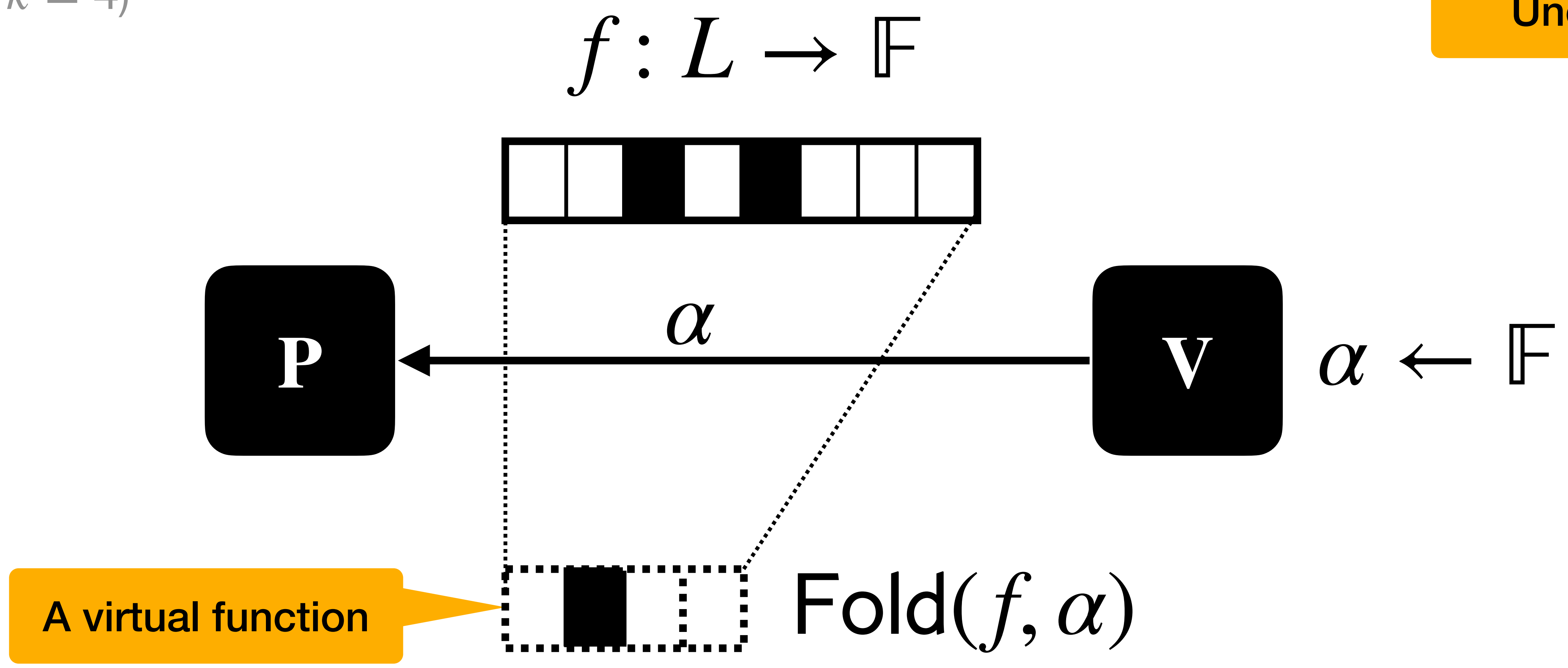
Can extend to every k that is a power of two.

FRI & STIR Folding

Reduce $\mathsf{RS}[n, m, \rho]$ **to** $\mathsf{RS}[n/2^k, m-k, \rho]$

Unchanged!

(Think $k = 4$)



How? Inspiration from FFTs, for $k = 1$:

$$\mathsf{Fold}(f, \alpha) := f_{\mathsf{odd}} + \alpha \cdot f_{\mathsf{even}}$$

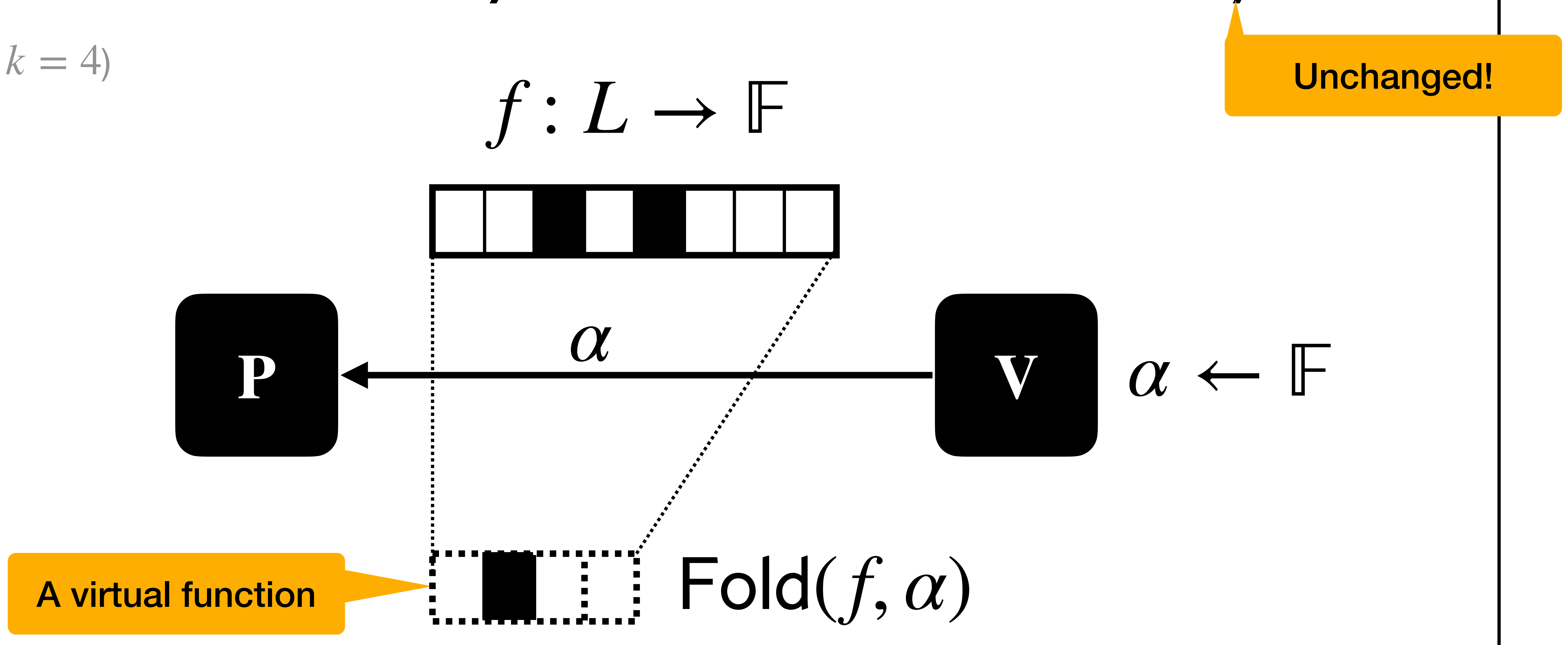
Can extend to every k that is a power of two.

Properties:

FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m - k, \rho]$

(Think $k = 4$)



How? Inspiration from FFTs, for $k = 1$:

$$\text{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

Can extend to every k that is a power of two.

Properties:

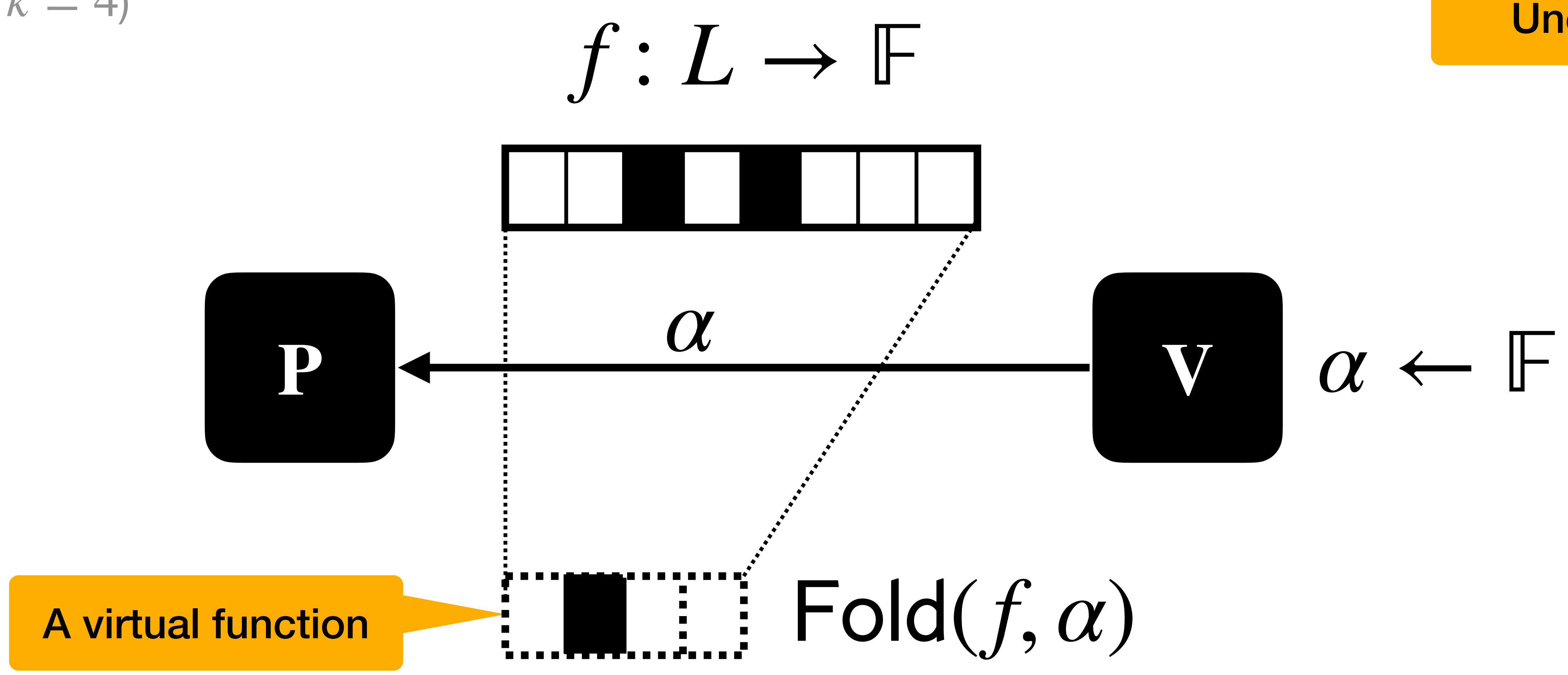
Local: compute Fold$(f, \alpha)(z)$ at any point $z \in L^{2^k}$ with 2^k queries to f.

FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m-k, \rho]$

Unchanged!

(Think $k = 4$)



How? Inspiration from FFTs, for $k = 1$:

$$\text{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

Can extend to every k that is a power of two.

Properties:

Local: compute Fold$(f, \alpha)(z)$ at any point $z \in L^{2^k}$ with 2^k queries to f.

$\delta \in \left(0, 1 - \sqrt{\rho}\right)$

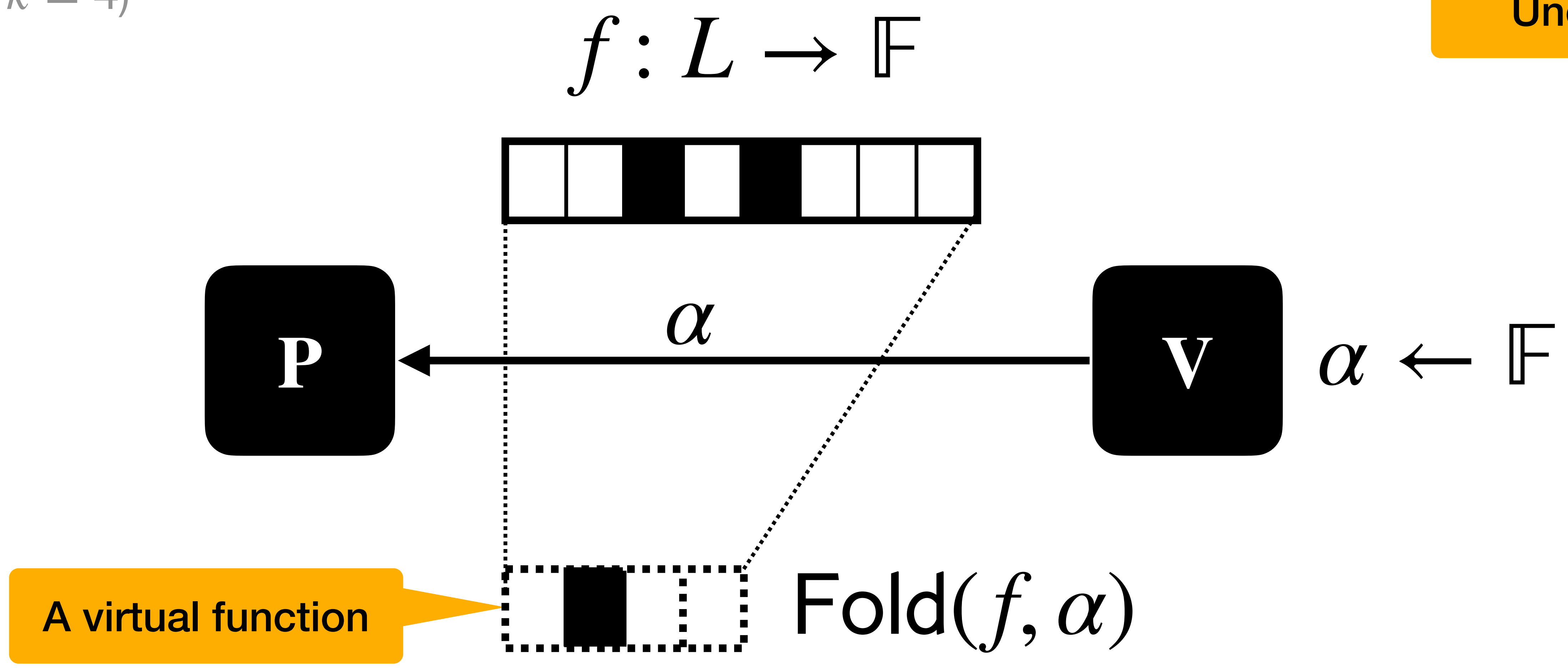
Distance preservation: if f is δ-far from RS$[n, m, \rho]$, then w.h.p. Fold(f, α) remains also δ-far from RS$[n/2^k, m-k, \rho]$

FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m - k, \rho]$

Unchanged!

(Think $k = 4$)



How? Inspiration from FFTs, for $k = 1$:

$$\text{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

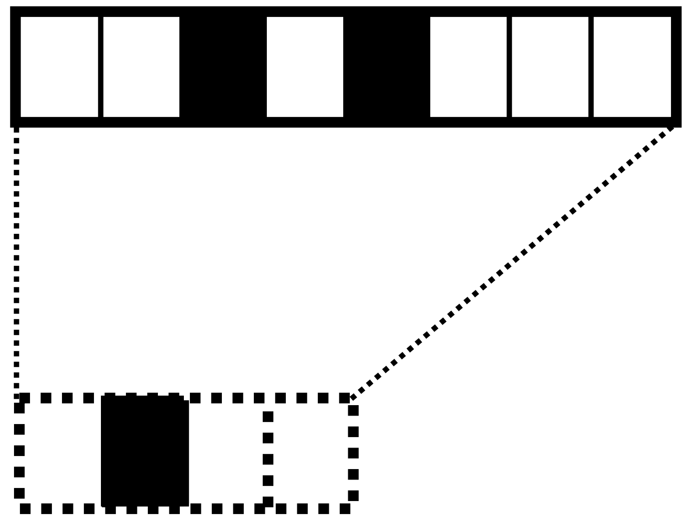
Can extend to every k that is a power of two.

Properties:

Local: compute Fold$(f, \alpha)(z)$ at any point $z \in L^{2^k}$ with 2^k queries to f.

$\delta \in \left(0, 1 - \sqrt{\rho}\right)$

Distance preservation: if f is δ-far from RS$[n, m, \rho]$, then w.h.p. Fold(f, α) remains also δ-far from RS$[n/2^k, m - k, \rho]$



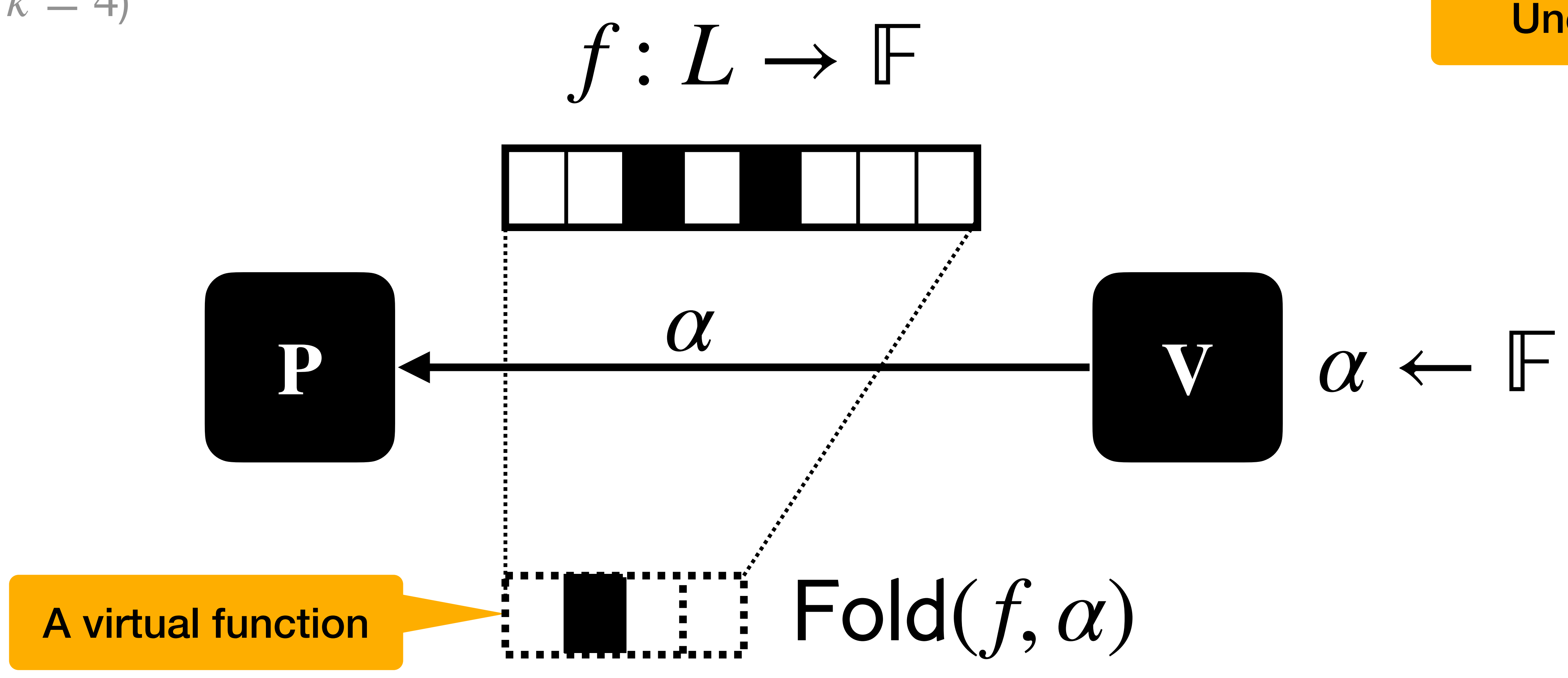
Unless w.p. $\approx \frac{\text{poly}(n, 2^m)}{|\mathbb{F}|}$, the fraction of "corrupted" entries does not decrease.

FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m-k, \rho]$

(Think $k = 4$)

Unchanged!



How? Inspiration from FFTs, for $k = 1$:

$$\text{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

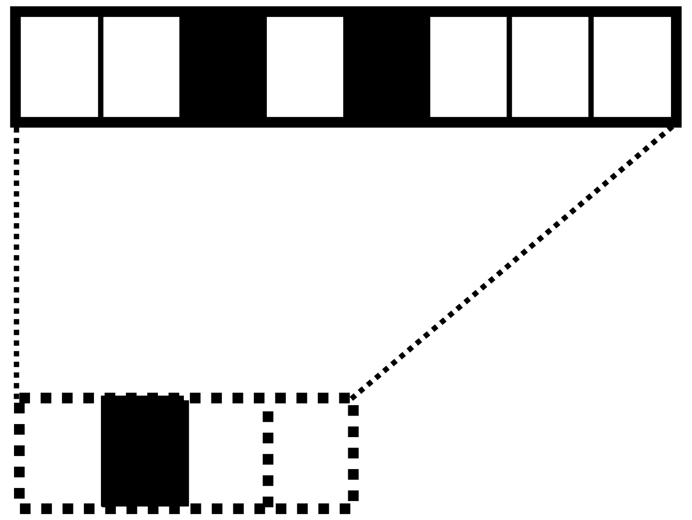
Can extend to every k that is a power of two.

Properties:

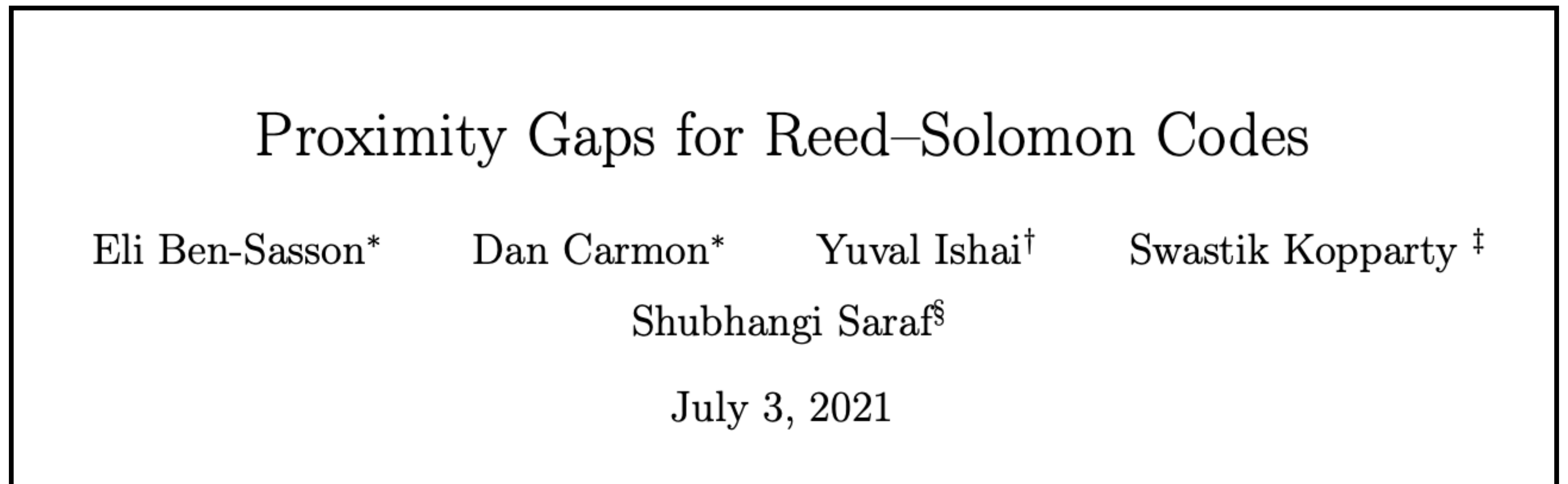
Local: compute Fold$(f, \alpha)(z)$ at any point $z \in L^{2^k}$ with 2^k queries to f.

$\delta \in \left(0, 1 - \sqrt{\rho}\right)$

Distance preservation: if f is δ-far from RS$[n, m, \rho]$, then w.h.p. Fold(f, α) remains also δ-far from RS$[n/2^k, m-k, \rho]$



Unless w.p. $\approx \frac{\text{poly}(n, 2^m)}{|\mathbb{F}|}$, the fraction of "corrupted" entries does not decrease.


Proximity Gaps for Reed–Solomon Codes

Eli Ben-Sasson* Dan Carmon* Yuval Ishai† Swastik Kopparty ‡
Shubhangi Saraf§

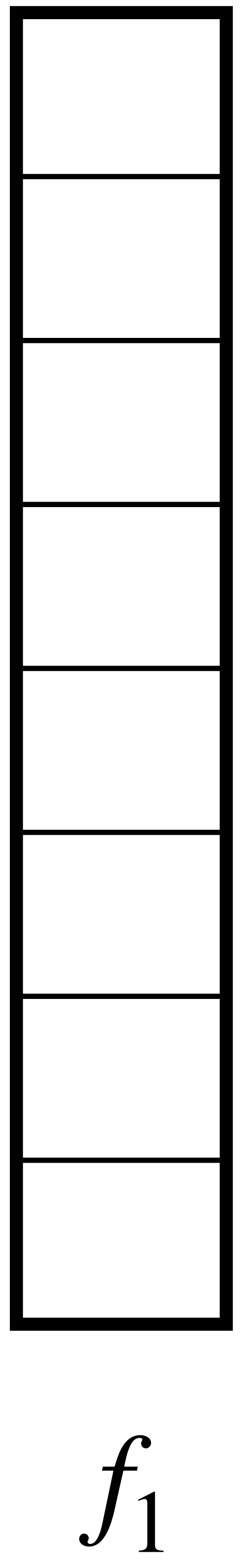
July 3, 2021

Mutual correlated agreement

Test a random linear combination

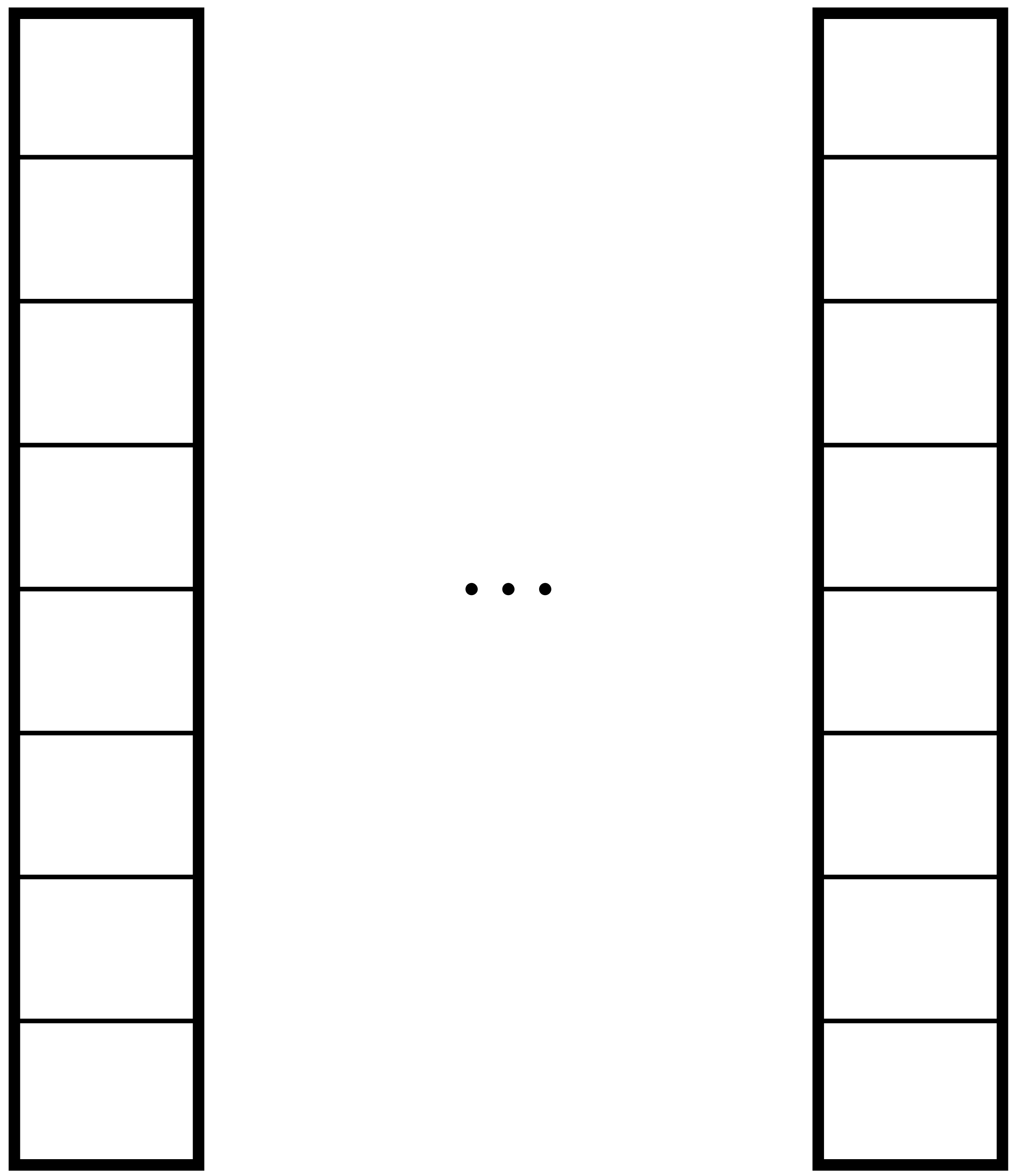
Mutual correlated agreement

Test a random linear combination



Mutual correlated agreement

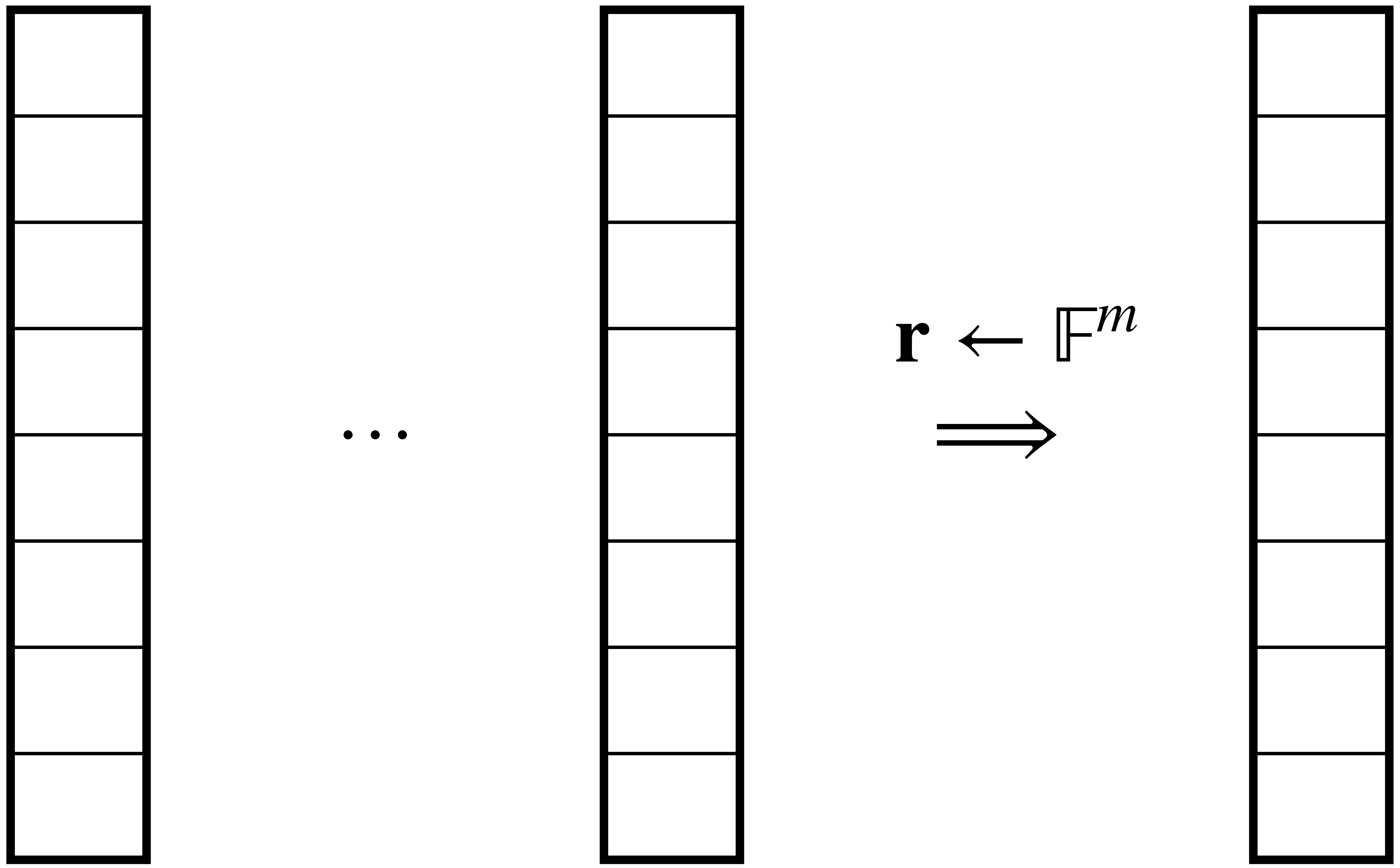
Test a random linear combination



f_1 $\quad\cdots\quad$ f_m

Mutual correlated agreement

Test a random linear combination



$$\mathbf{r} \leftarrow \mathbb{F}^m \quad \Longrightarrow$$

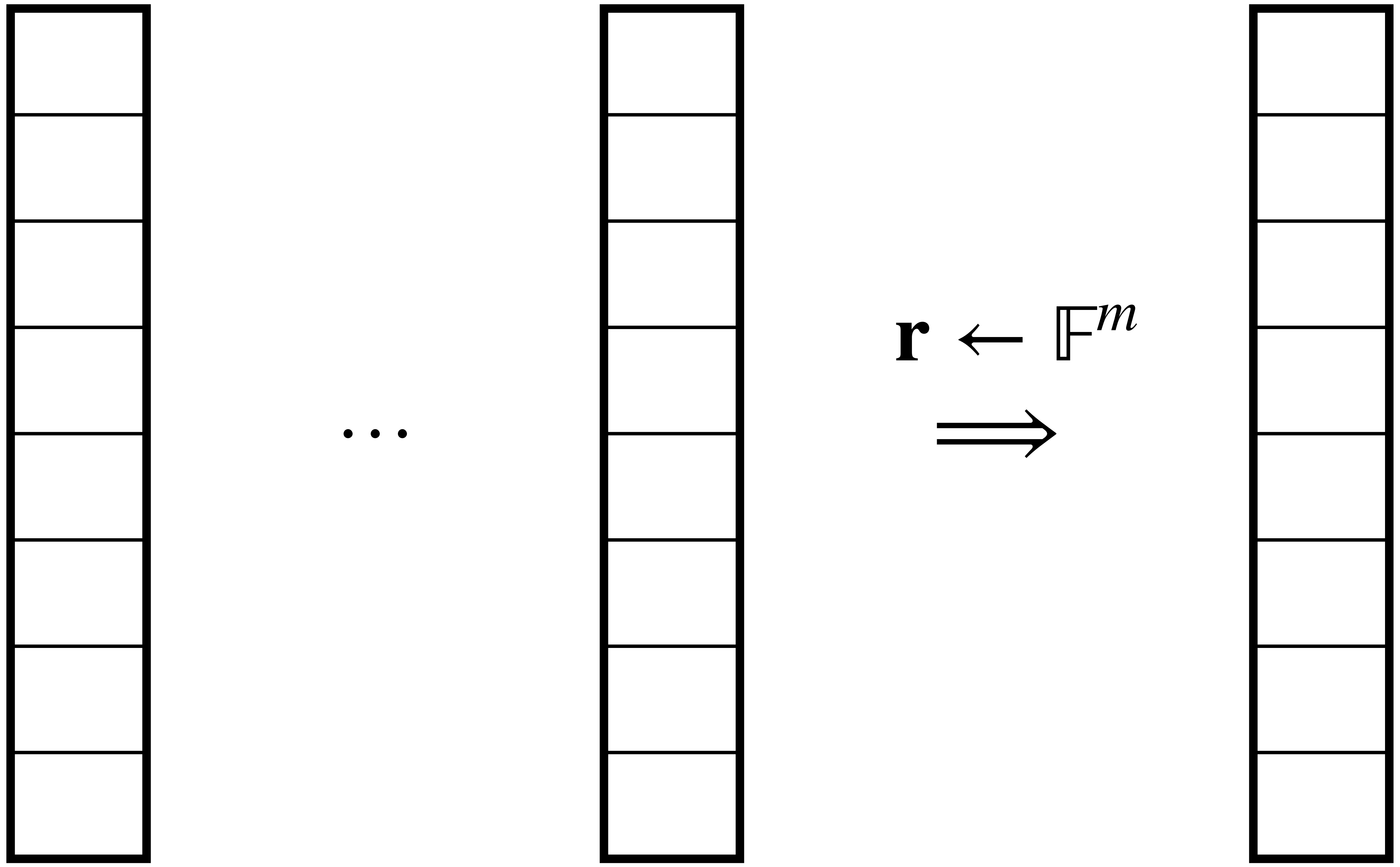
$f_1 \quad \cdots \quad f_m$

$$f^* := \sum_i r_i f_i$$

Mutual correlated agreement

Test a random linear combination

if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

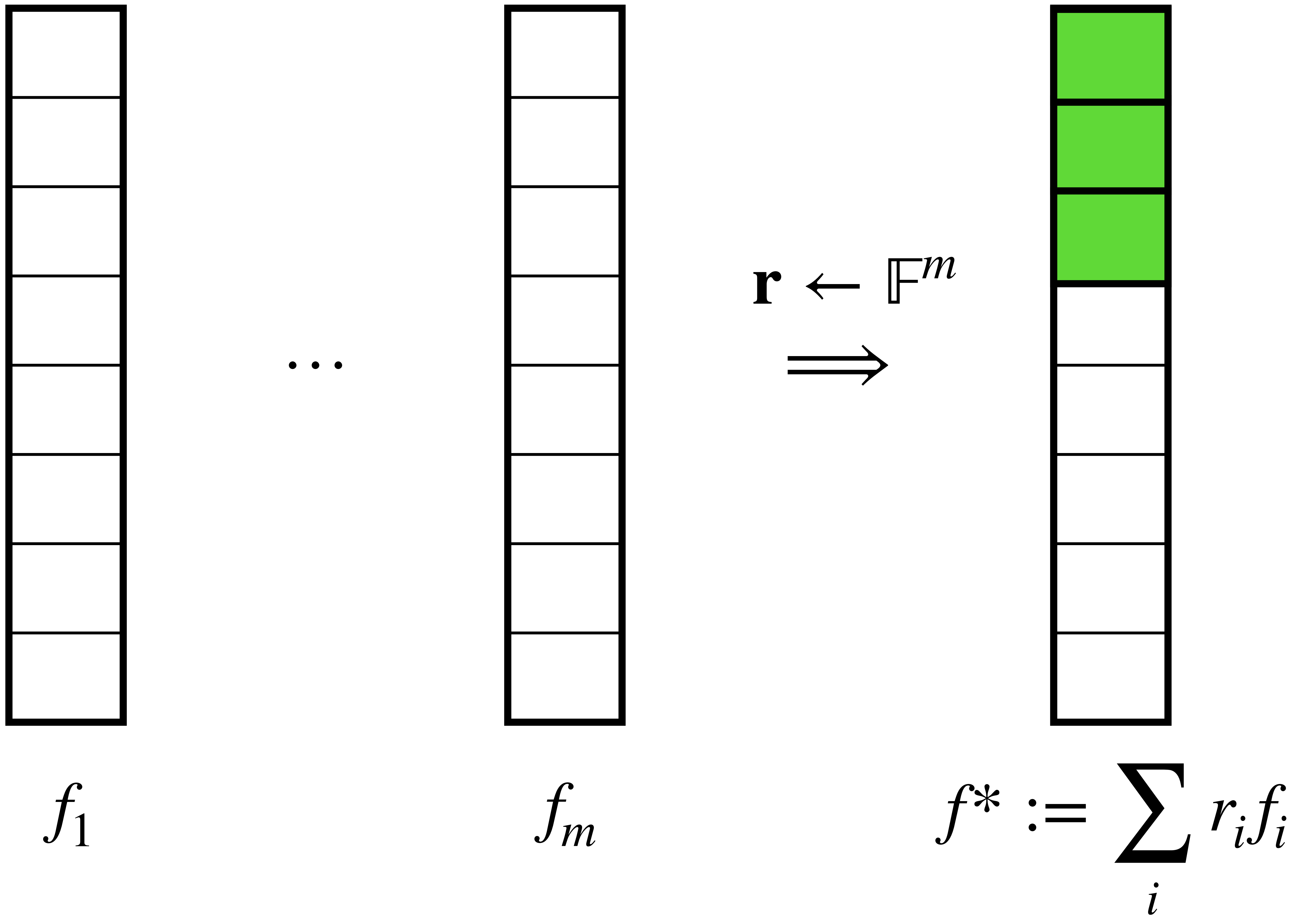


f_1 $\cdots$ f_m $\mathbf{r} \leftarrow \mathbb{F}^m \implies$ $f^* := \sum_i r_i f_i$

Mutual correlated agreement

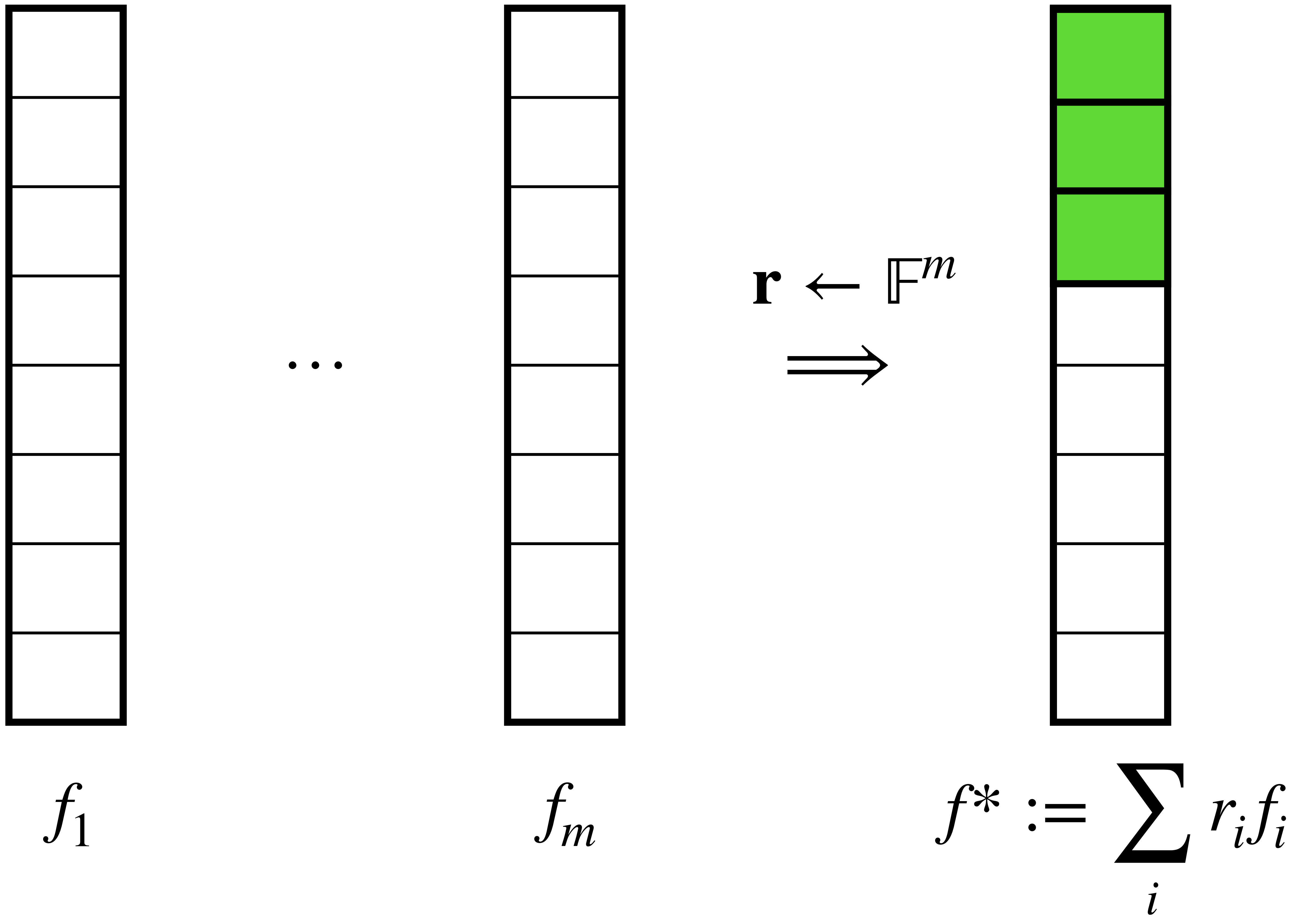
Test a random linear combination

if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:



Mutual correlated agreement

Test a random linear combination

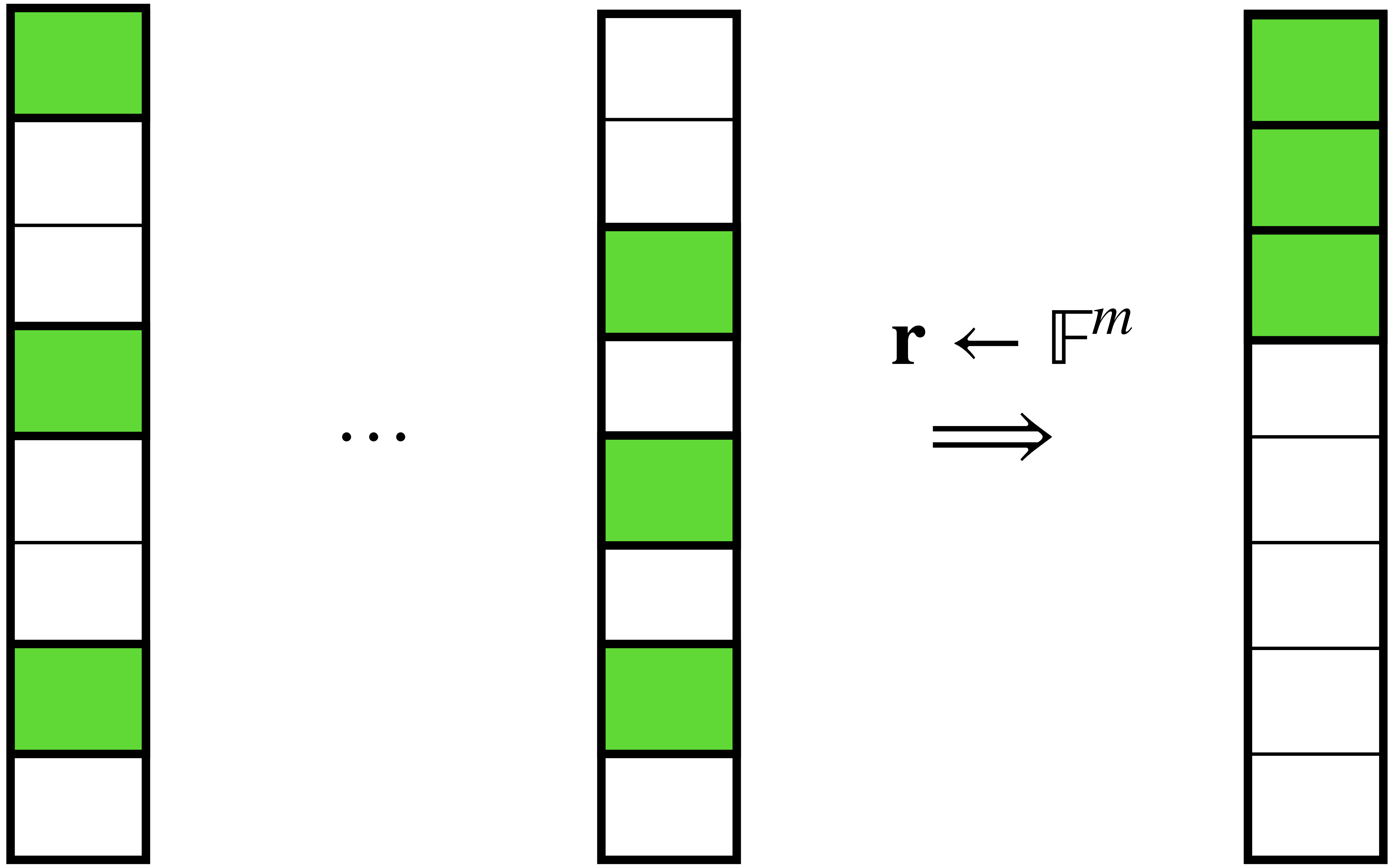


if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Mutual correlated agreement

Test a random linear combination



$\mathbf{r} \leftarrow \mathbb{F}^m$ $\Longrightarrow$

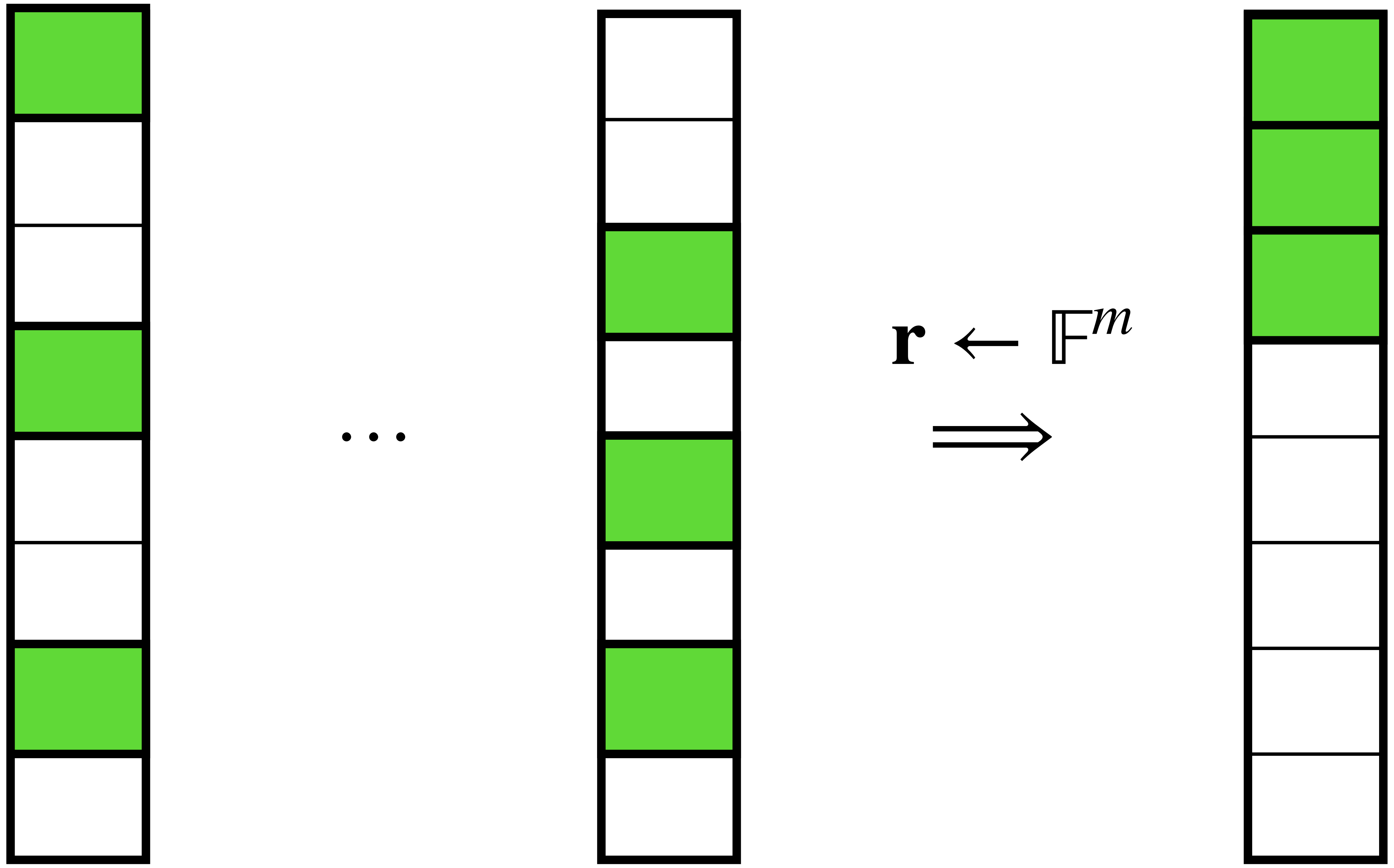
f_1 $\cdots$ f_m $\qquad f^* := \sum_i r_i f_i$

if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Mutual correlated agreement

Test a random linear combination



f_1 $\cdots$ f_m $f^* := \sum_i r_i f_i$

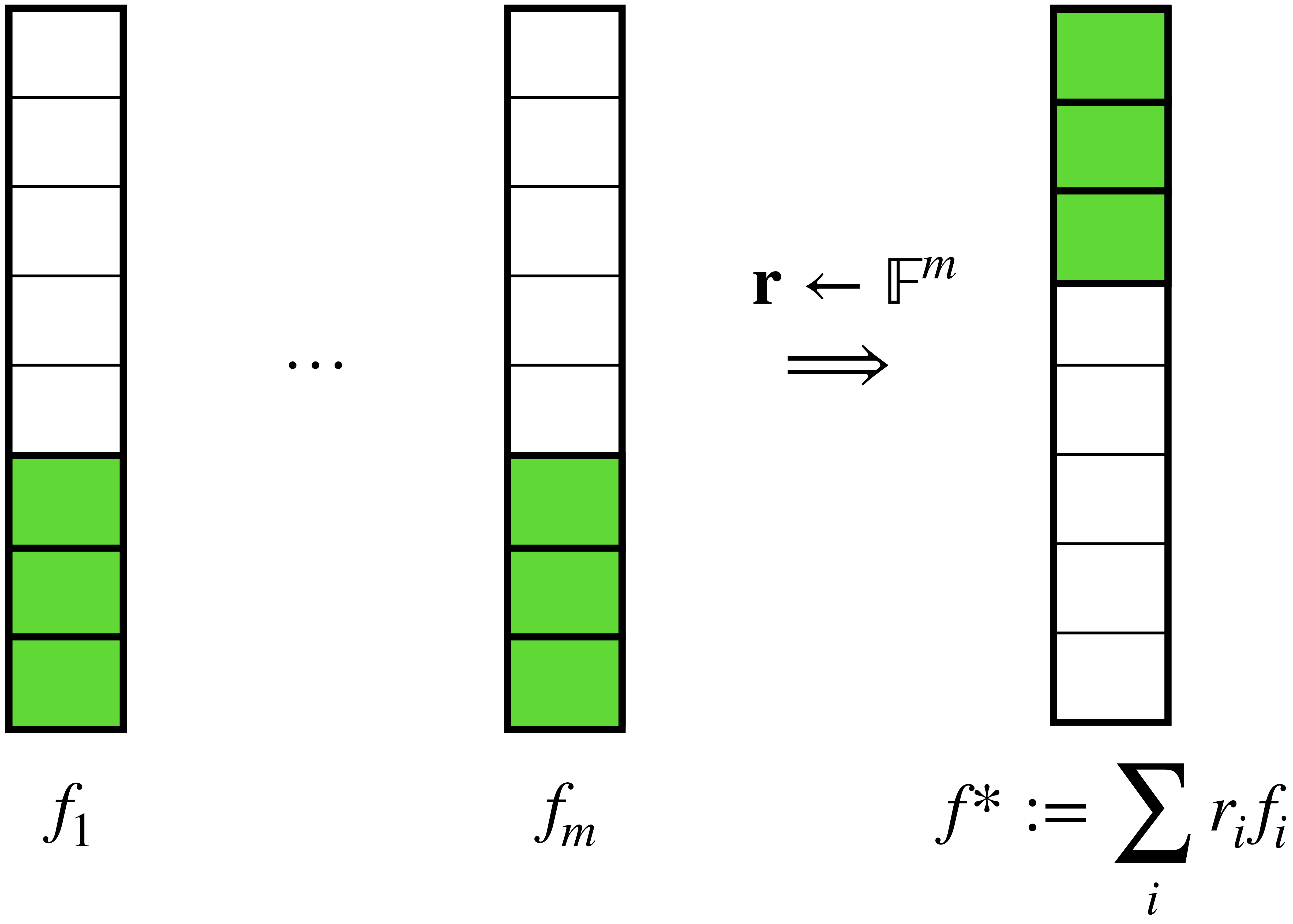
if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Correlated agreement: then $f_1, \ldots, f_m$ agree with $\mathscr{C}$ on the same “stripe”

Mutual correlated agreement

Test a random linear combination



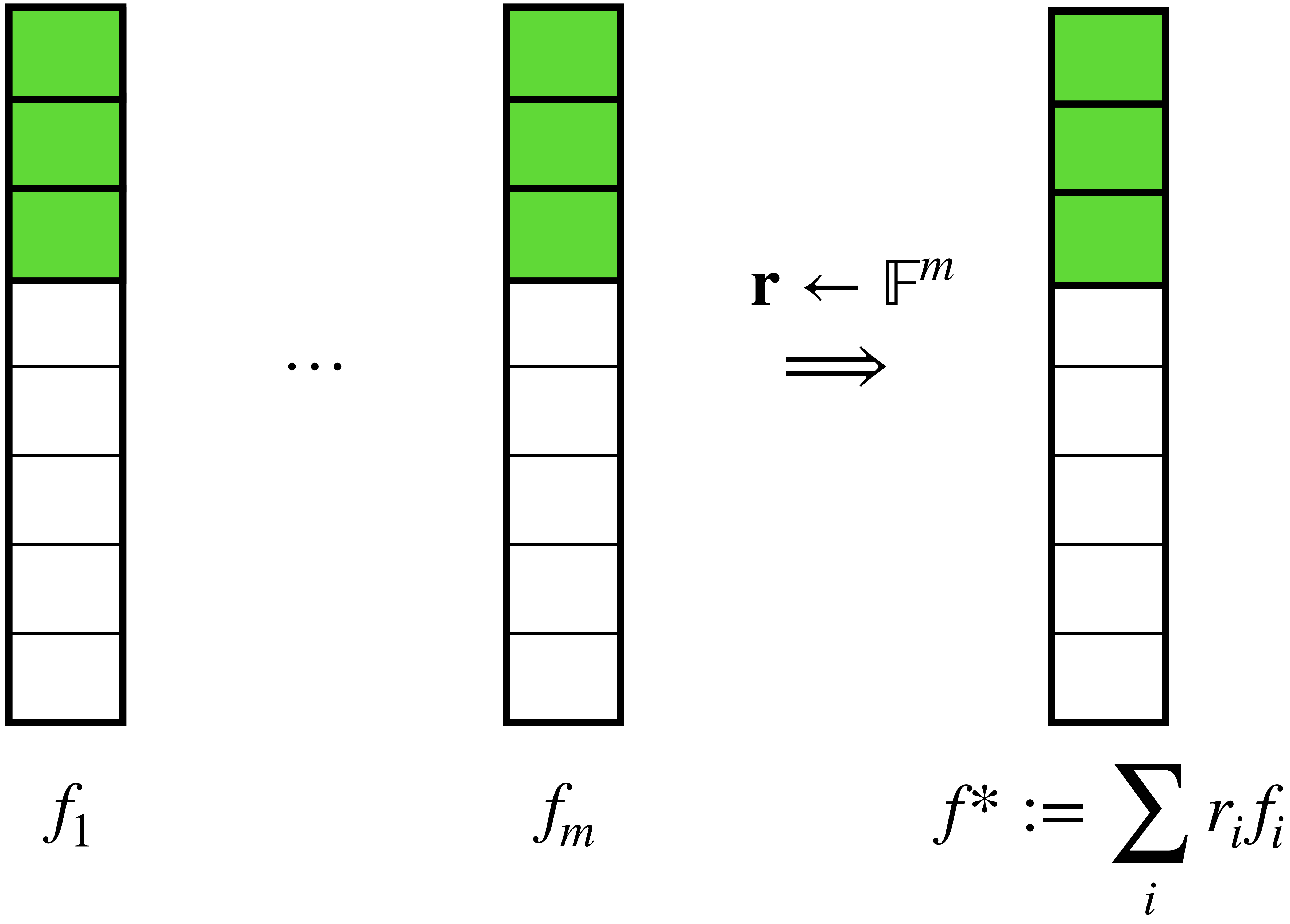
if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Correlated agreement: then $f_1, \ldots, f_m$ agree with $\mathscr{C}$ on the same "stripe"

Mutual correlated agreement

Test a random linear combination



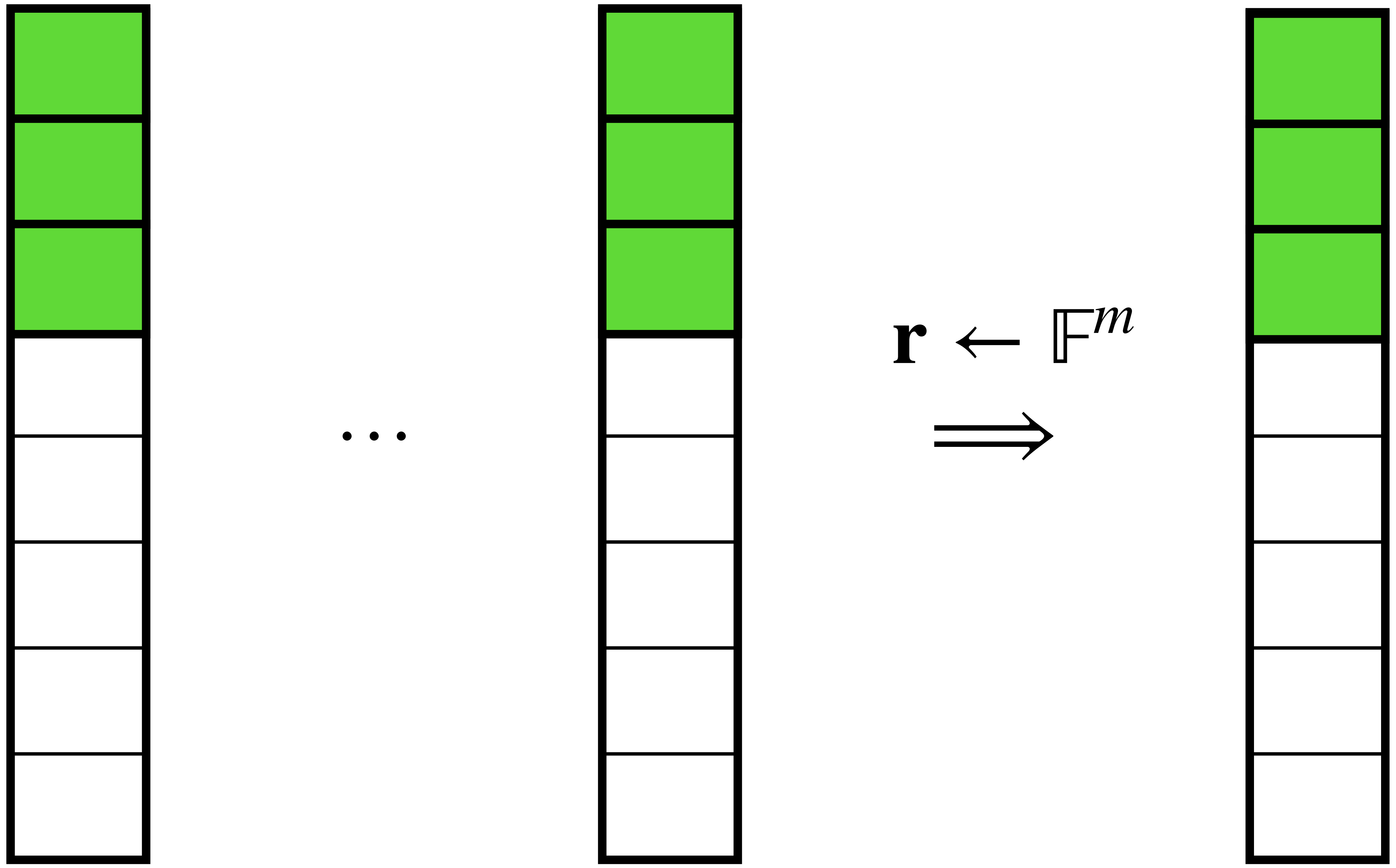
if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Correlated agreement: then $f_1, \ldots, f_m$ agree with $\mathscr{C}$ on the same "stripe"

Mutual correlated agreement

Test a random linear combination



f_1 $\quad$ f_m $\quad$ $f^* := \sum_i r_i f_i$

if w.h.p. $\Delta(f^*, \mathscr{C}) \leq \delta$:

Agreement: then $\Delta(f_i, \mathscr{C}) \leq \delta$.

Correlated agreement: then $f_1, \dots, f_m$ agree with $\mathscr{C}$ on the same "stripe"

Mutual correlated agreement: the stripe in which $f_1, \dots, f_m$ agree with $\mathscr{C}$ is the same on which f^* does:

"*No new correlated domains appear*"

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

$f_1, \ldots, f_m : L \to \mathbb{F}$

●

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).

$f_1, \ldots, f_m : L \to \mathbb{F}$

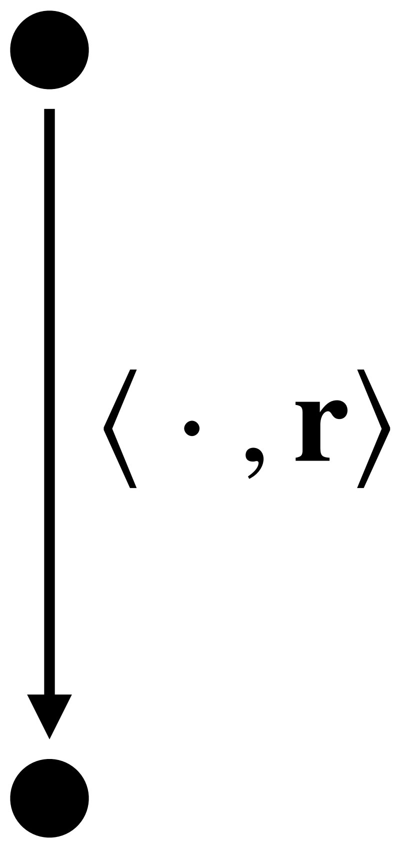
●

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

$f_1, \ldots, f_m : L \to \mathbb{F}$



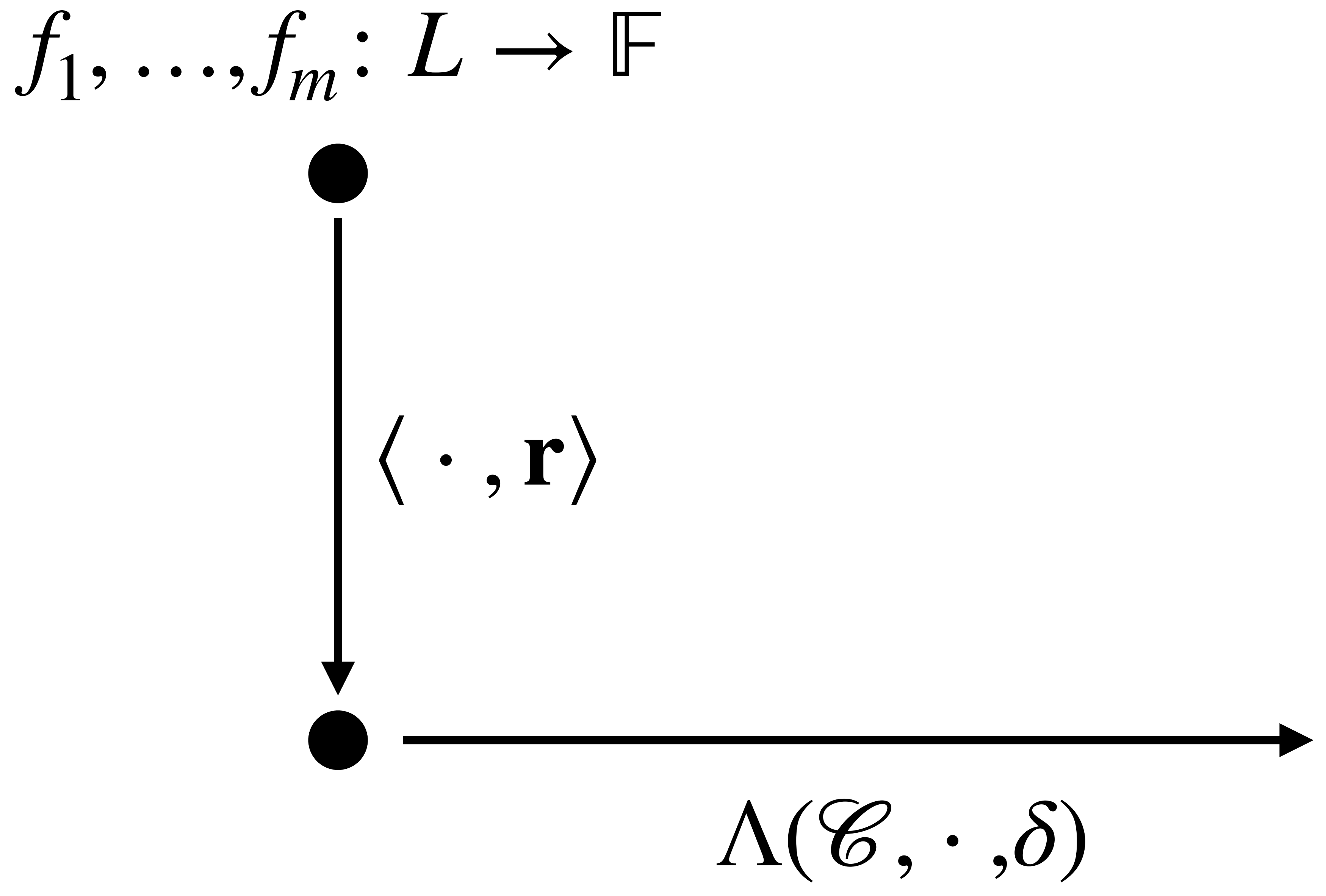
- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

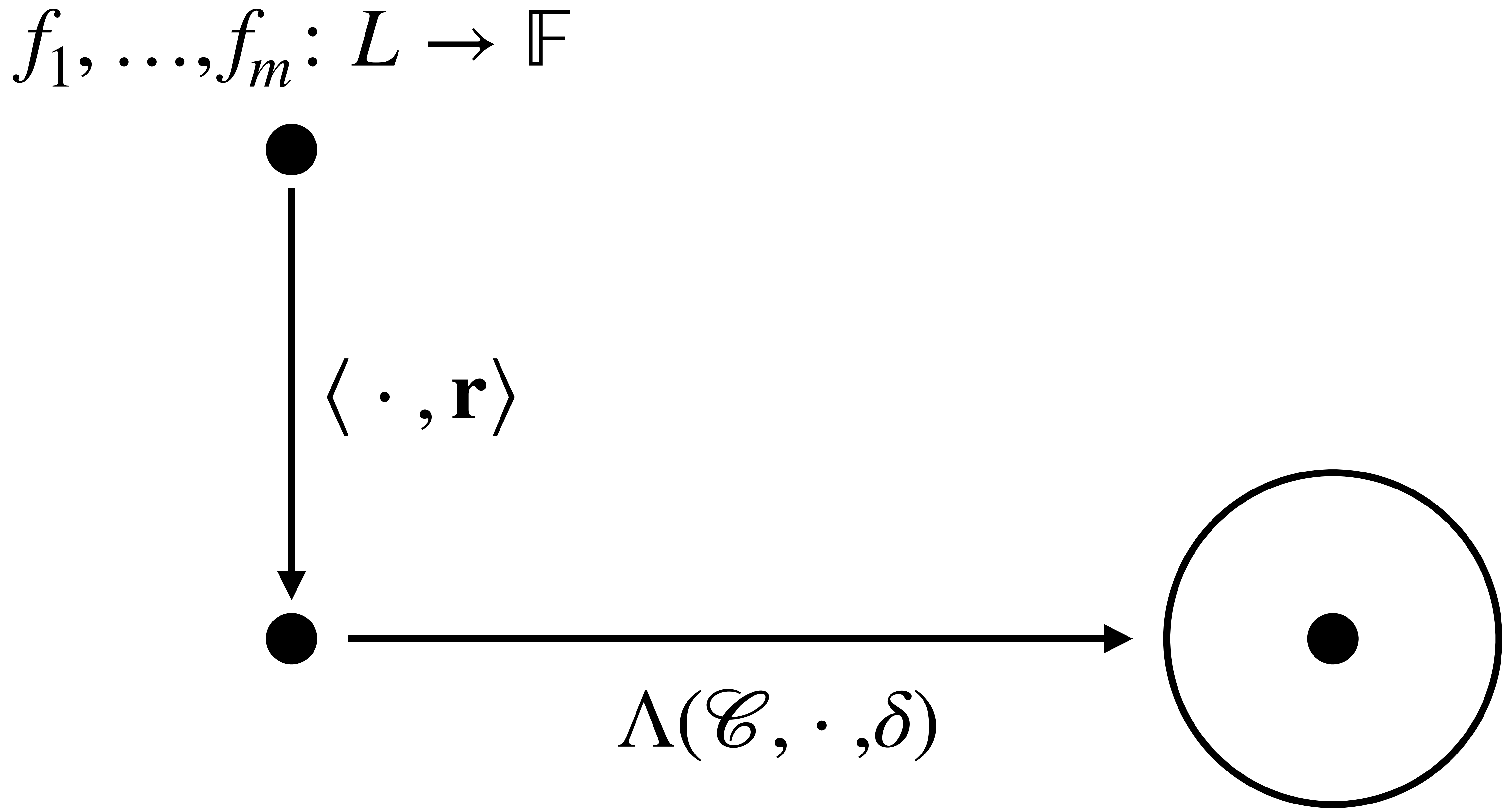
- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).



List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f



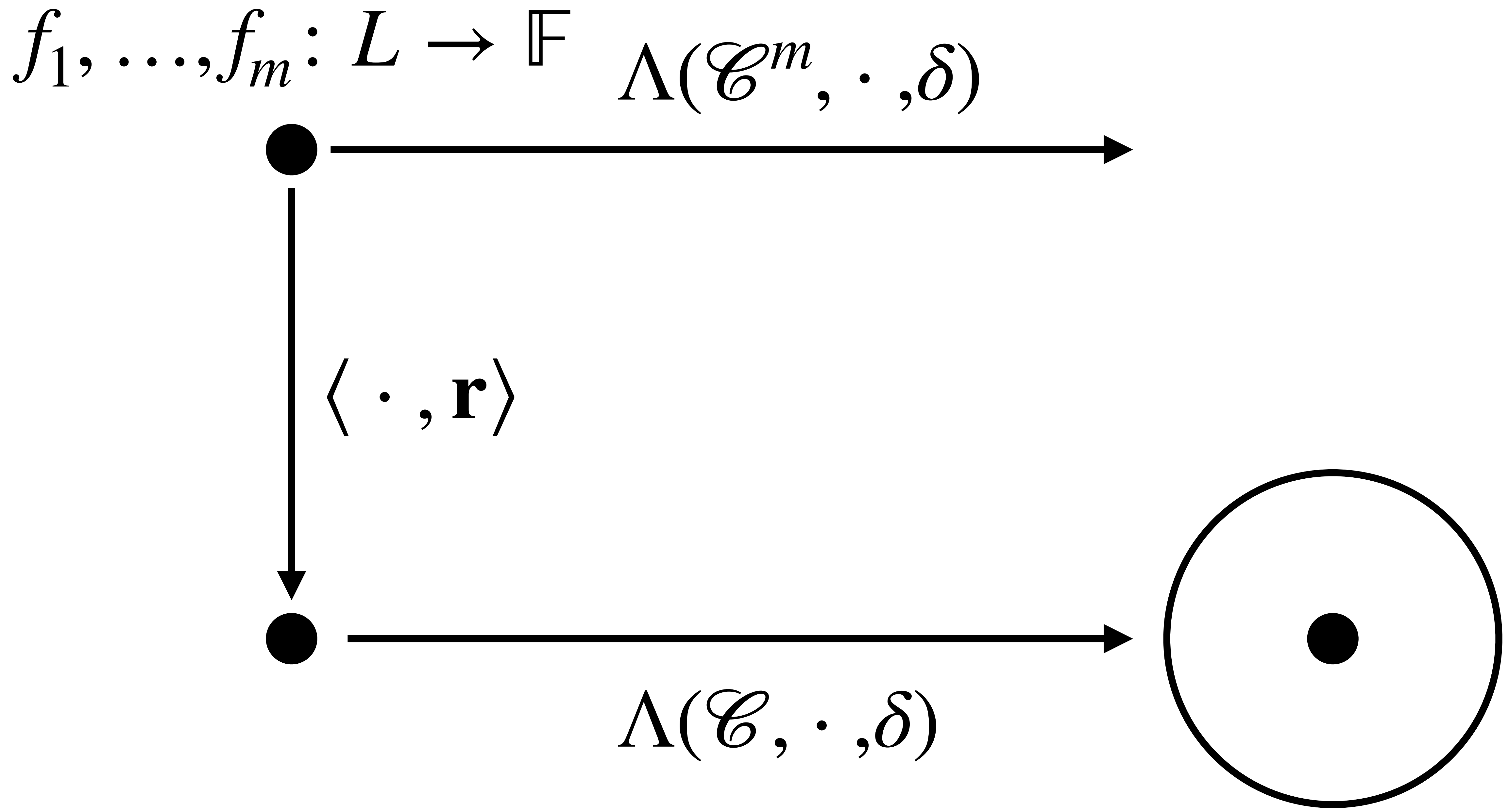
- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).

List-RLC lemma and List-Fold

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

Implied by mutual correlated agreement

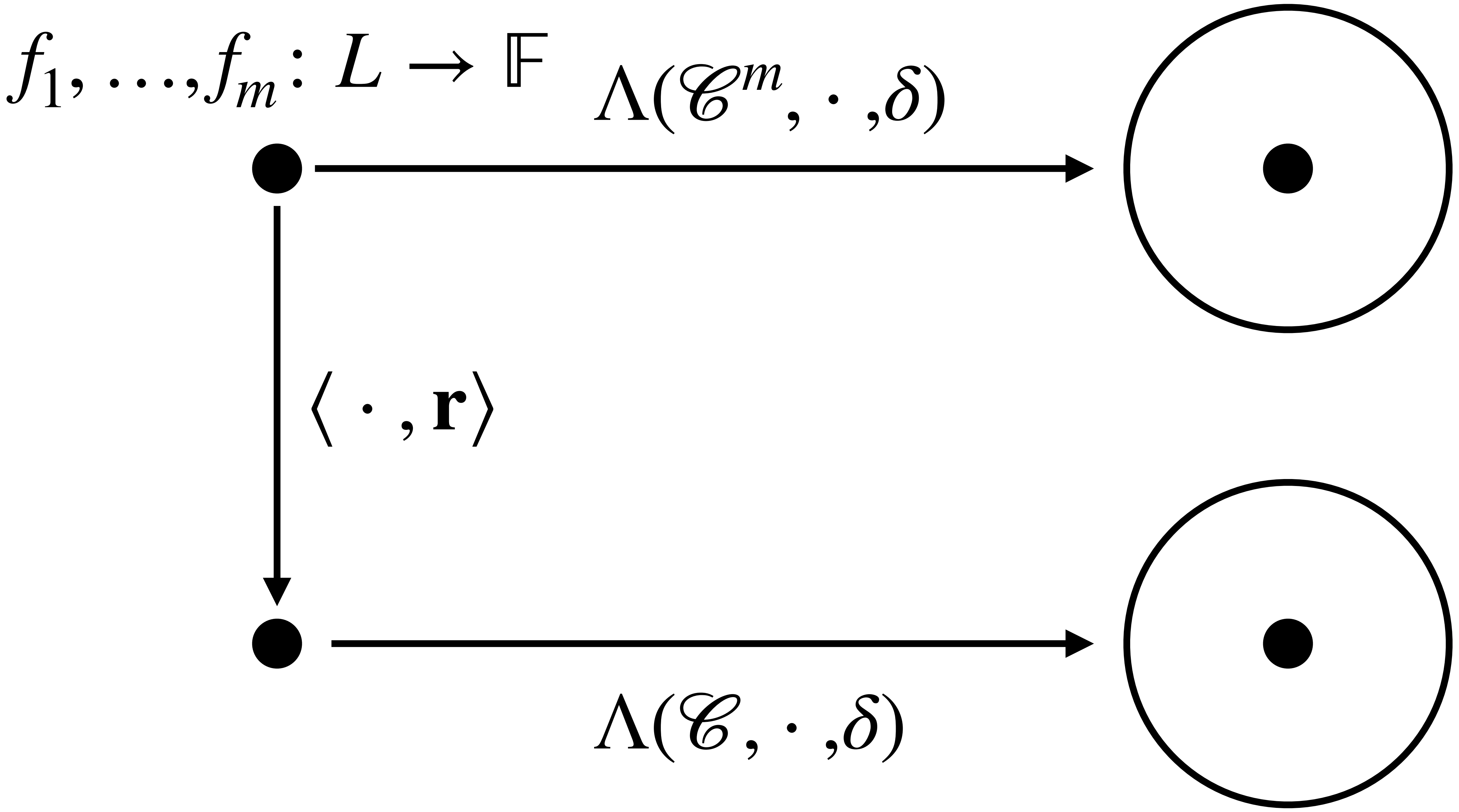
- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).



List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

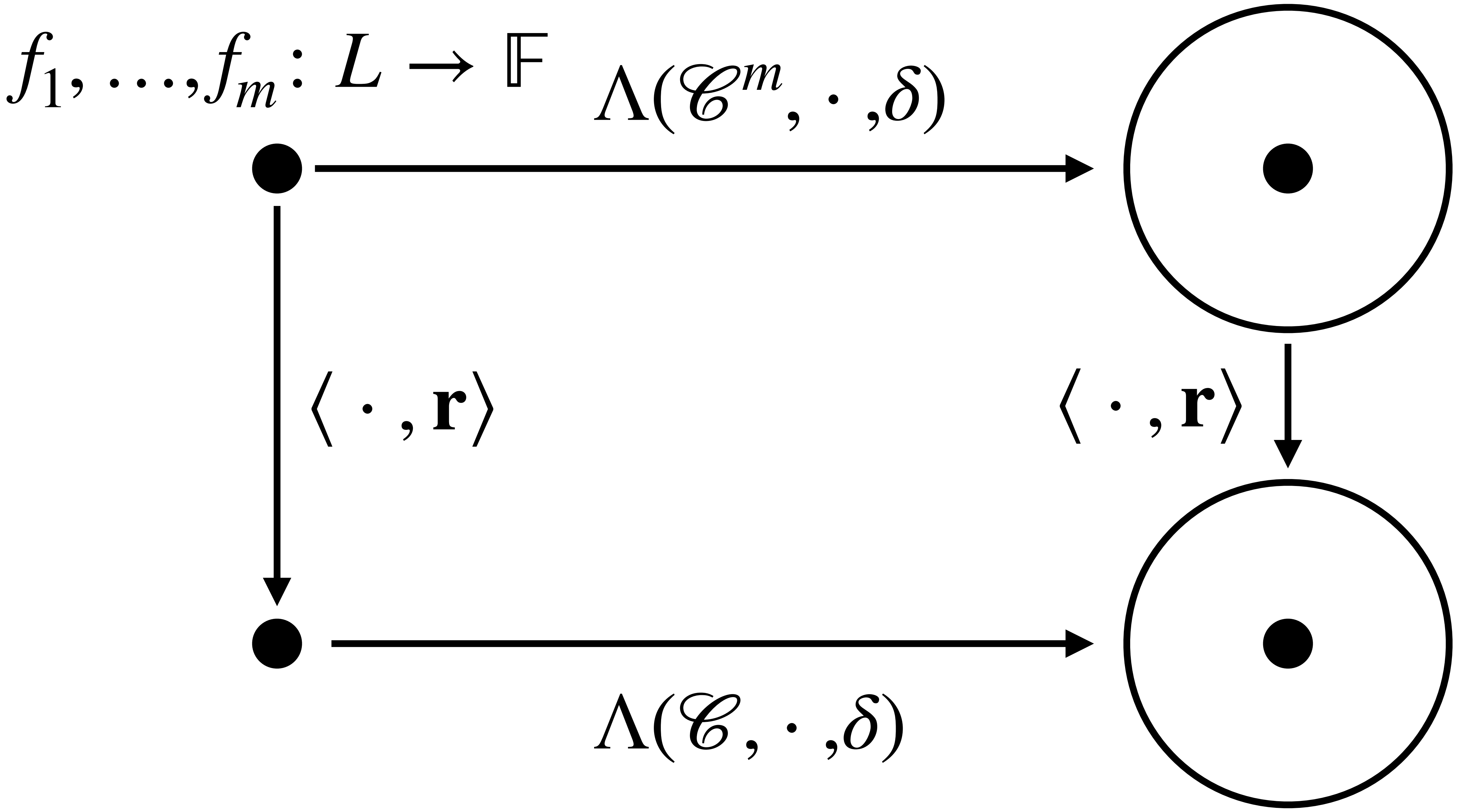


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

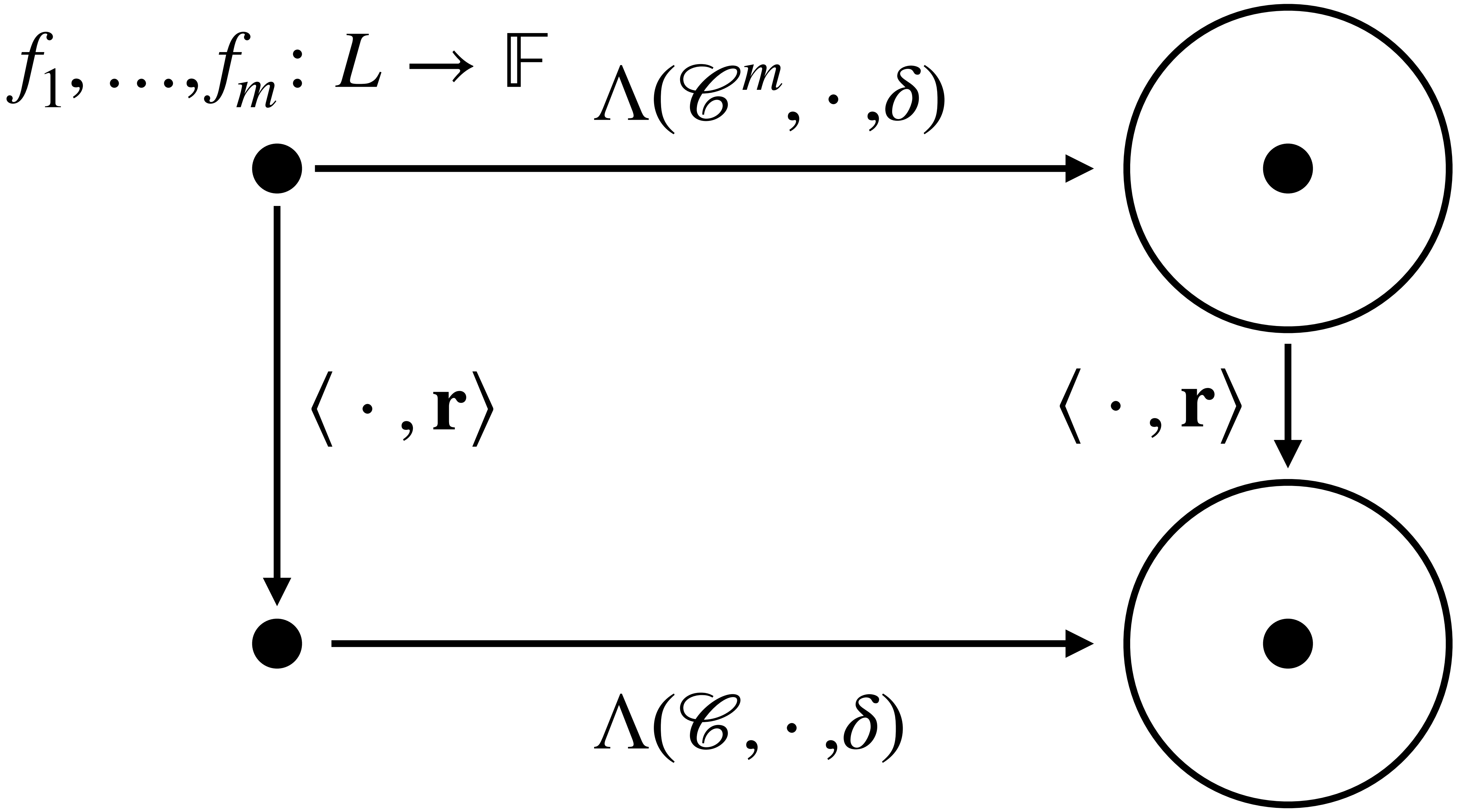


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

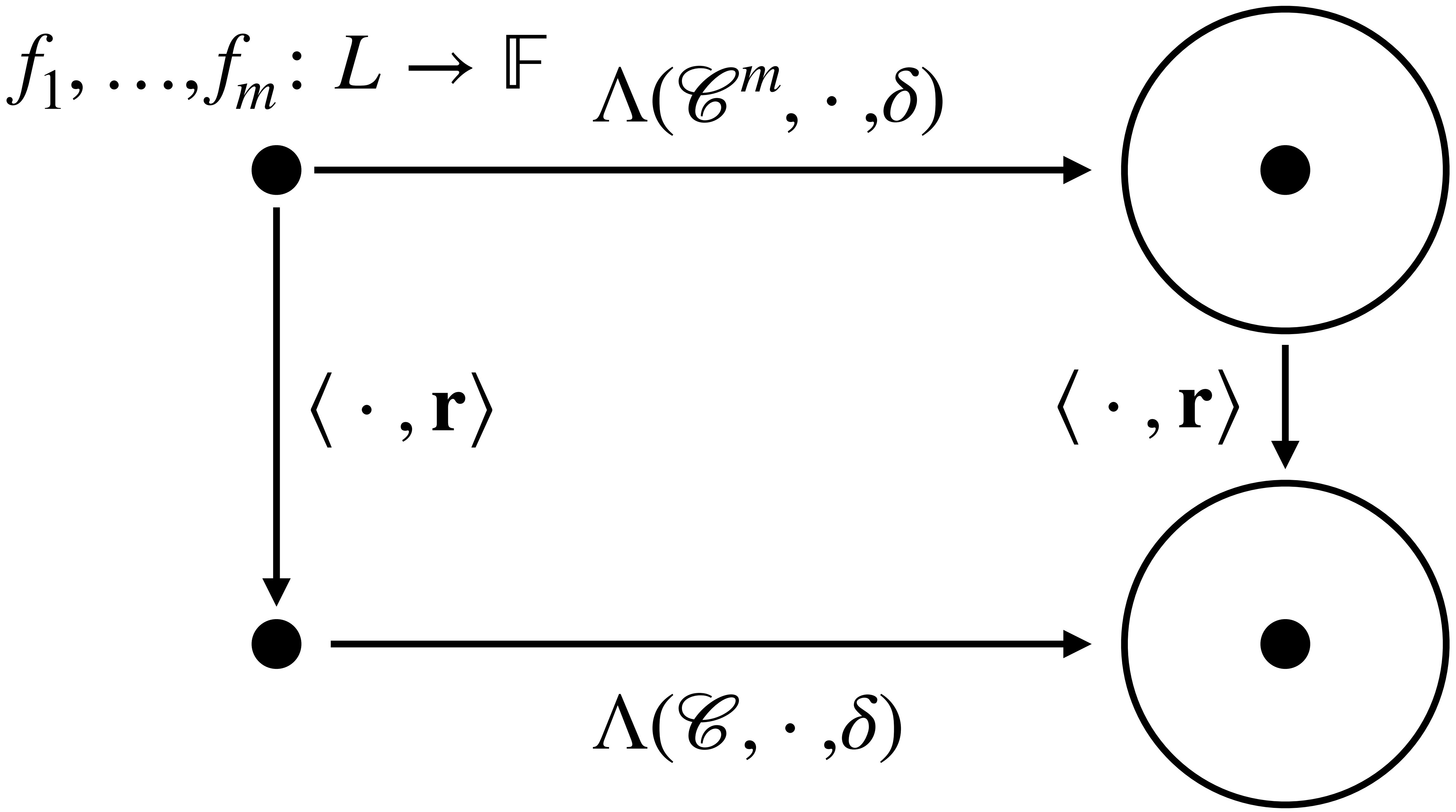


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

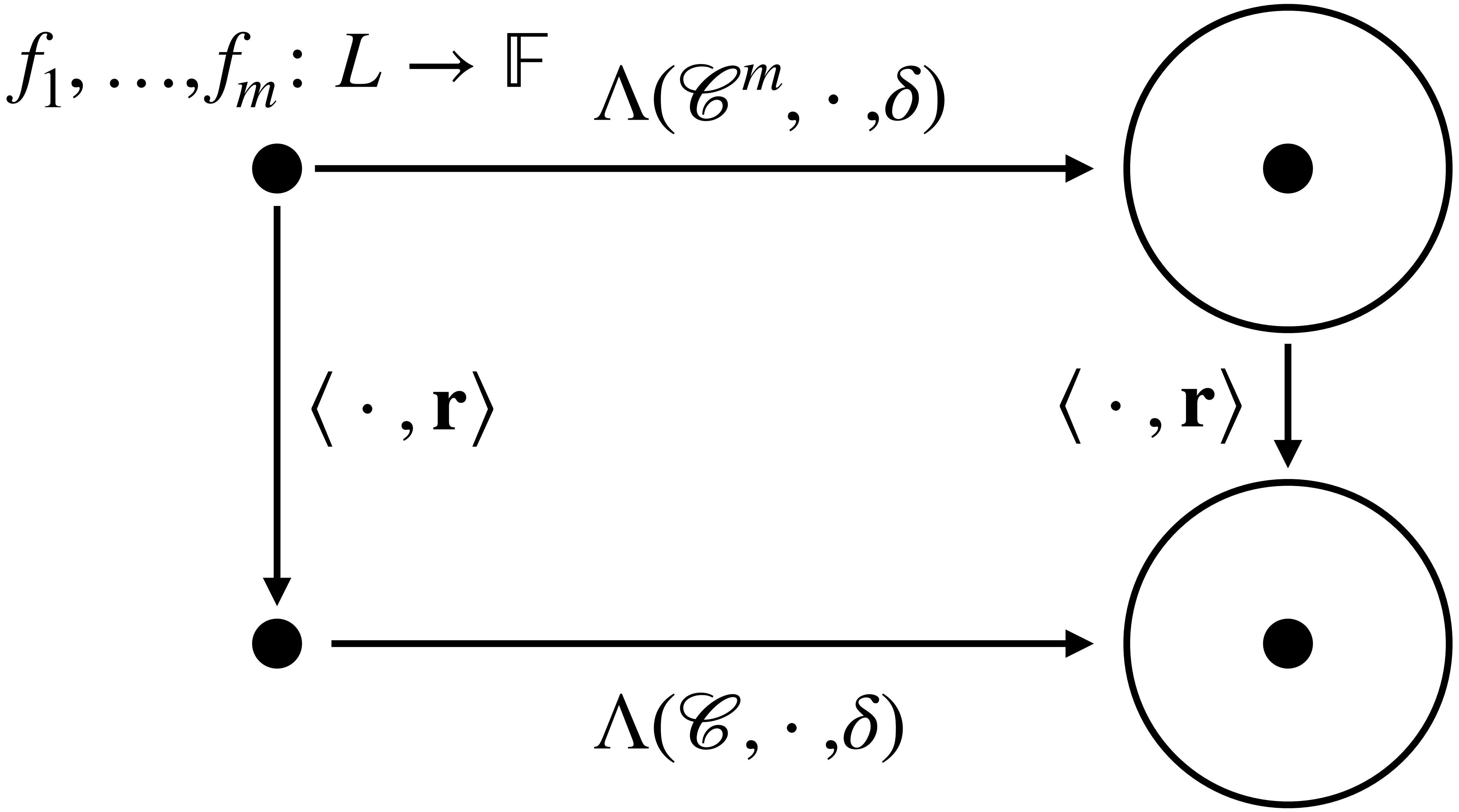


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$
- Folding version: w.h.p. over α: $\Lambda(\mathscr{C}, \mathsf{Fold}(f, \alpha), \delta) = \left\{ \mathsf{Fold}(u, \alpha) : u \in \Lambda(\mathscr{C}, f, \delta) \right\}$

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

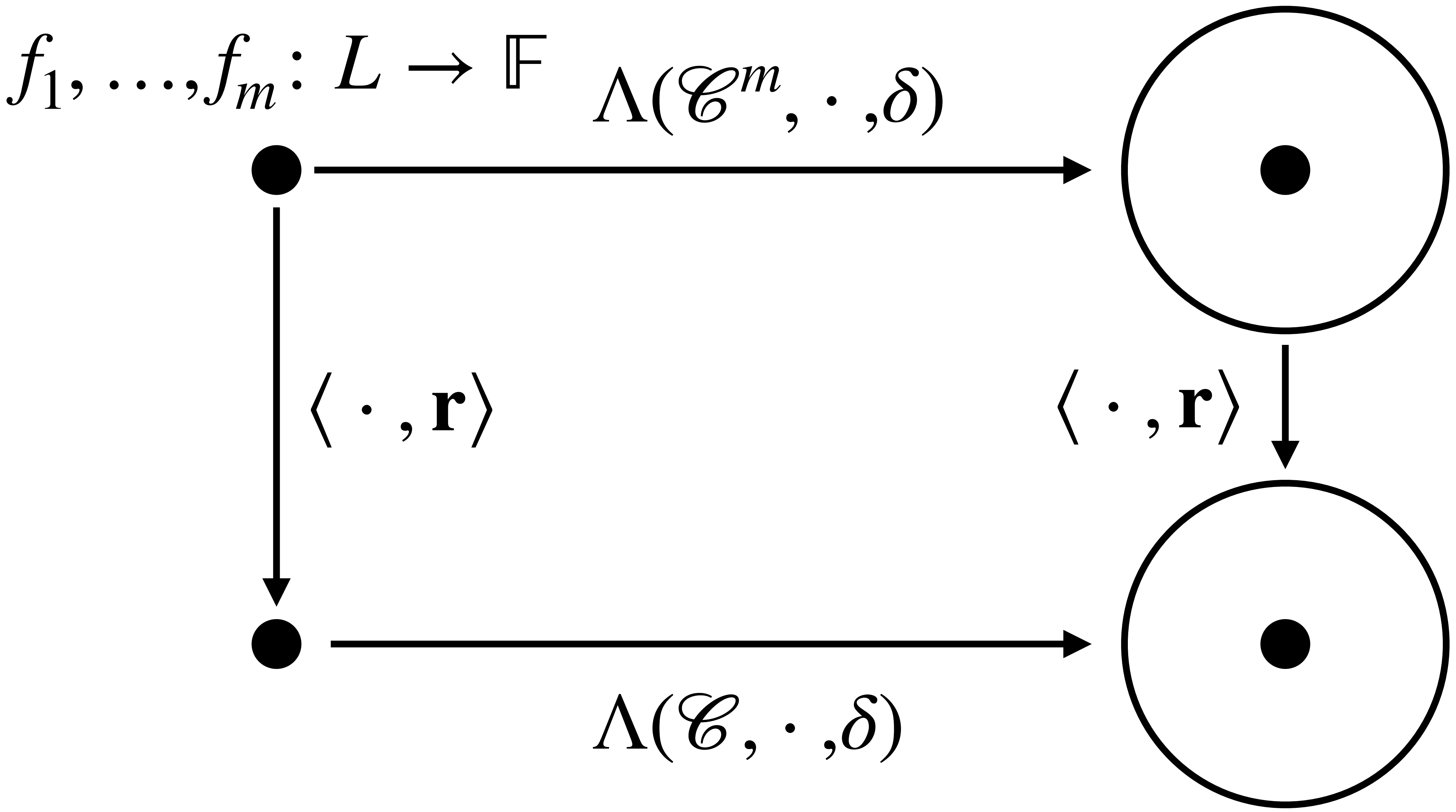


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$
- Folding version: w.h.p. over α: $\Lambda(\mathscr{C}, \mathsf{Fold}(f, \alpha), \delta) = \left\{ \mathsf{Fold}(u, \alpha) : u \in \Lambda(\mathscr{C}, f, \delta) \right\}$
- Alternatively, each term in the l.h.s can be “explained" by terms in the r.h.s.

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f

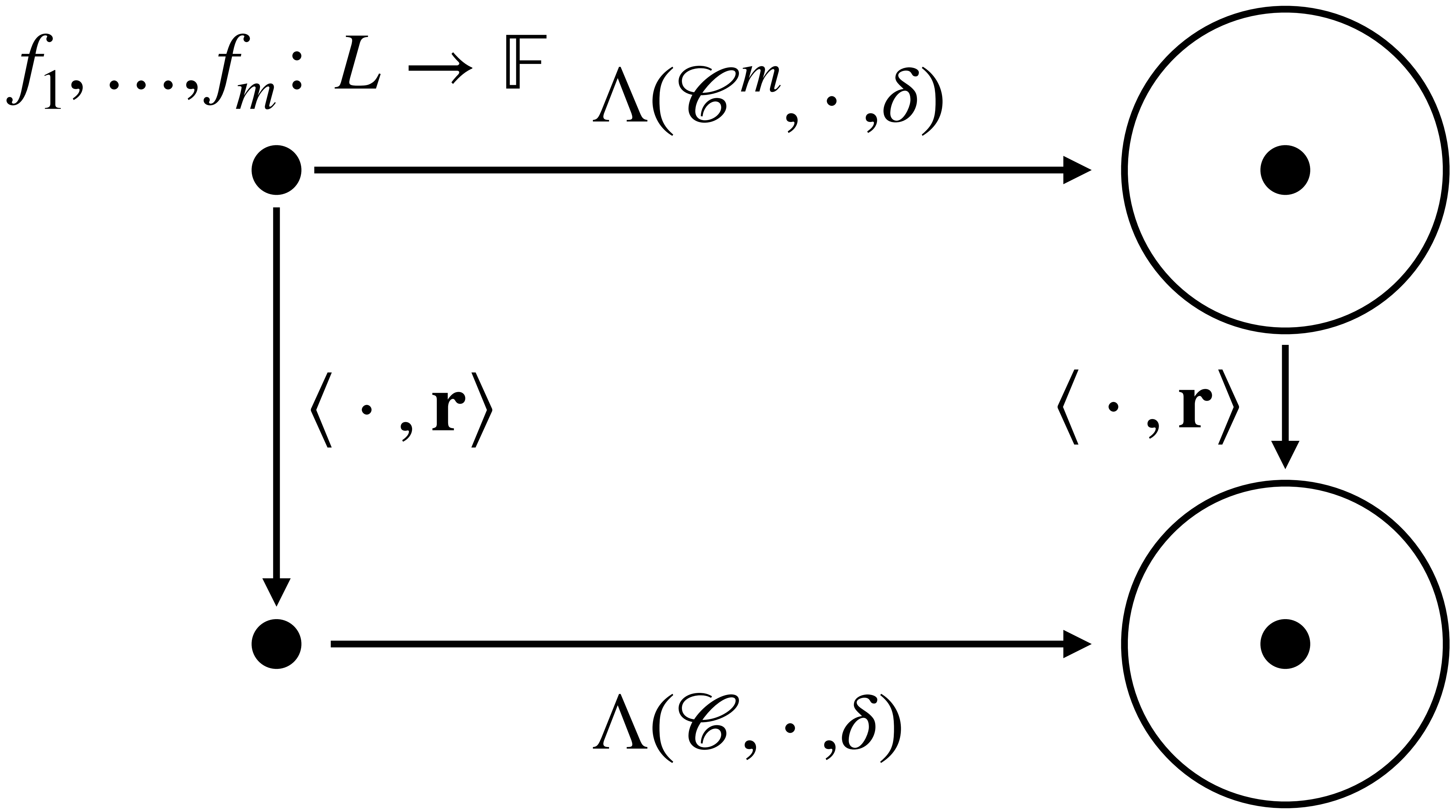


- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$
- Folding version: w.h.p. over α: $\Lambda(\mathscr{C}, \mathsf{Fold}(f, \alpha), \delta) = \left\{ \mathsf{Fold}(u, \alpha) : u \in \Lambda(\mathscr{C}, f, \delta) \right\}$
- Alternatively, each term in the l.h.s can be “explained" by terms in the r.h.s.
- We show correlated agreement implies mutual correlated agreement in *unique decoding.*

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f



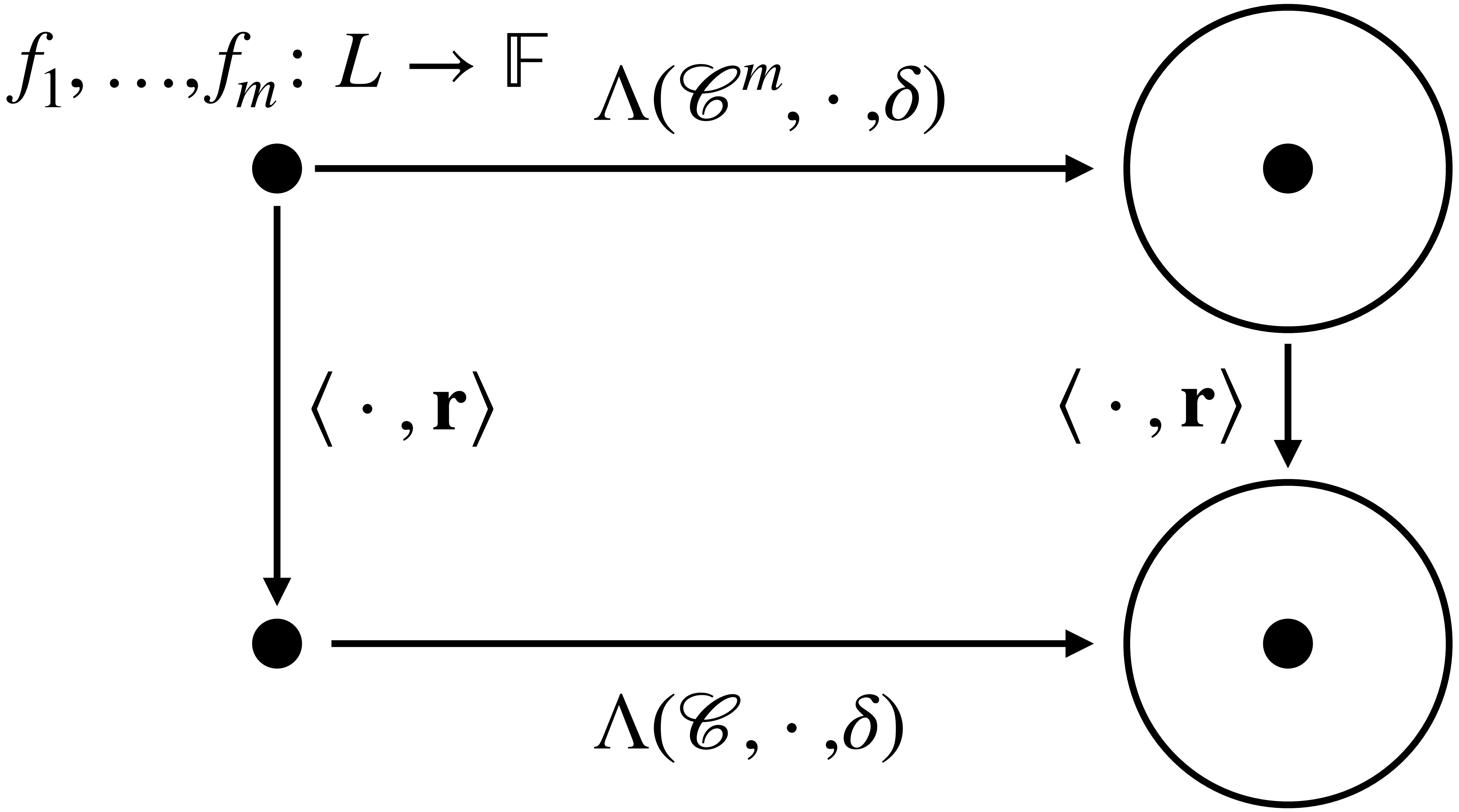
Stronger than what is required for STIR's soundness

- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$
- Folding version: w.h.p. over α: $\Lambda(\mathscr{C}, \mathsf{Fold}(f, \alpha), \delta) = \left\{ \mathsf{Fold}(u, \alpha) : u \in \Lambda(\mathscr{C}, f, \delta) \right\}$
- Alternatively, each term in the l.h.s can be “explained" by terms in the r.h.s.
- We show correlated agreement implies mutual correlated agreement in *unique decoding.*

List-RLC lemma and List-Fold

Implied by mutual correlated agreement

$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f



Stronger than what is required for STIR's soundness

- Taking lists and (random) combinations **commute** (if mutual correlated agreement holds).
- Random linear combination version: w.h.p. over $\mathbf{r}$: $\Lambda(\mathscr{C}, \langle \mathbf{f}, \mathbf{r} \rangle, \delta) = \left\{ \langle \mathbf{u}, \mathbf{r} \rangle : \mathbf{u} \in \Lambda(\mathscr{C}^m, \mathbf{f}, \delta) \right\}$
- Folding version: w.h.p. over α: $\Lambda(\mathscr{C}, \mathsf{Fold}(f, \alpha), \delta) = \left\{ \mathsf{Fold}(u, \alpha) : u \in \Lambda(\mathscr{C}, f, \delta) \right\}$
- Alternatively, each term in the l.h.s can be “explained" by terms in the r.h.s.
- We show correlated agreement implies mutual correlated agreement in *unique decoding.*

Recent results show it holds up to 1.5 Johnson for general linear codes!

WHIR Folding

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

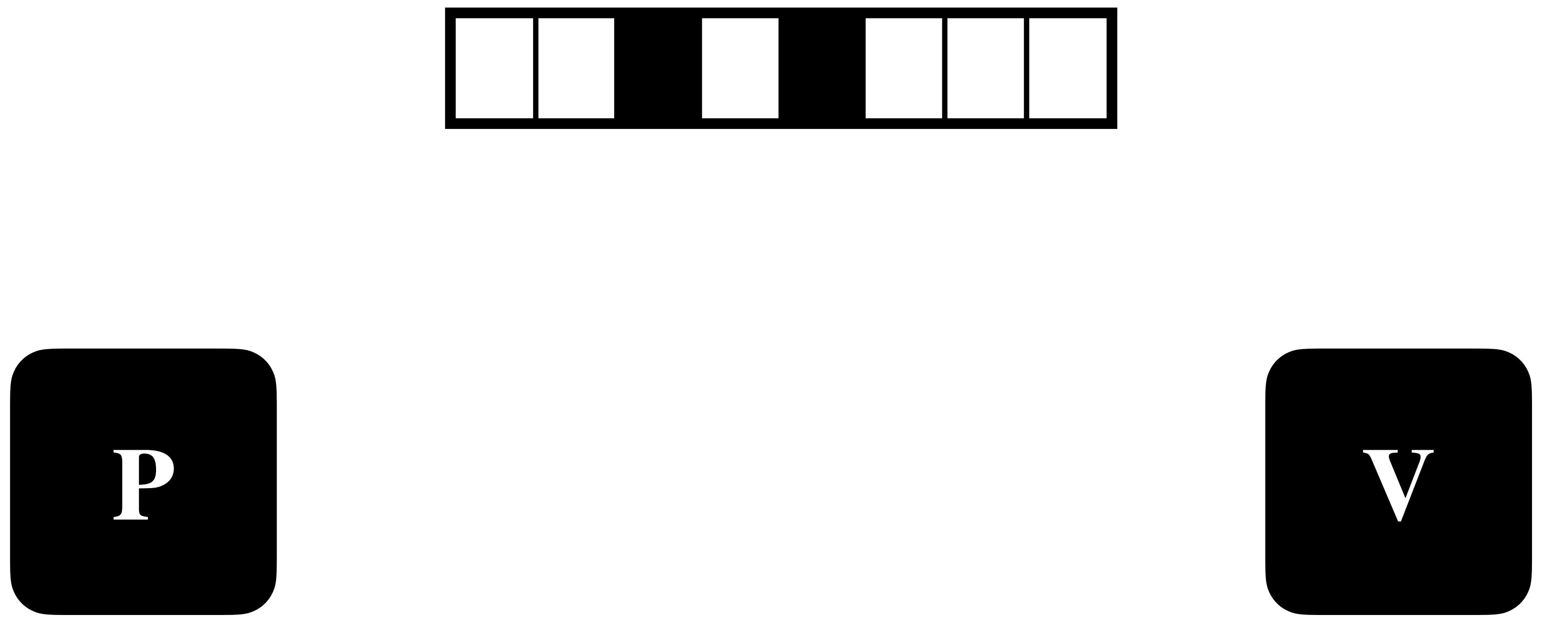
Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$f : L \to \mathbb{F}$$



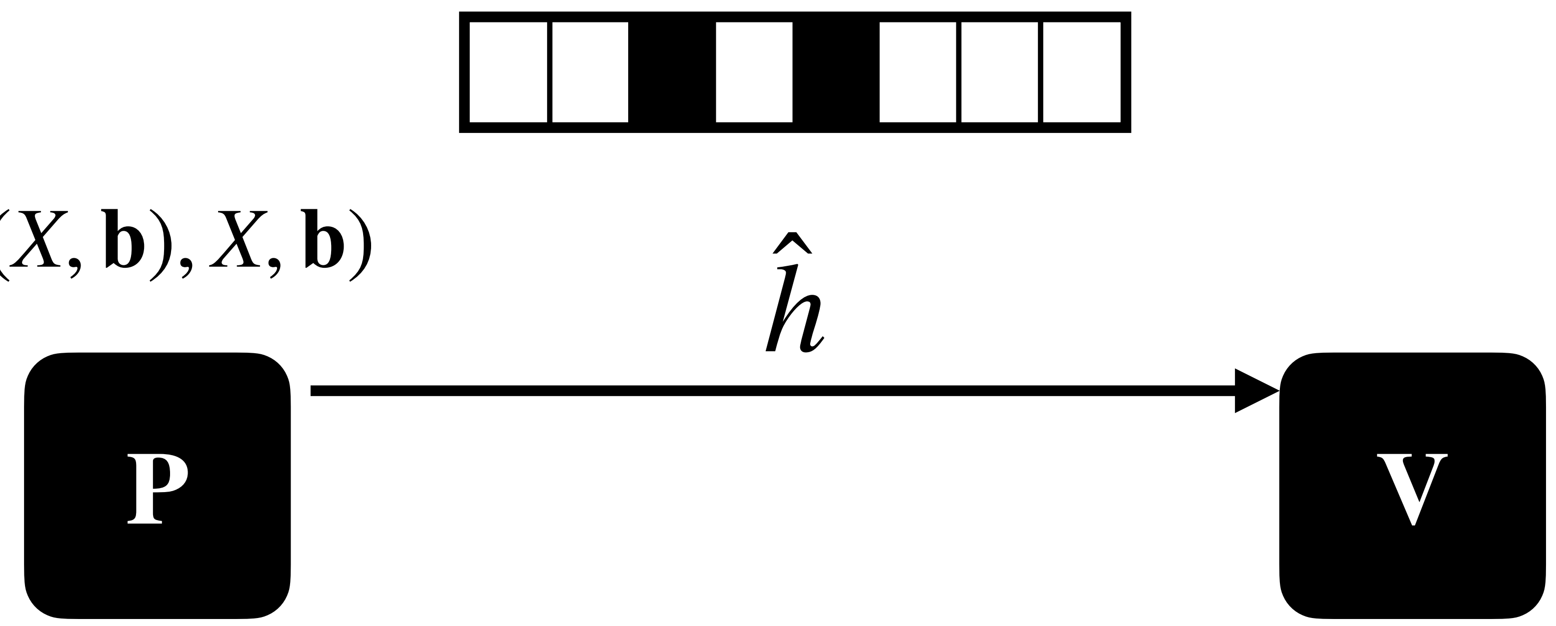
WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$f : L \to \mathbb{F}$$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



$$h(0) + h(1) =_? \sigma$$

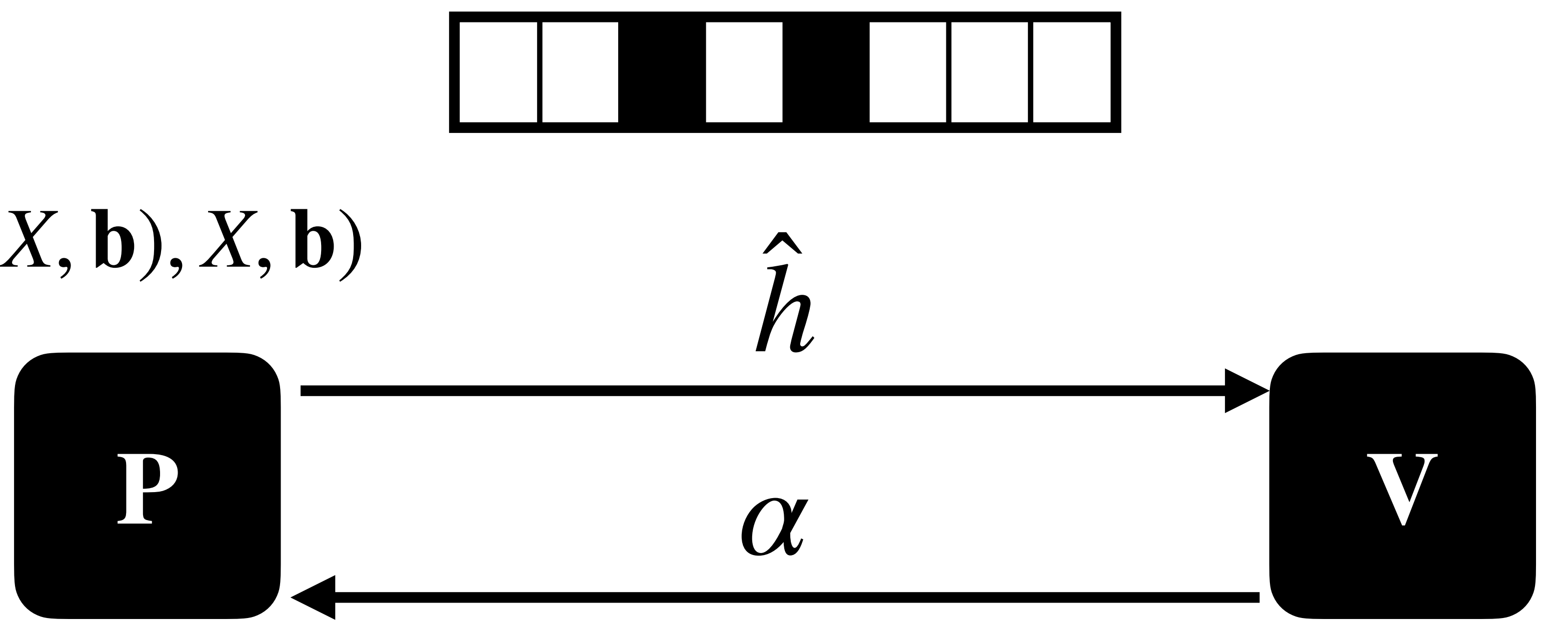
WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$f : L \to \mathbb{F}$$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



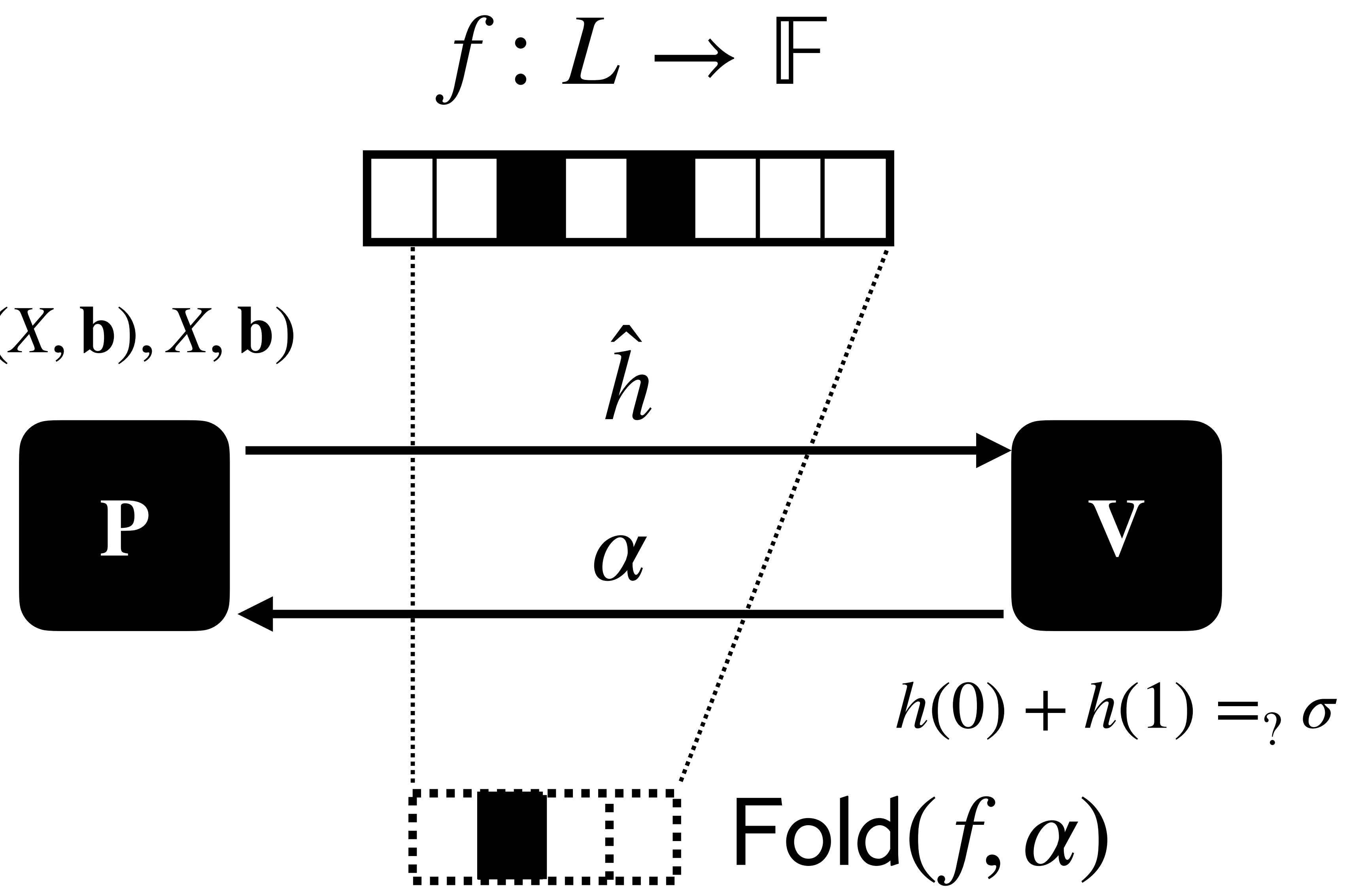
$$h(0) + h(1) =_? \sigma$$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b}\in\{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



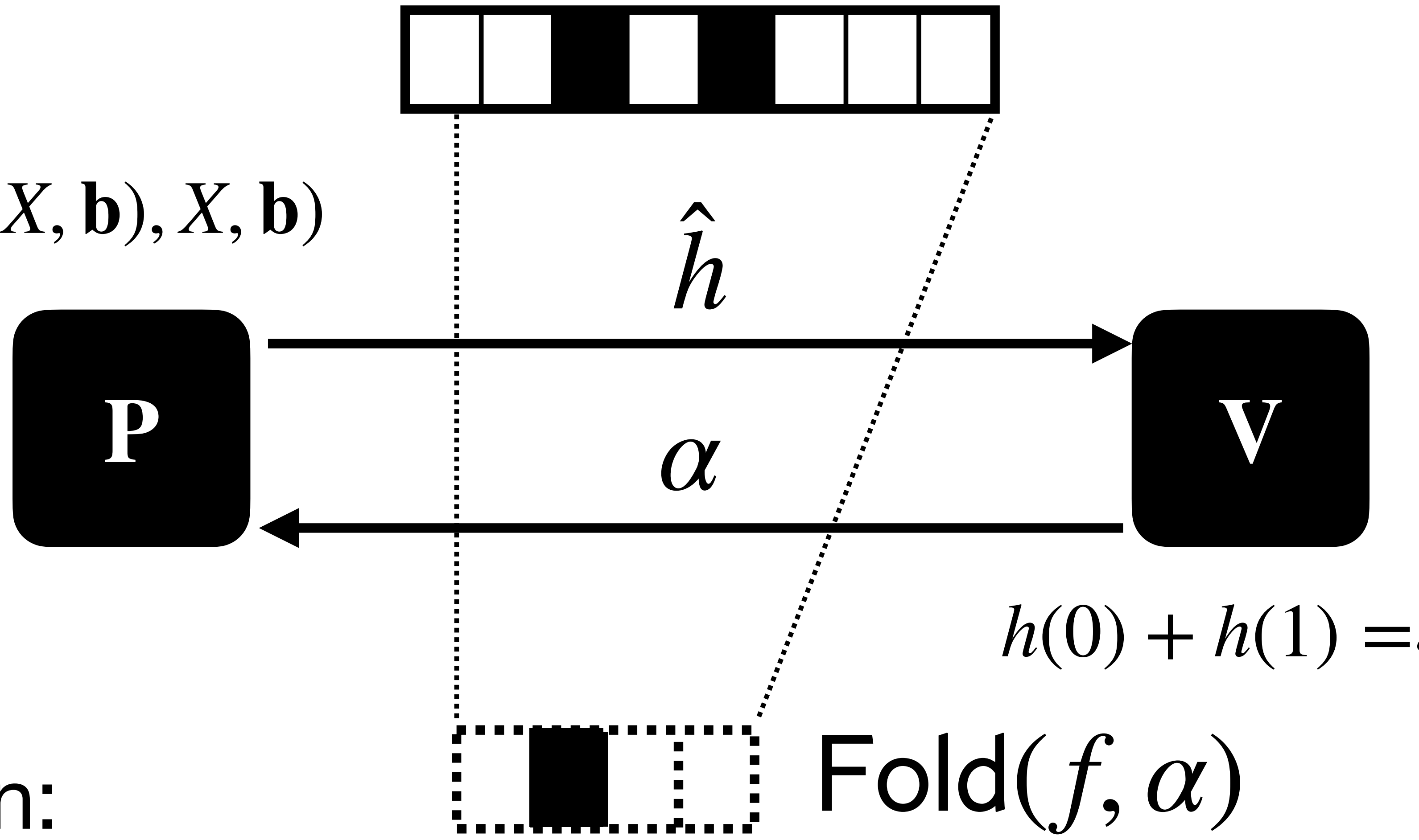
WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$f : L \to \mathbb{F}$$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



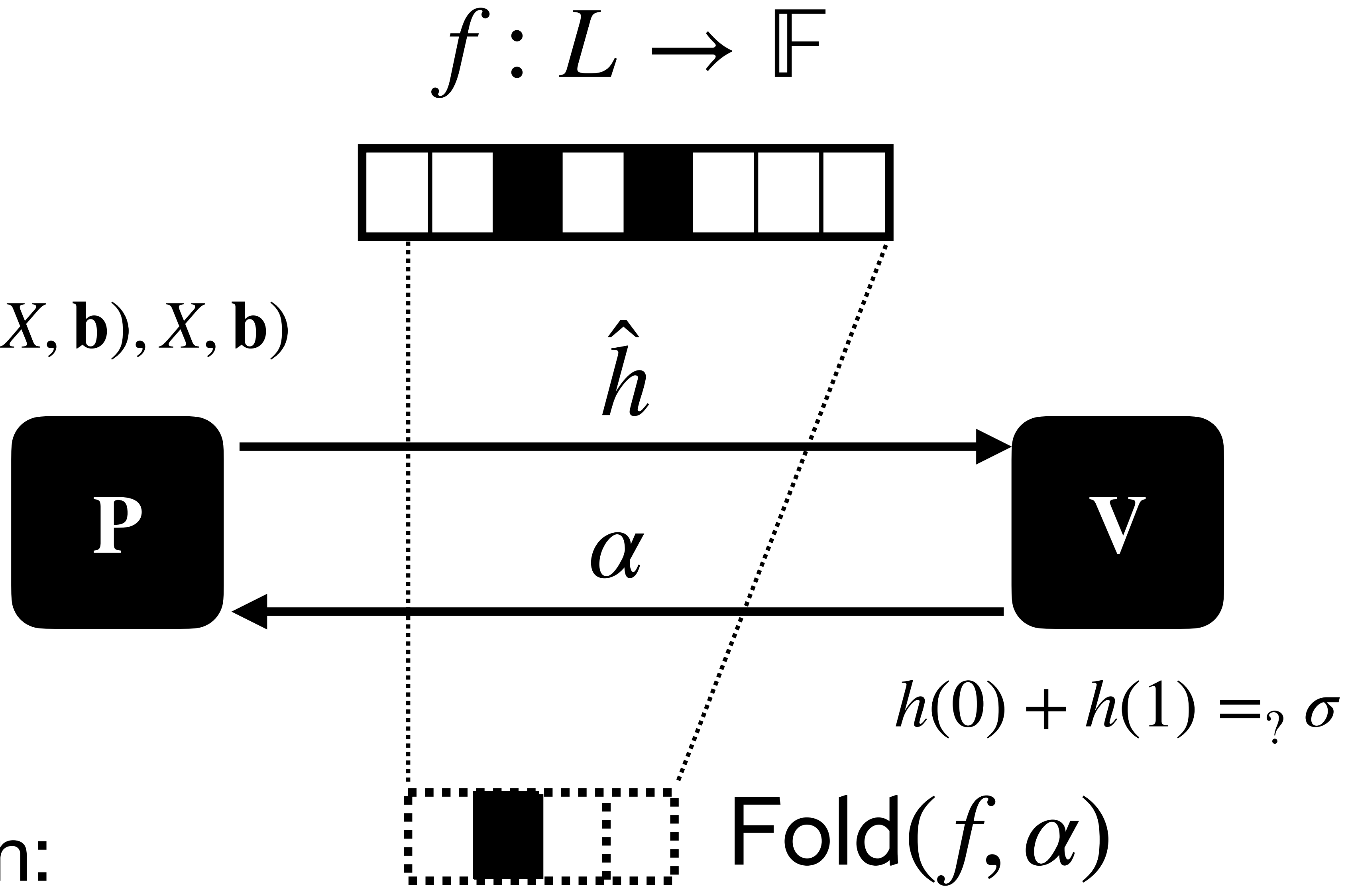
Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

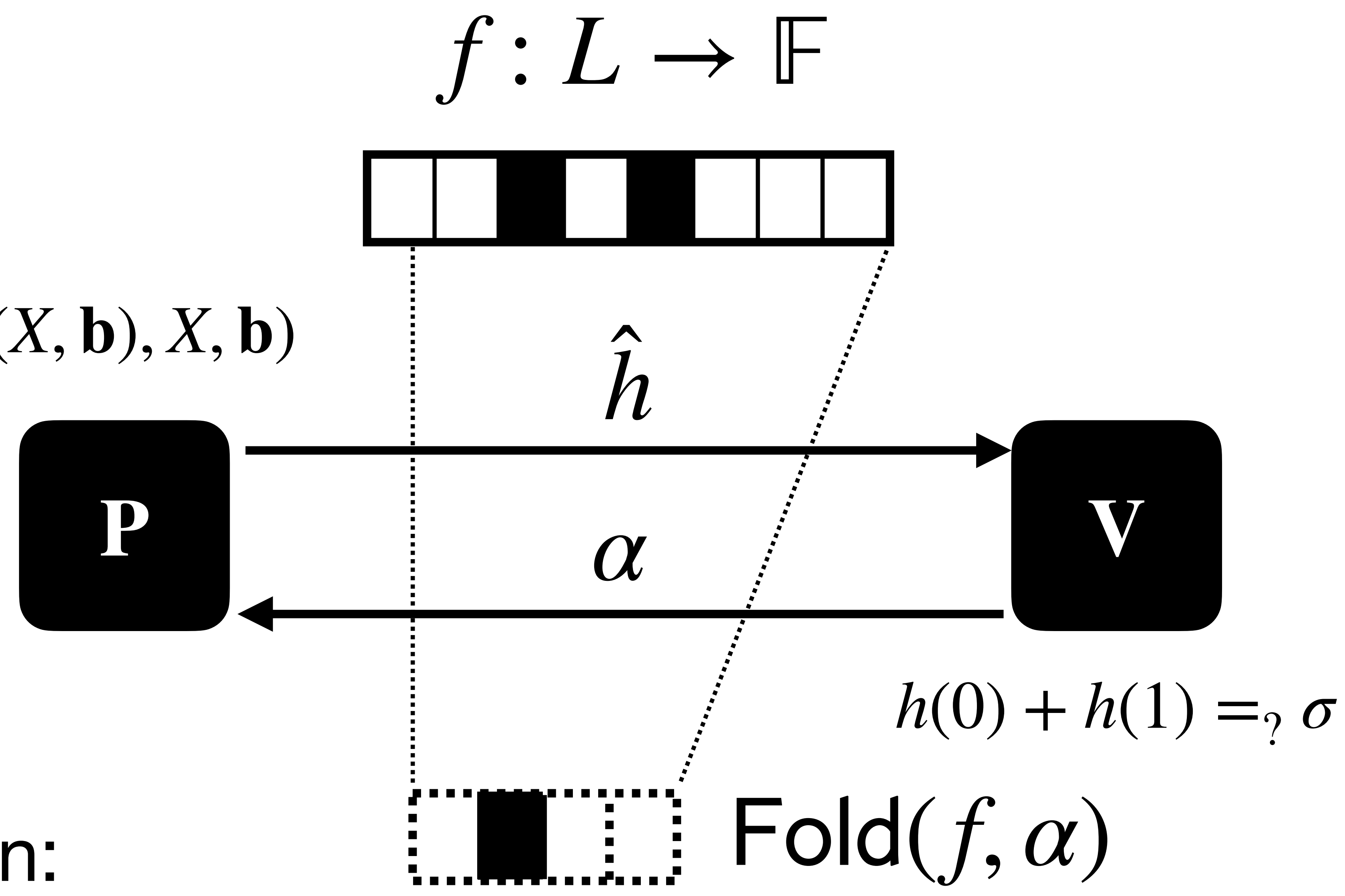
- $h(0) + h(1) = \sigma$,

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1,\ \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b}\in\{0,1\}^{m-1}} \hat{w}(\hat{f}(X,\mathbf{b}), X, \mathbf{b})$$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$

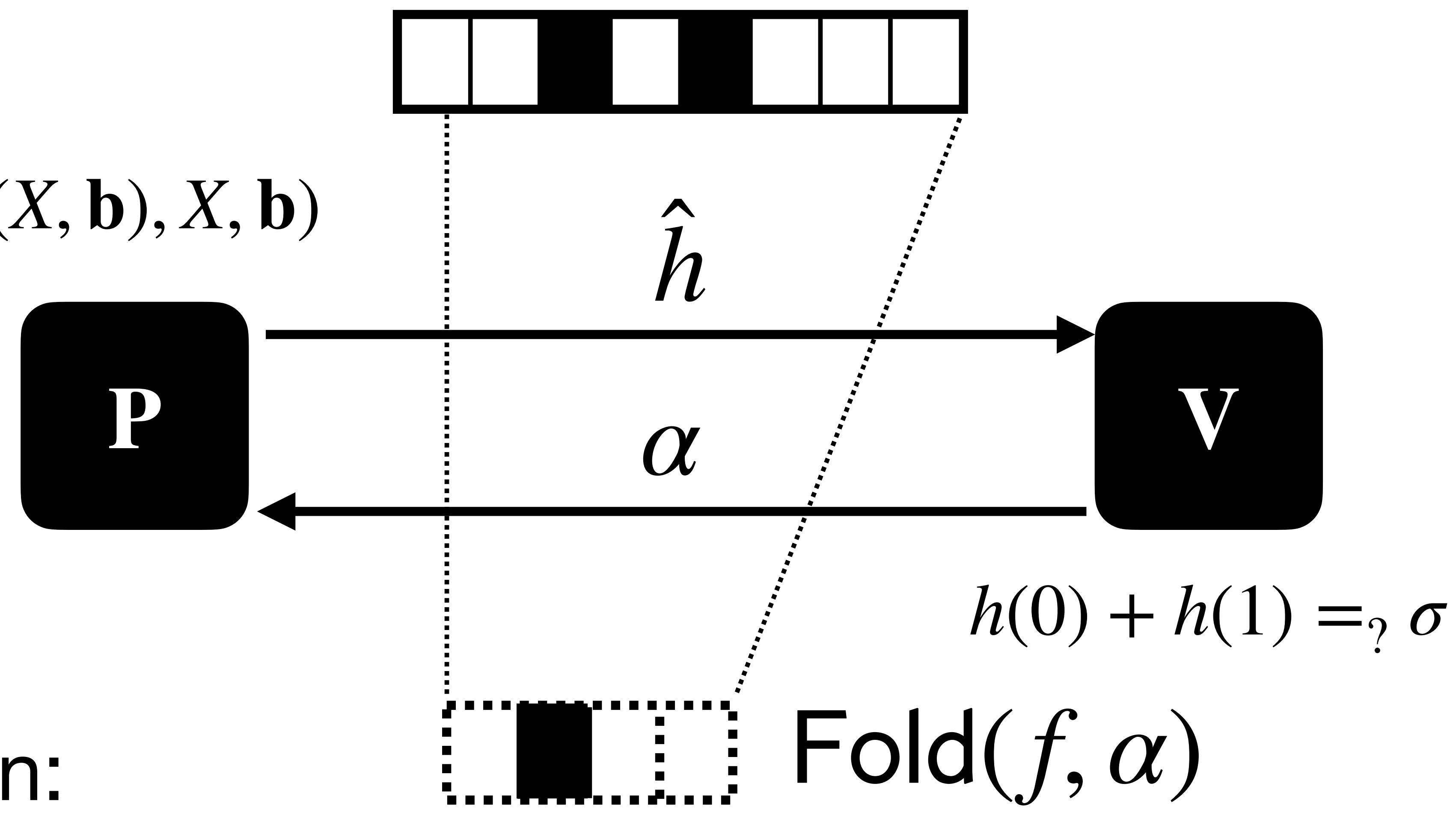
WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$f : L \to \mathbb{F}$

$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

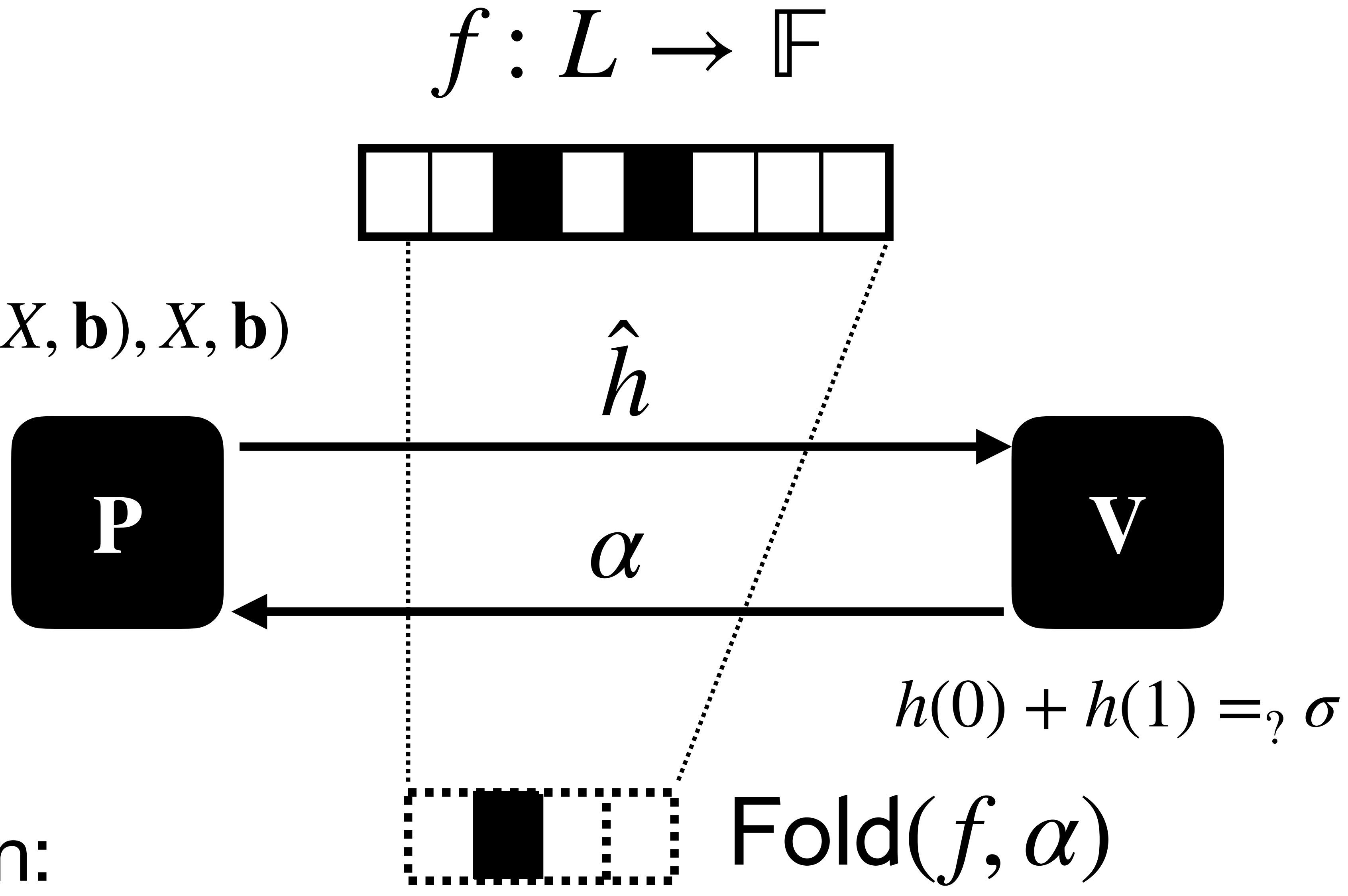
- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$
- $\widehat{\text{Fold}(f, \alpha)} = \hat{f}(\alpha, \cdot)$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$
- $\widehat{\mathsf{Fold}(f, \alpha)} = \hat{f}(\alpha, \cdot)$

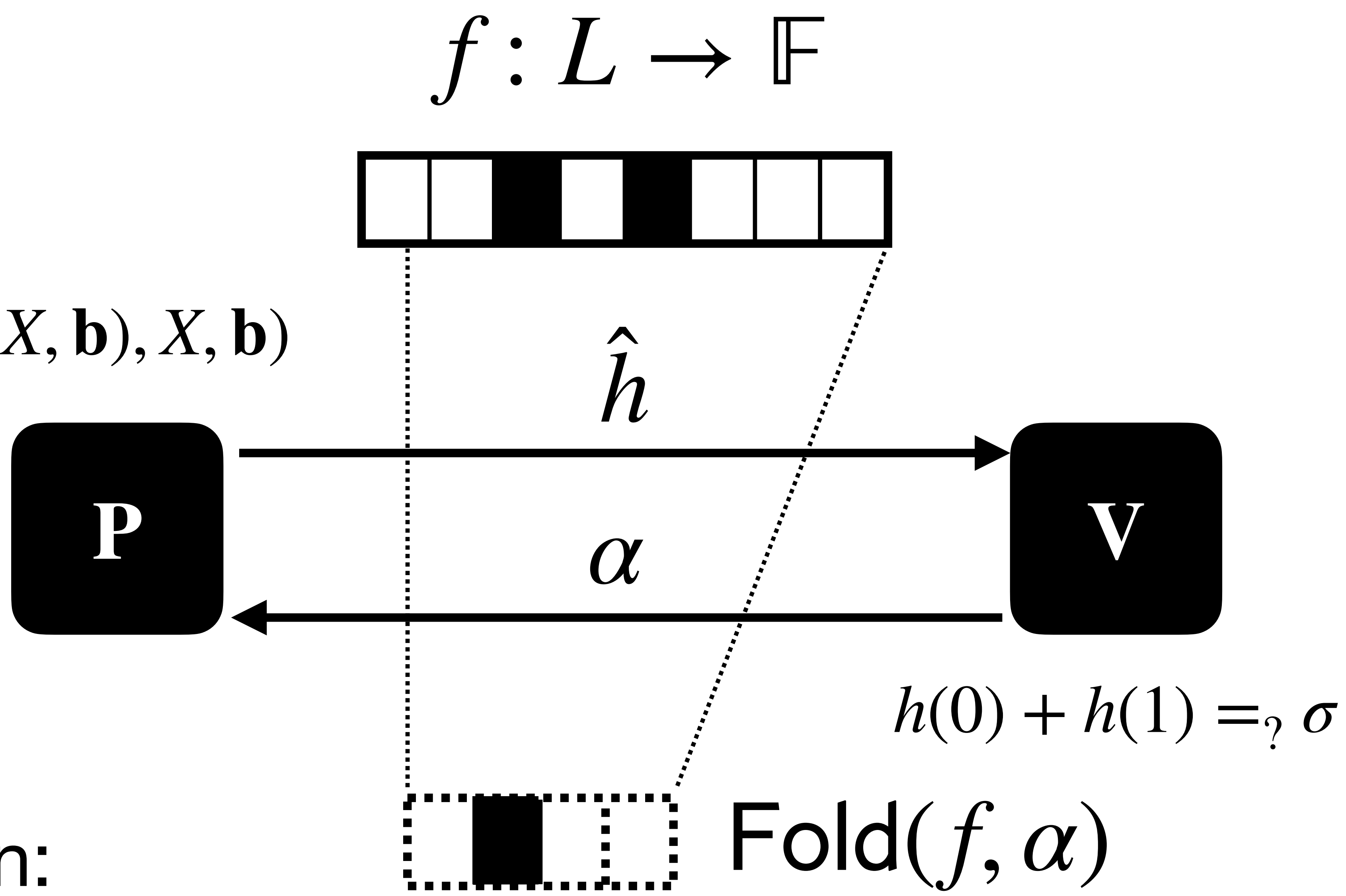
Soundness: by mutual correlated agreement, w.h.p. if $\Delta(f, \mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]) > \delta$ then $\Delta(\mathsf{Fold}(f, \alpha), \mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \hat{h}(\alpha)]) > \delta$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$
- $\widehat{\text{Fold}(f, \alpha)} = \hat{f}(\alpha, \cdot\,)$

Soundness: by mutual correlated agreement, w.h.p. if $\Delta(f, \text{CRS}[n, m, \rho, \hat{w}, \sigma]) > \delta$ then $\Delta(\text{Fold}(f, \alpha), \text{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \hat{h}(\alpha)]) > \delta$

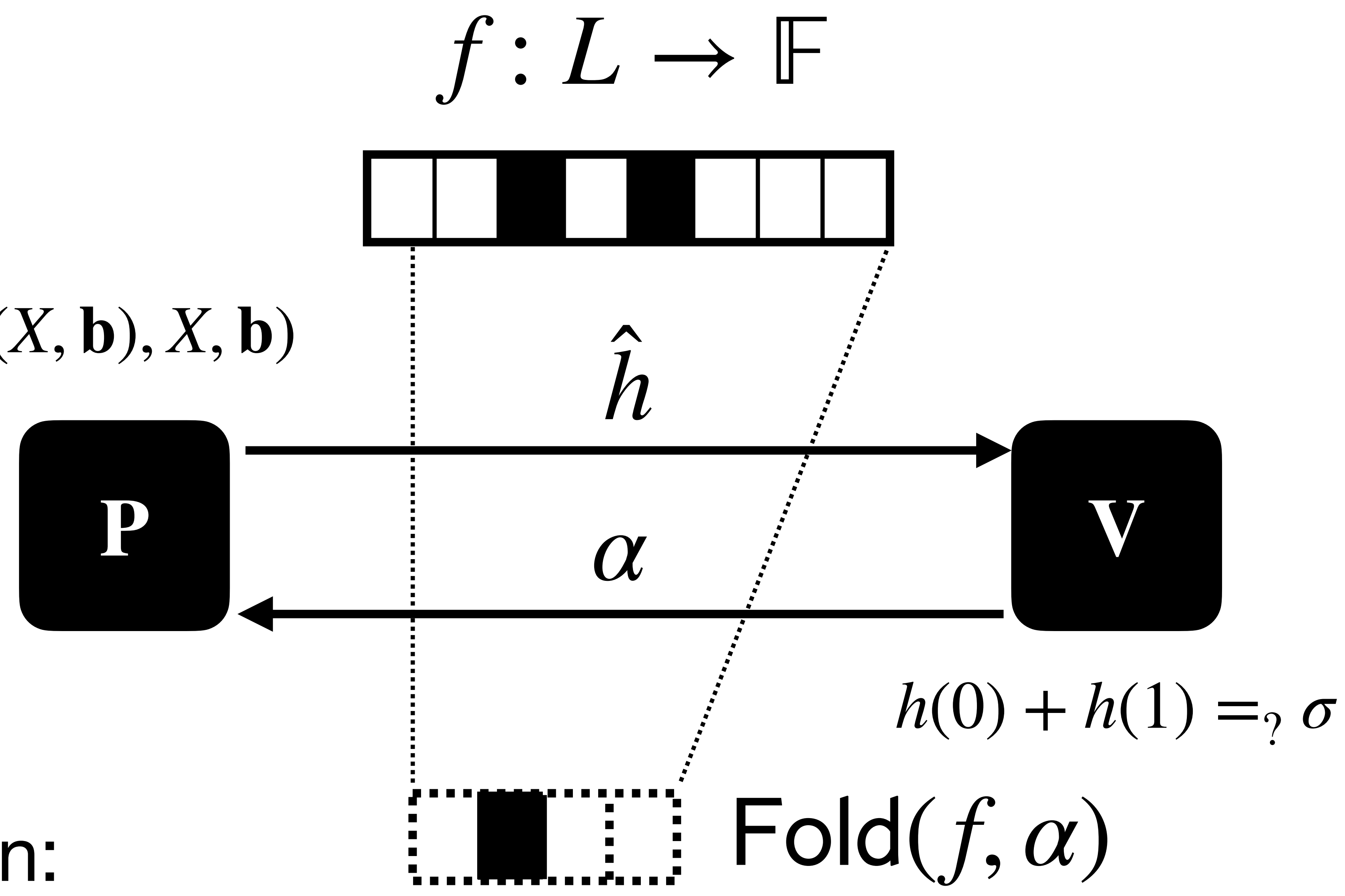
$\hat{w}_\alpha(Z, \mathbf{X}) = \hat{w}(Z, \alpha, \mathbf{X})$

WHIR Folding

Interleave sumcheck with FRI folding, similar to BaseFold, Hyperplonk, Gemini

Reduce CRS$[n, m, \rho, \hat{w}, \sigma]$ **to** CRS$[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$
- $\widehat{\text{Fold}(f, \alpha)} = \hat{f}(\alpha, \cdot\,)$

Soundness: by mutual correlated agreement, w.h.p. if $\Delta(f, \text{CRS}[n, m, \rho, \hat{w}, \sigma]) > \delta$ then $\Delta(\text{Fold}(f, \alpha), \text{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \hat{h}(\alpha)]) > \delta$

$\hat{w}_\alpha(Z, \mathbf{X}) = \hat{w}(Z, \alpha, \mathbf{X})$

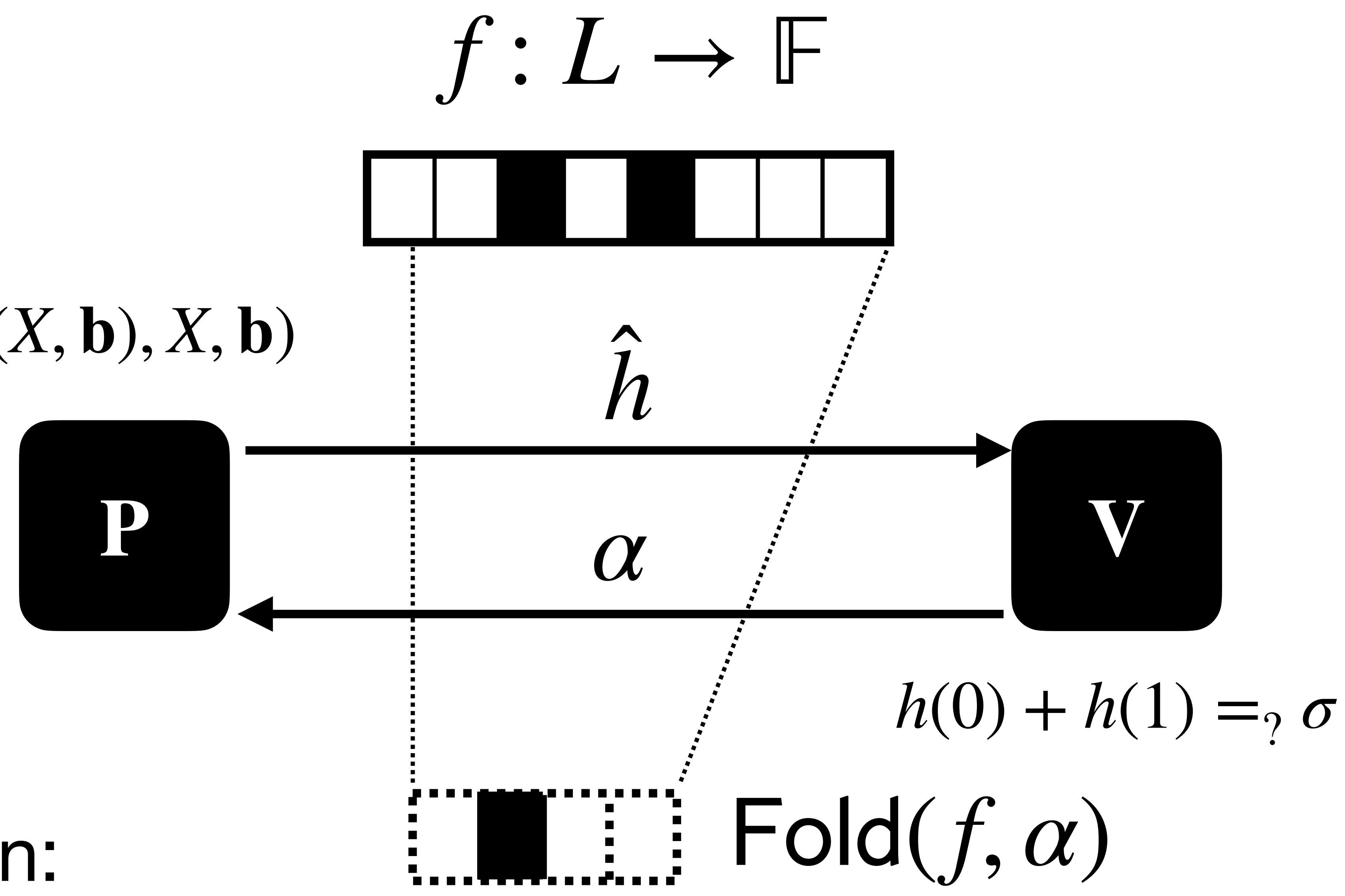
Unchanged!

WHIR Folding

Interleave sumcheck with FRI folding,
similar to BaseFold, Hyperplonk, Gemini

Reduce $\mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]$ **to** $\mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \sigma_\alpha]$

$$\hat{h}(X) := \sum_{\mathbf{b} \in \{0,1\}^{m-1}} \hat{w}(\hat{f}(X, \mathbf{b}), X, \mathbf{b})$$



In the full protocol, we fold by 2-by-2 k times. Can also fold by 2^k at a time (nice for first round!)

Completeness: $\sum_{\mathbf{b}} \hat{w}(f(\mathbf{b}), \mathbf{b}) = \sigma$ then:

- $h(0) + h(1) = \sigma$,
- $\sum_{\mathbf{b}} \hat{w}(f(\alpha, \mathbf{b}), \alpha, \mathbf{b}) = \hat{h}(\alpha)$
- $\widehat{\mathsf{Fold}(f, \alpha)} = \hat{f}(\alpha, \cdot\,)$

Soundness: by mutual correlated agreement, w.h.p. if $\Delta(f, \mathsf{CRS}[n, m, \rho, \hat{w}, \sigma]) > \delta$ then $\Delta(\mathsf{Fold}(f, \alpha), \mathsf{CRS}[n/2, m-1, \rho, \hat{w}_\alpha, \hat{h}(\alpha)]) > \delta$

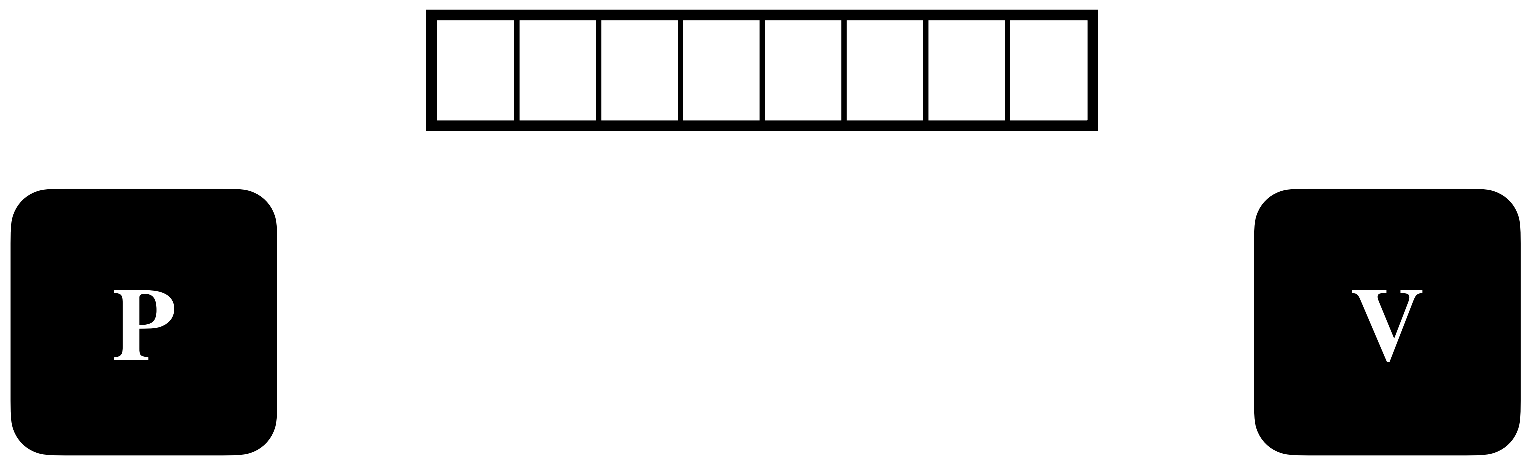
$\hat{w}_\alpha(Z, \mathbf{X}) = \hat{w}(Z, \alpha, \mathbf{X})$

Unchanged!

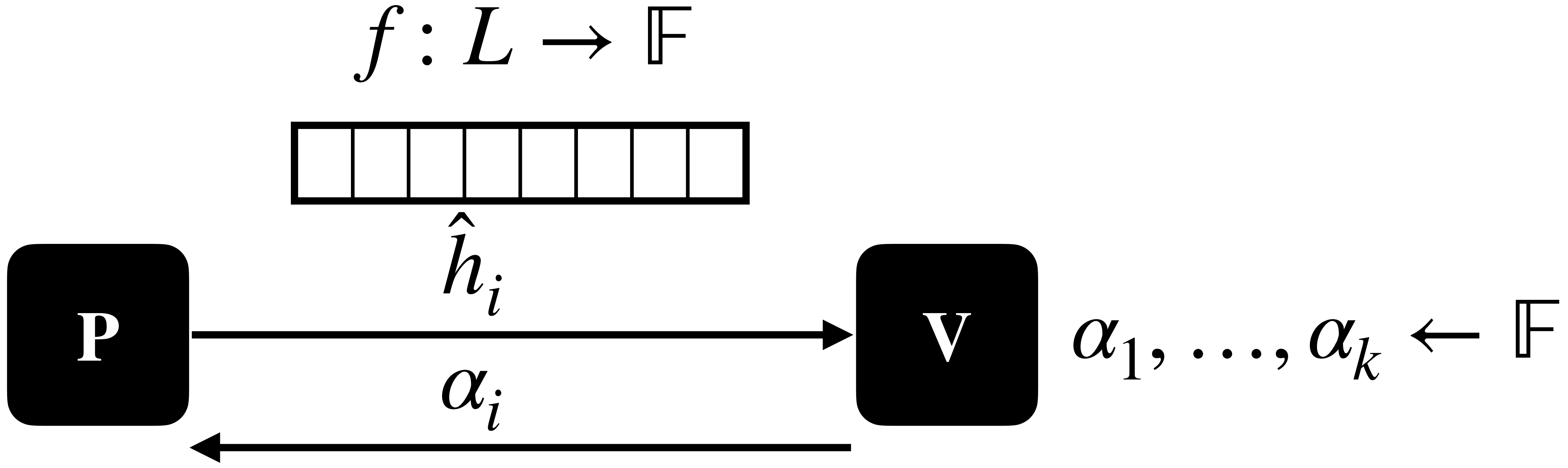
WHIR iteration

WHIR iteration

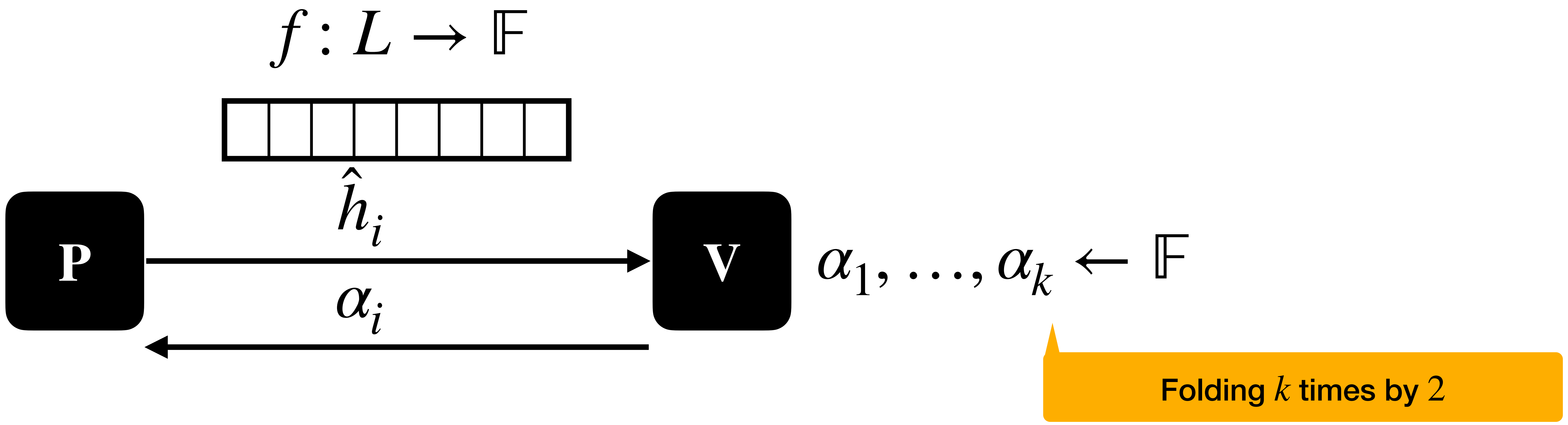
$f : L \to \mathbb{F}$



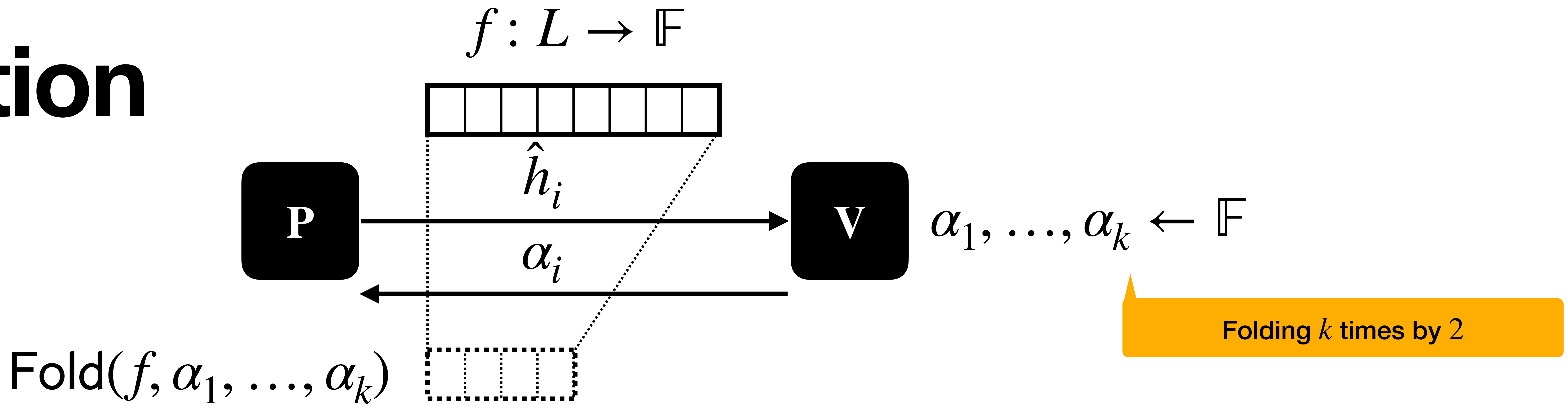
WHIR iteration



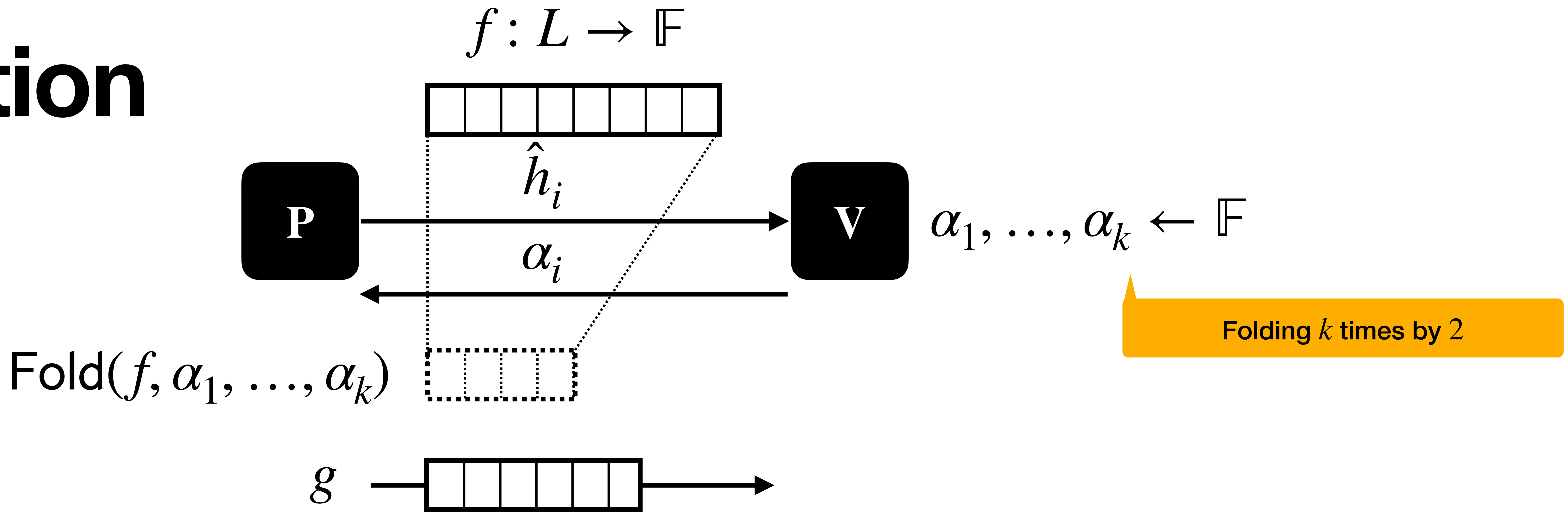
WHIR iteration



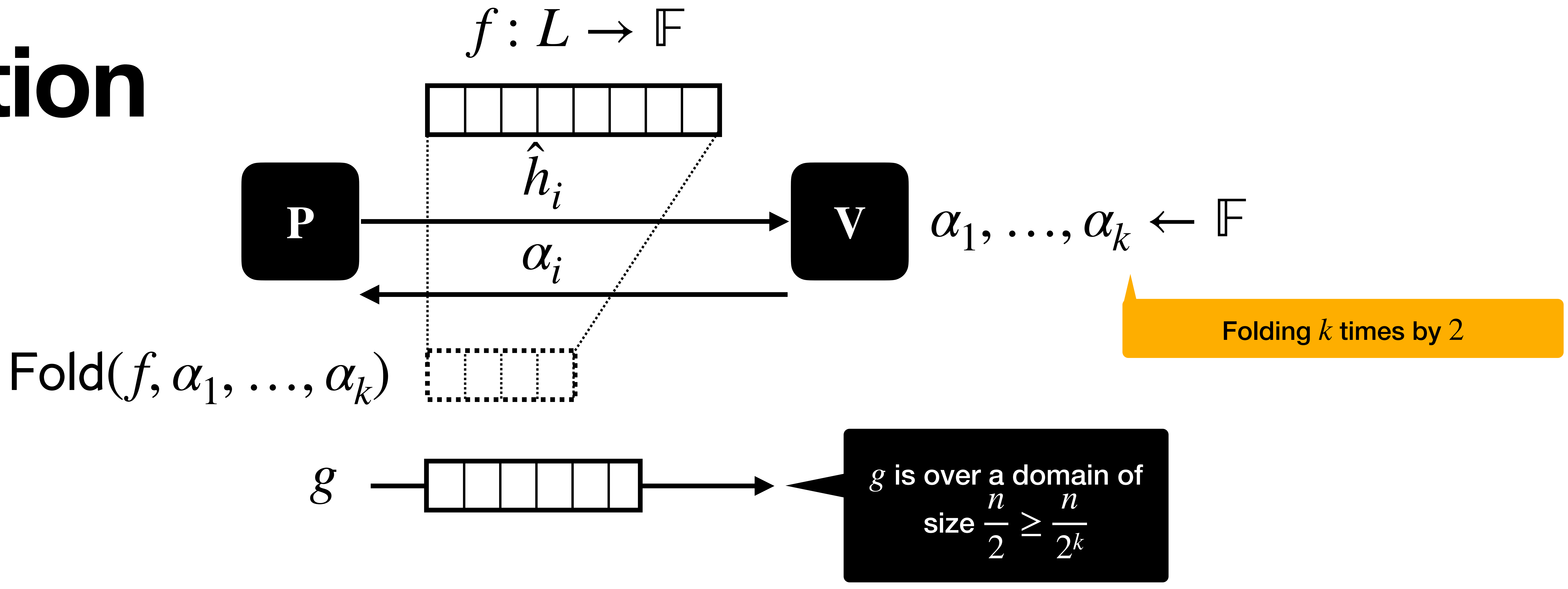
WHIR iteration



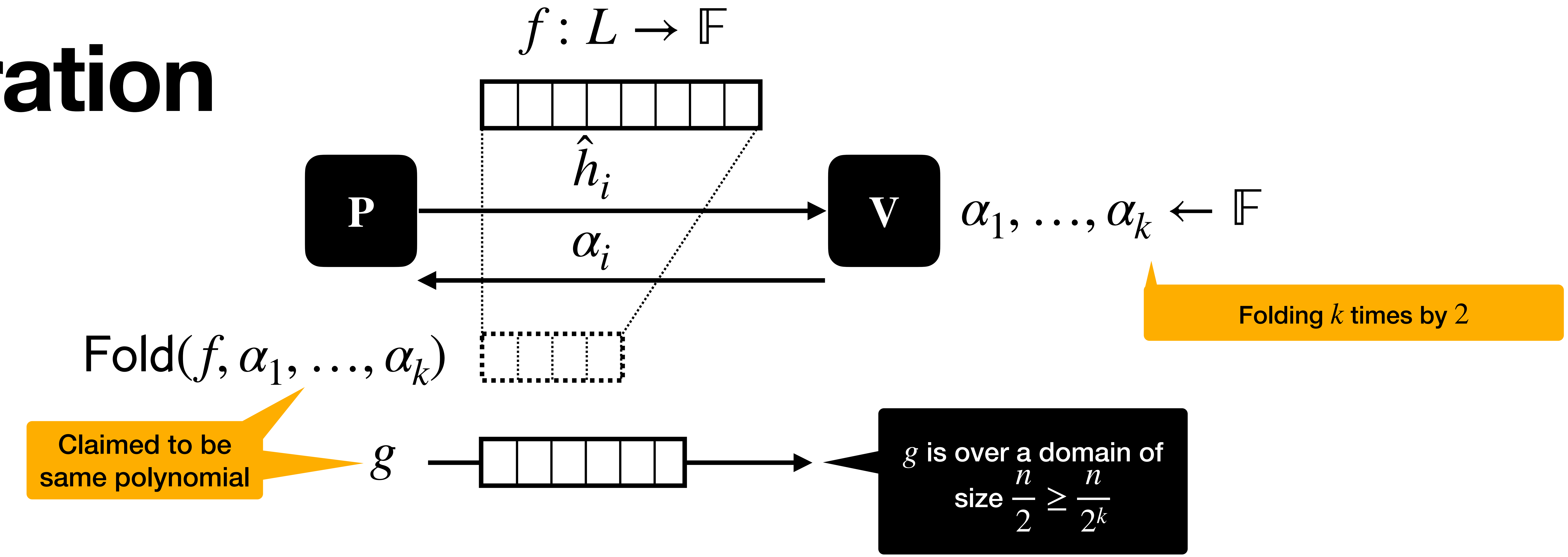
WHIR iteration



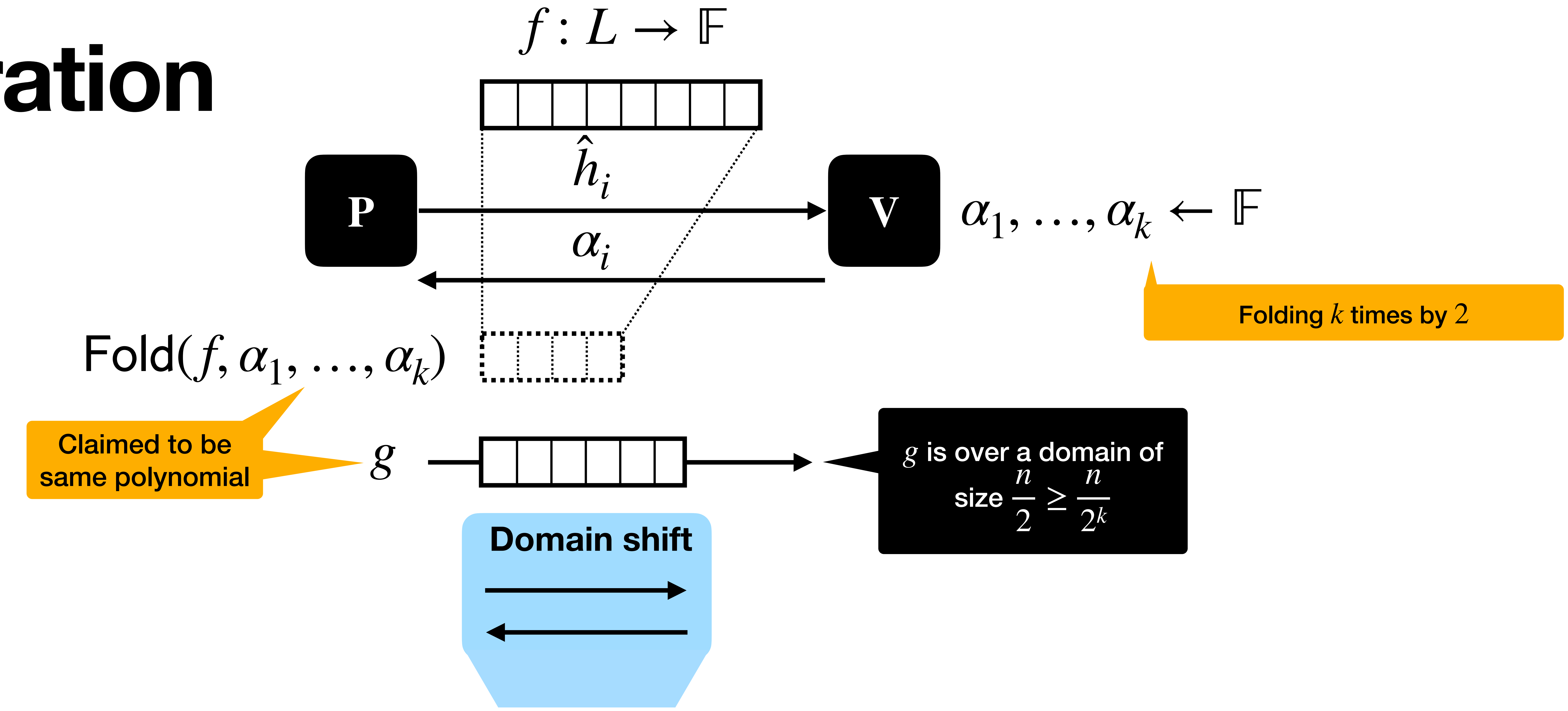
WHIR iteration



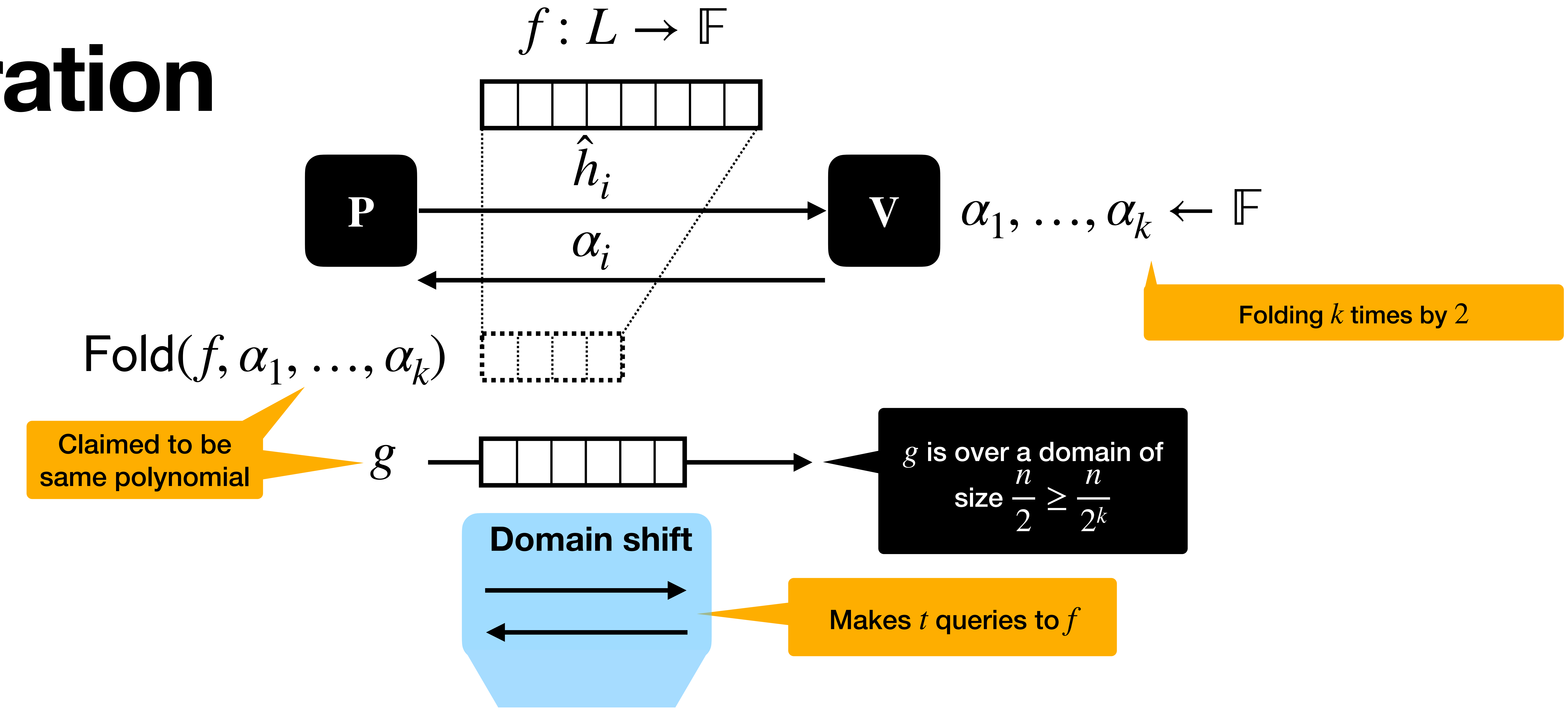
WHIR iteration



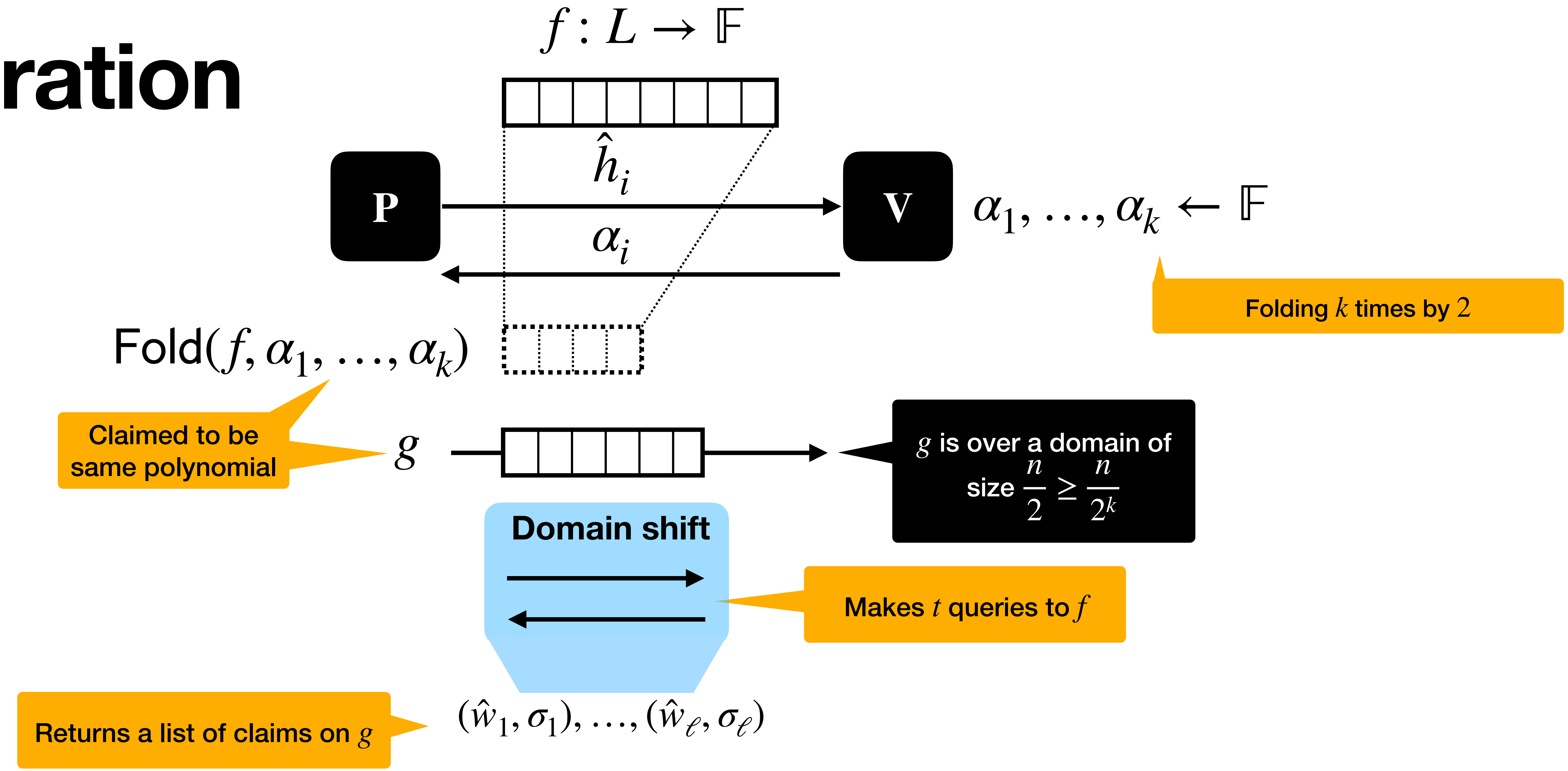
WHIR iteration



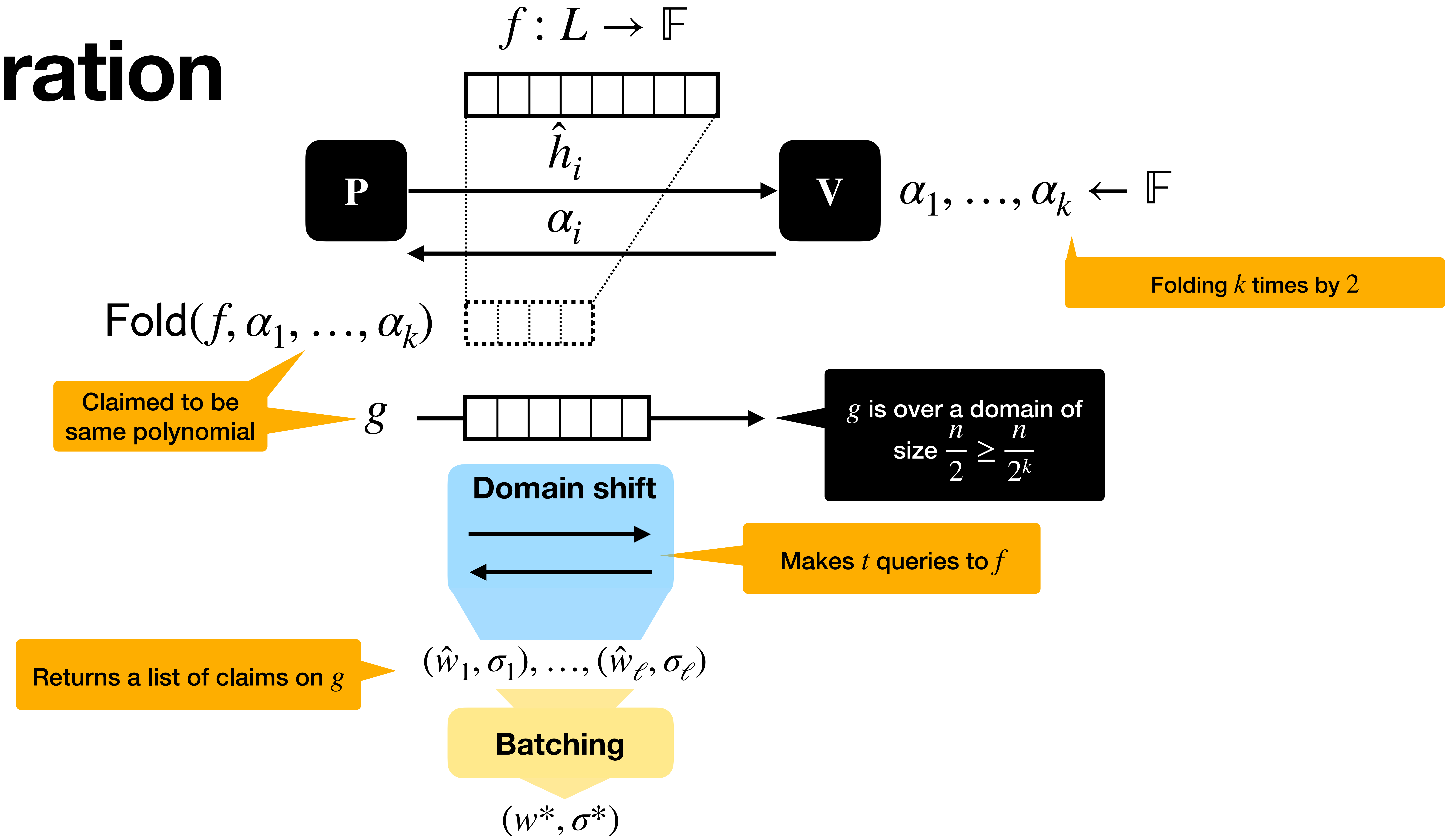
WHIR iteration



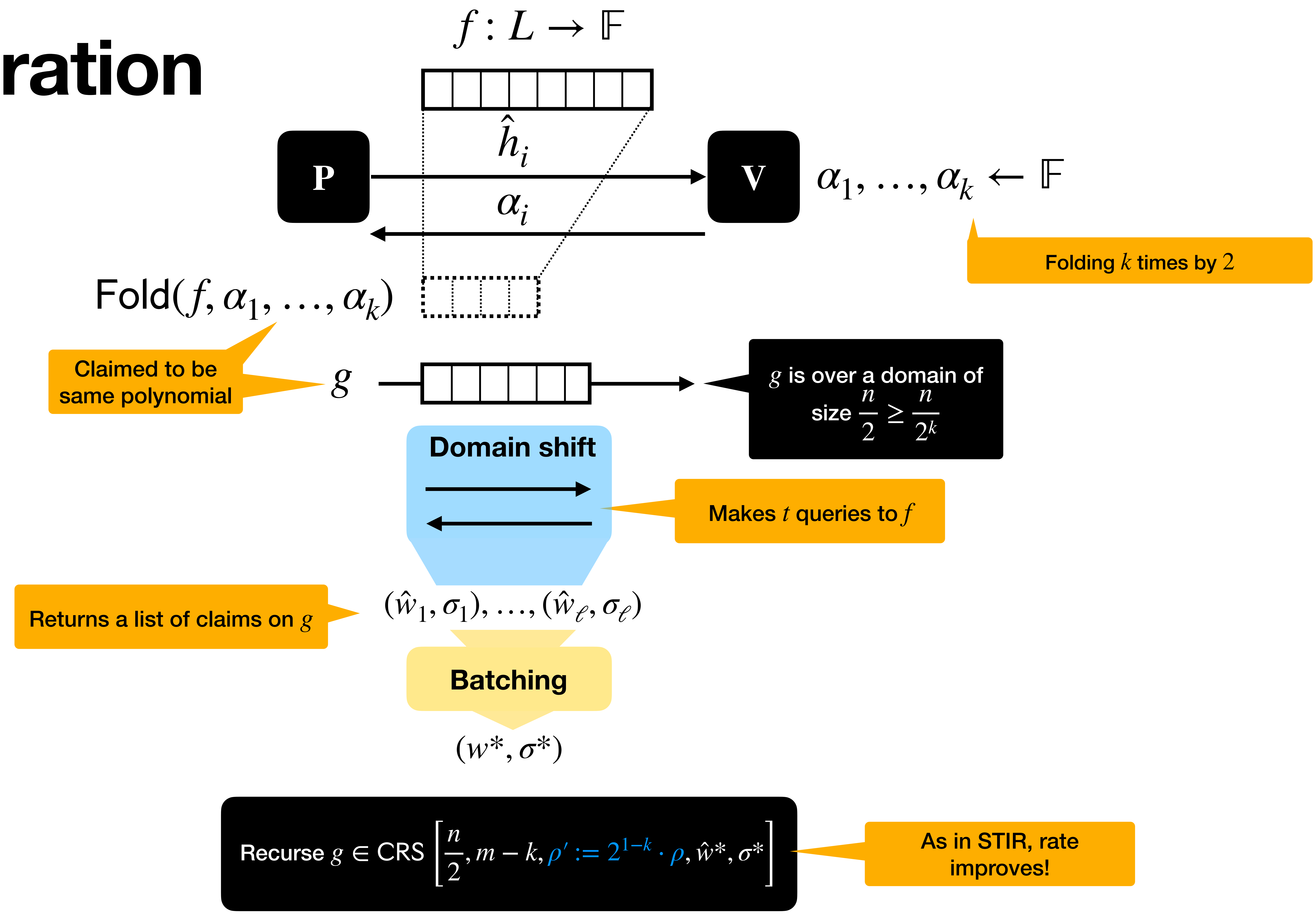
WHIR iteration



WHIR iteration

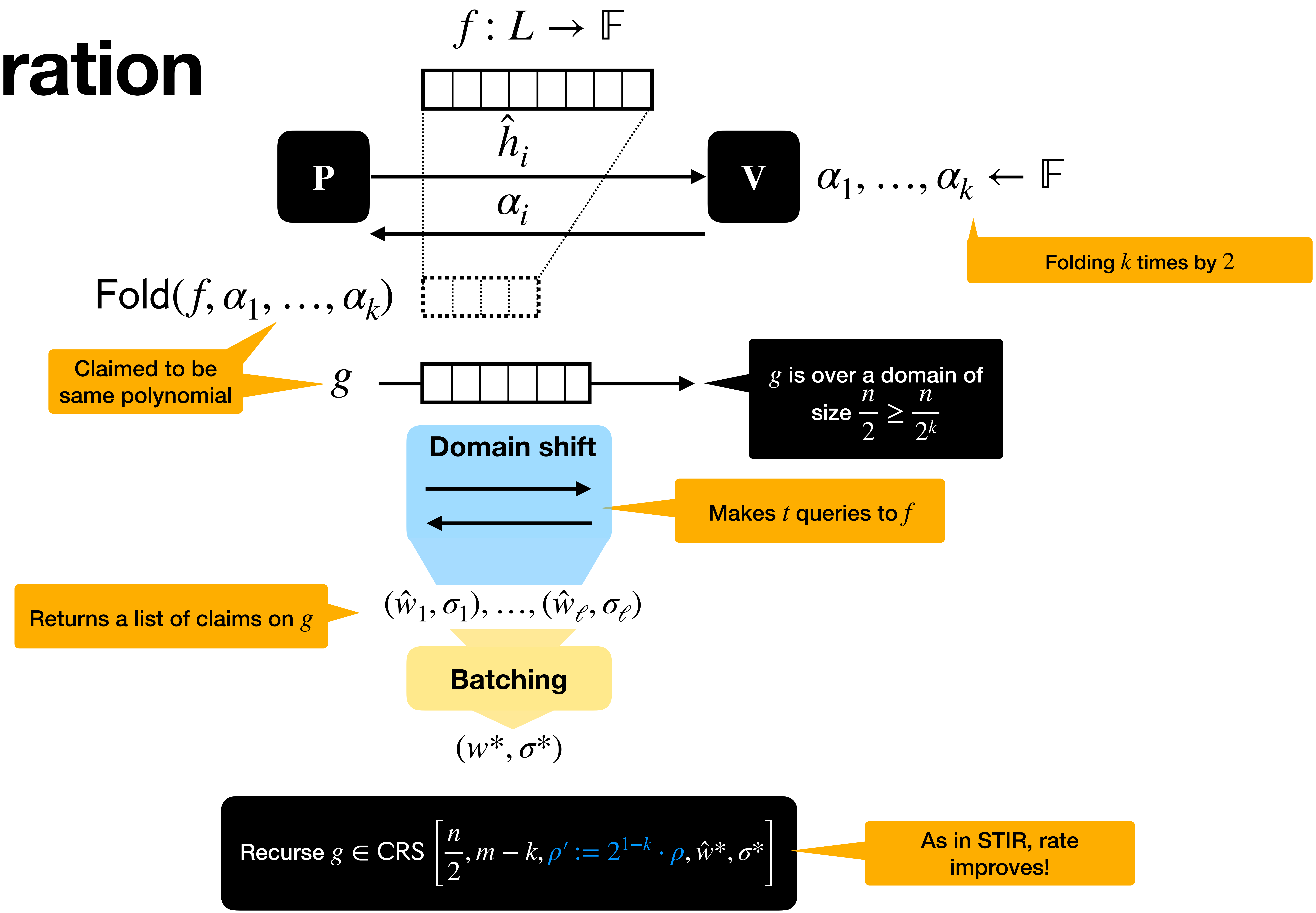


WHIR iteration



WHIR iteration

Similar structure to STIR! Multilinear structure forbids using quotients: we need new ideas to domain shift!

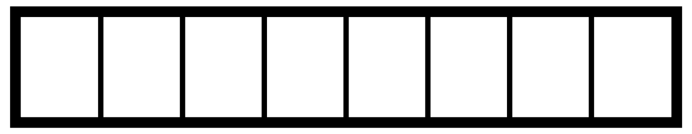


Domain shifting

Domain shifting

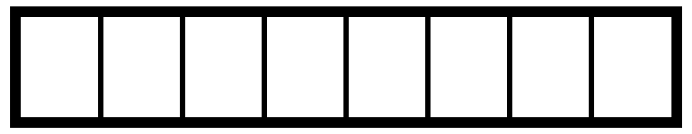
$f : L \to \mathbb{F}$

Claim on f**:** $(\hat{w}, \sigma)$

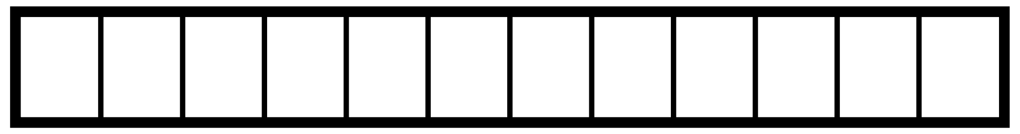


Domain shifting

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$

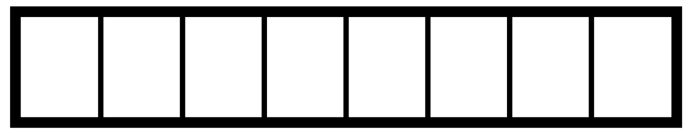


Claim on f: $(\hat{w}, \sigma)$

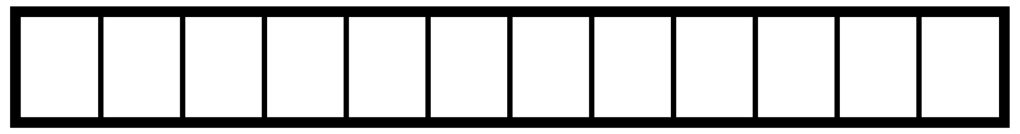
Domain shifting

Claim on f**:** $(\hat{w}, \sigma)$

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$

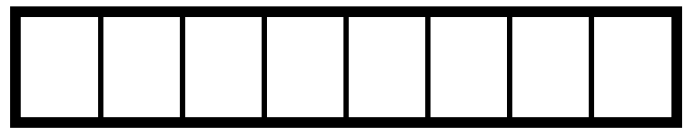


Output claims on g**:**
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

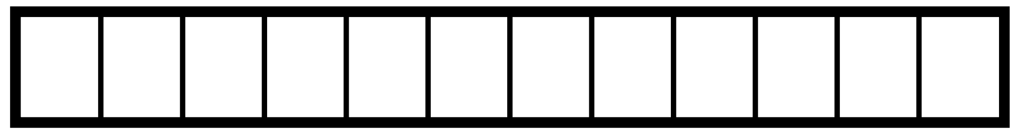
Domain shifting

Claim on f**:** $(\hat{w}, \sigma)$

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$



Output claims on g**:**
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Domain shifting

$f : L \to \mathbb{F}$ $\qquad$ $g : L^* \to \mathbb{F}$

Claim on f**:** $(\hat{w}, \sigma)$

Output claims on g**:** $(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

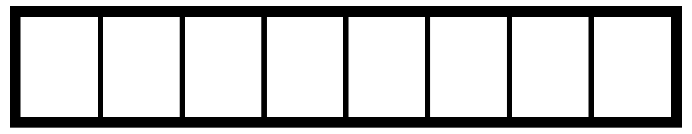
Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

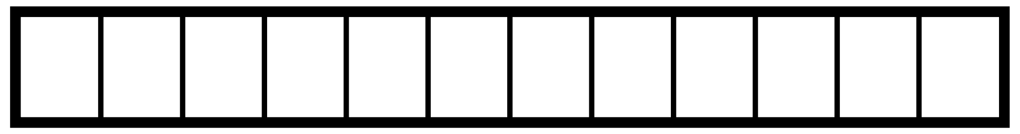
OOD subprotocol (next)

Domain shifting

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$



Claim on f: $(\hat{w}, \sigma)$

Output claims on g: $(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

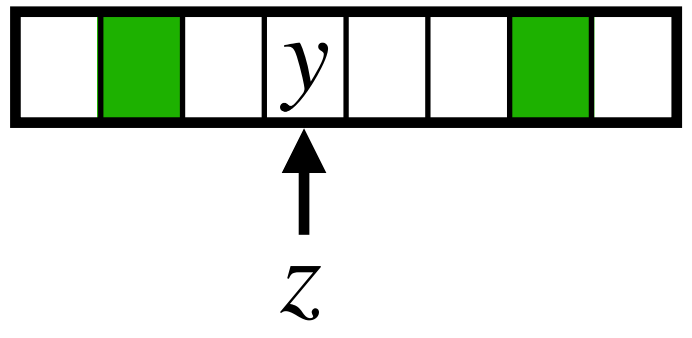
OOD subprotocol (next)

Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

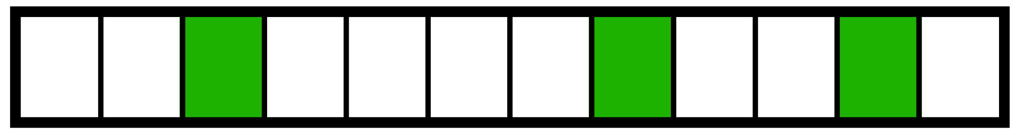
Domain shifting

Claim on f: $(\hat{w}, \sigma)$

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$



Output claims on g: $(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

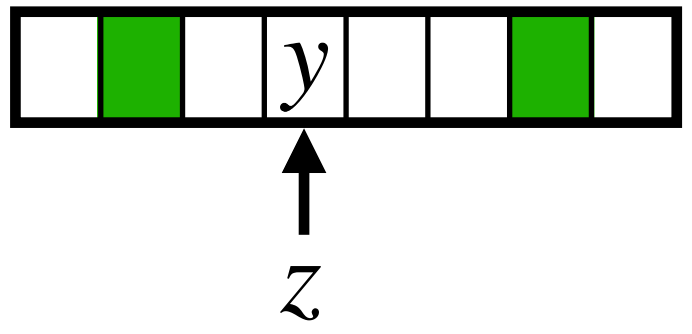
OOD subprotocol (next)

Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

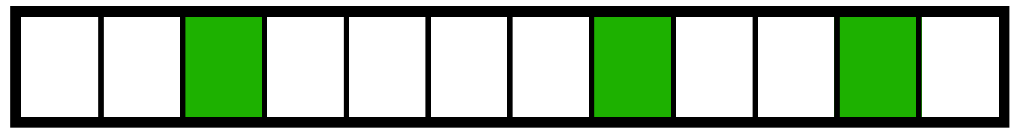
Domain shifting

Claim on f: $(\hat{w}, \sigma)$

$f : L \to \mathbb{F}$



$g : L^* \to \mathbb{F}$



Output claims on g: $(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

OOD subprotocol (next)

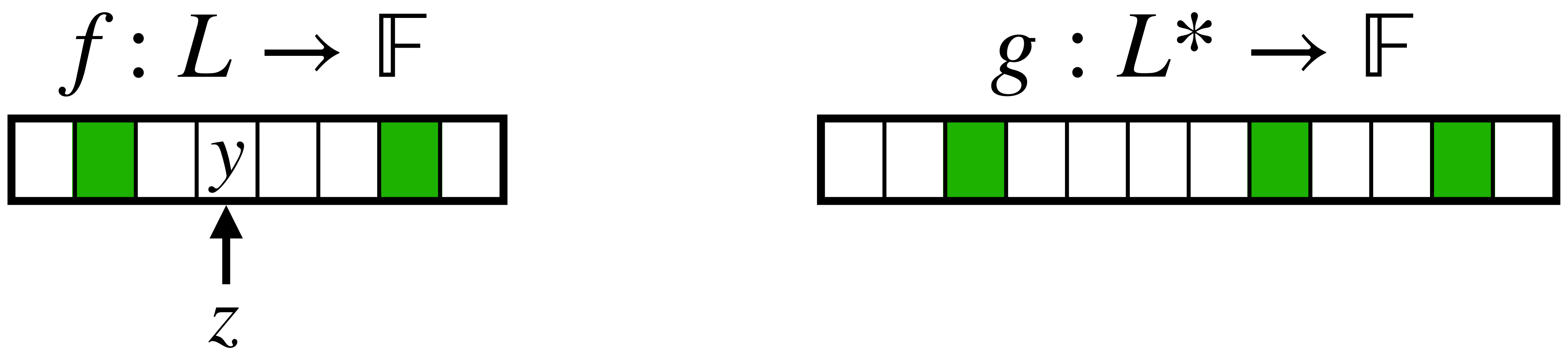
Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

Domain shifting

Claim on f: $(\hat{w}, \sigma)$



Output claims on g: $(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from $\mathrm{CRS}[\,|L|\,, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from $\mathrm{CRS}[\,|L^*|\,, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

OOD subprotocol (next)

Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

So, except with probability $\sqrt{\rho}$, g is $\left(1 - \sqrt{\rho'}\right)$-far from $\mathrm{CRS}[\,|L^*|\,, m, \rho', (\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)]$.

Can amplify to $\sqrt{\rho}^t$

Out Of Domain

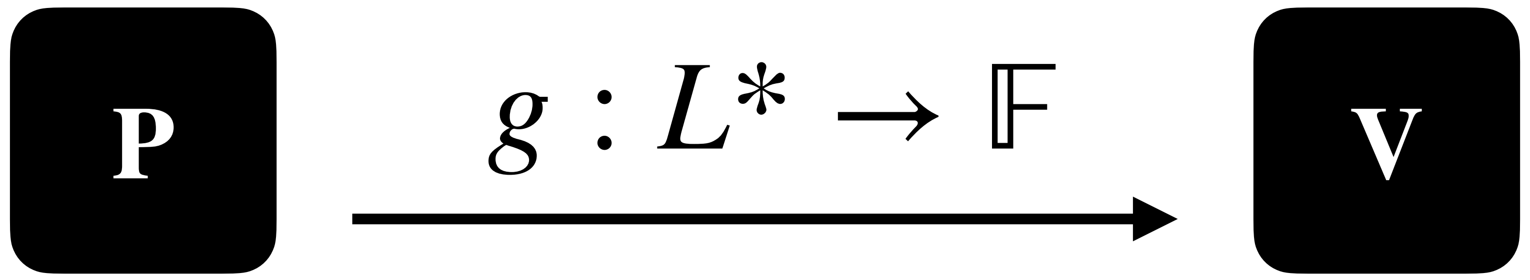
Subprotocol to force unique

Out Of Domain

Subprotocol to force unique

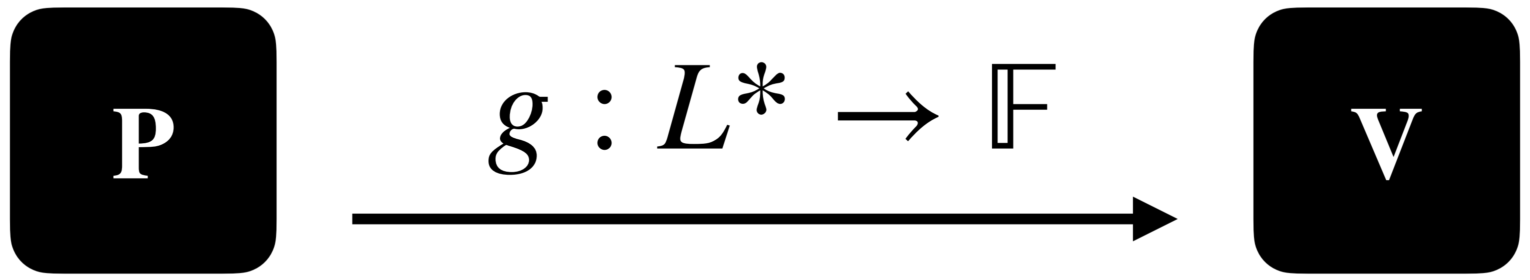
Out Of Domain

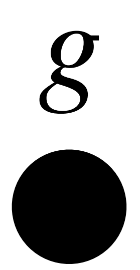
Subprotocol to force unique



Out Of Domain

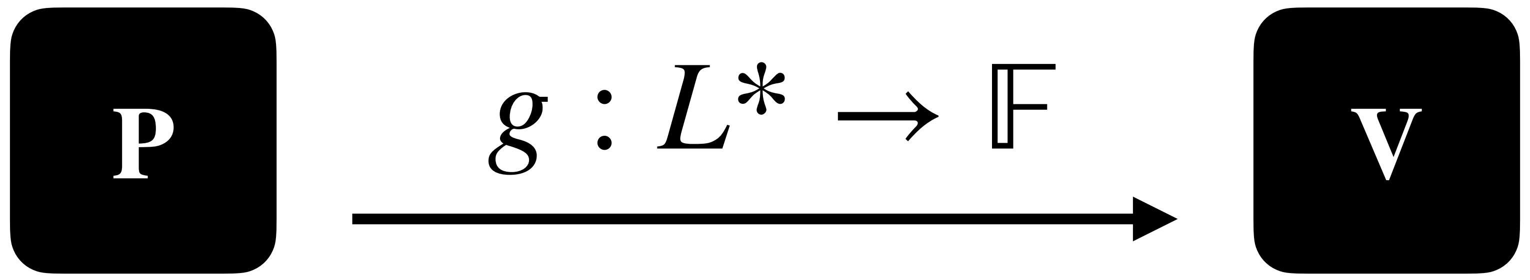
Subprotocol to force unique

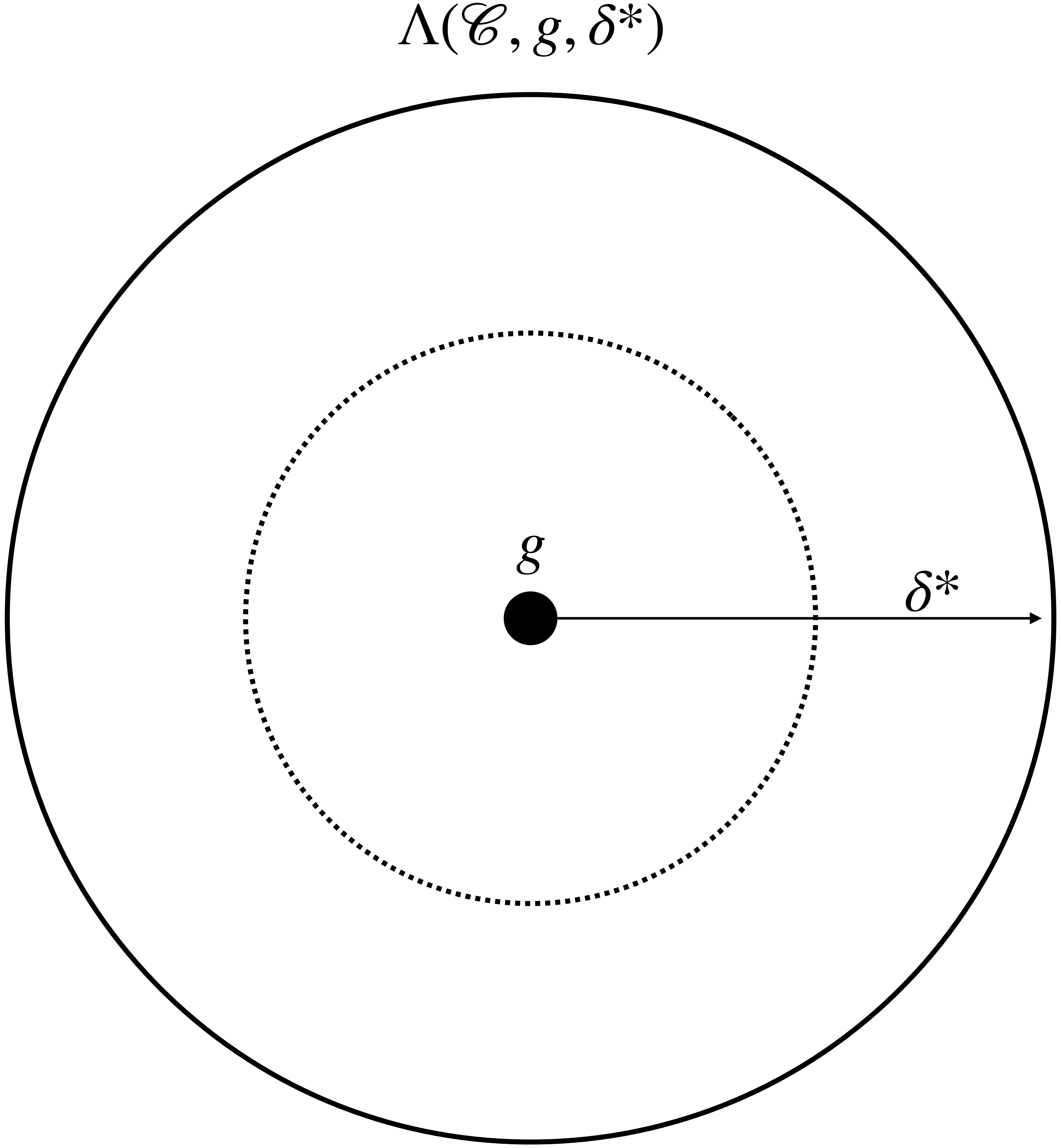




Out Of Domain

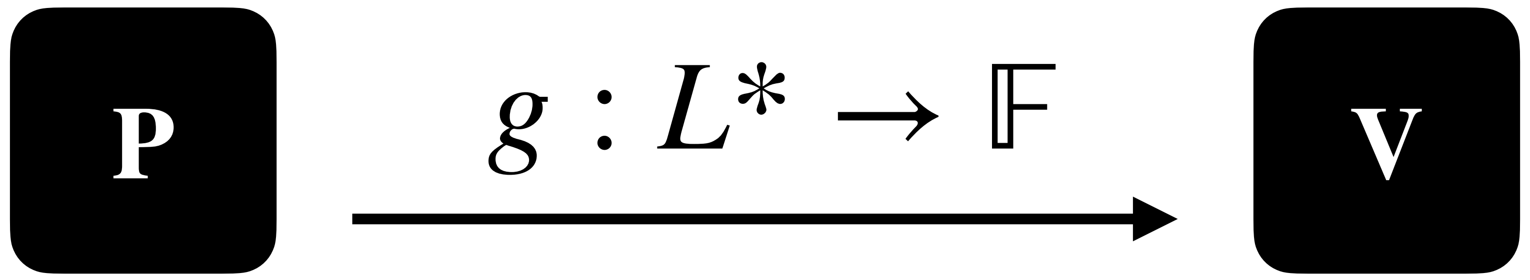
Subprotocol to force unique

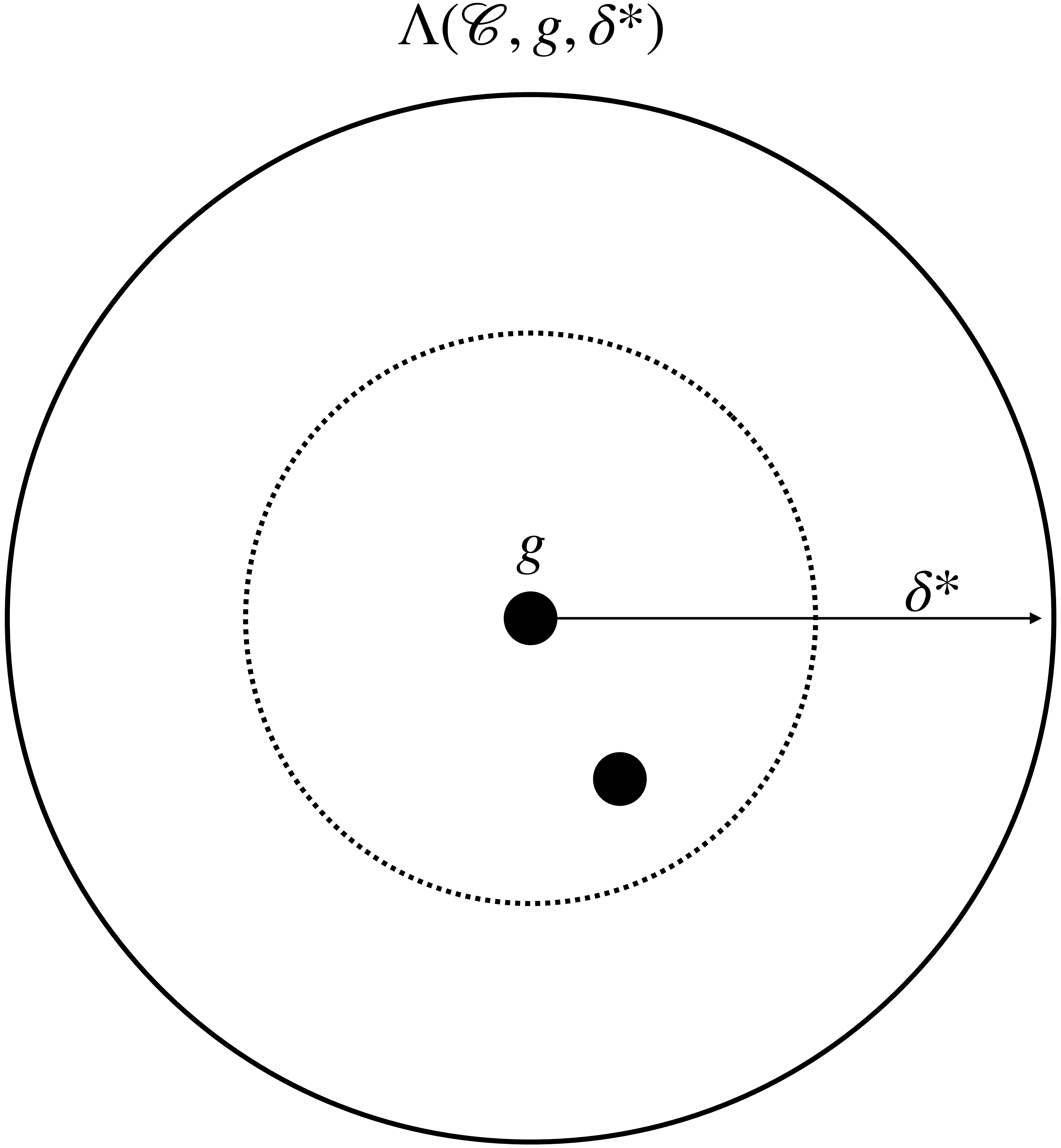




Out Of Domain

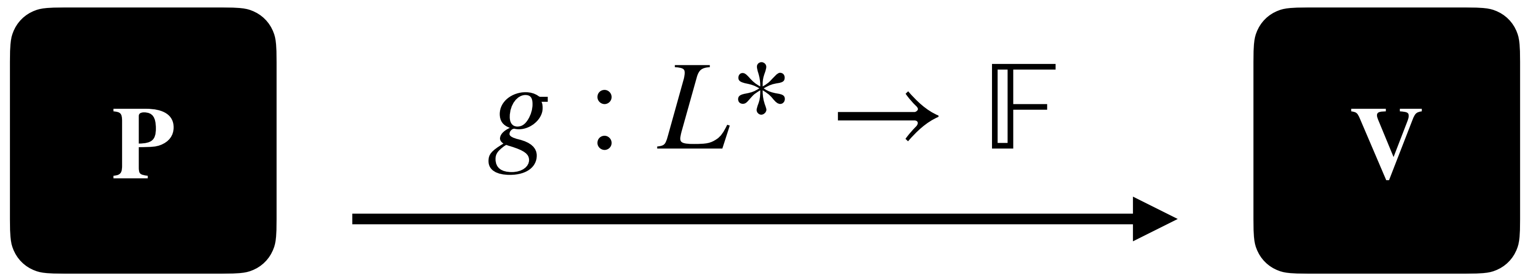
Subprotocol to force unique

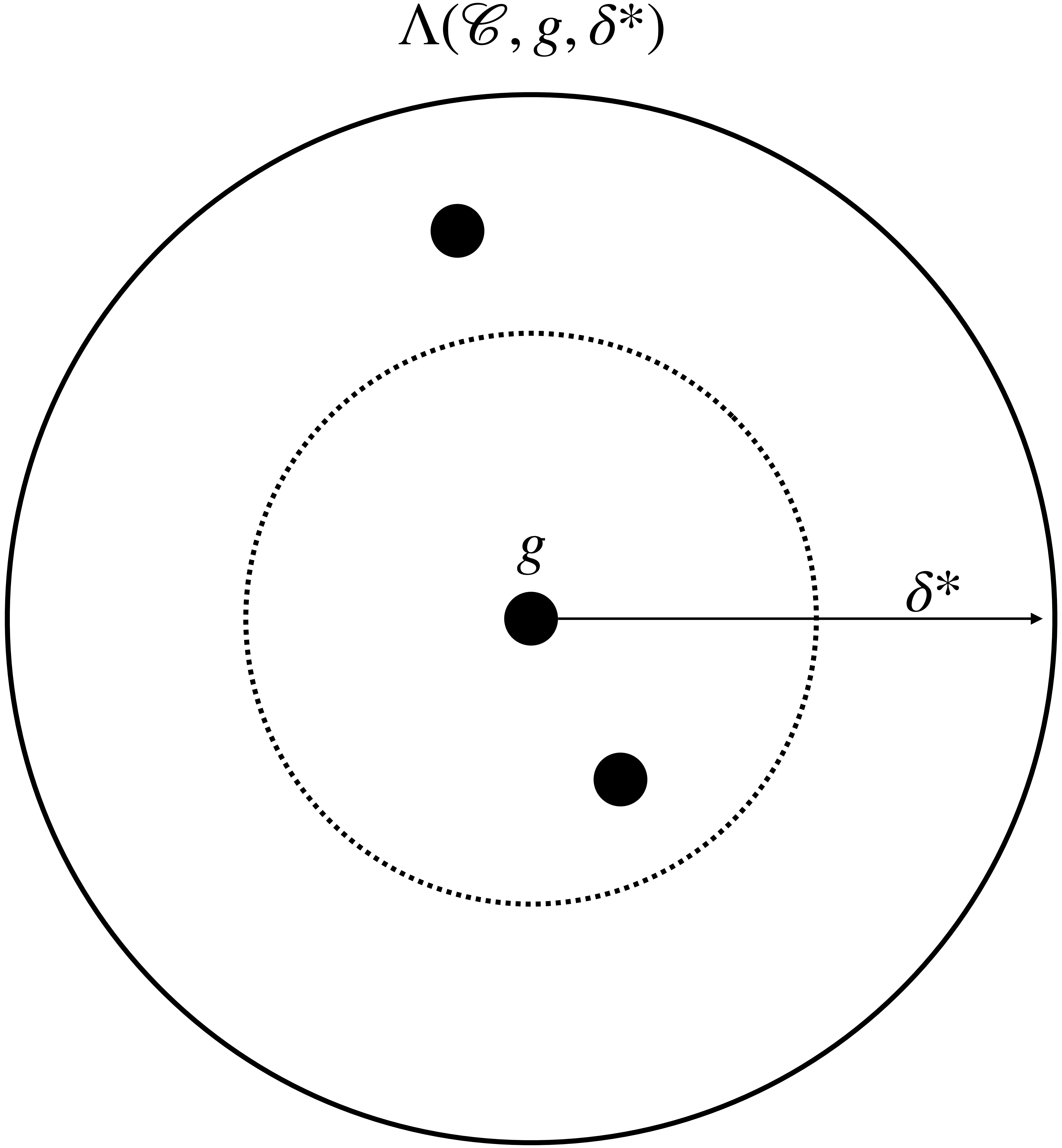




Out Of Domain

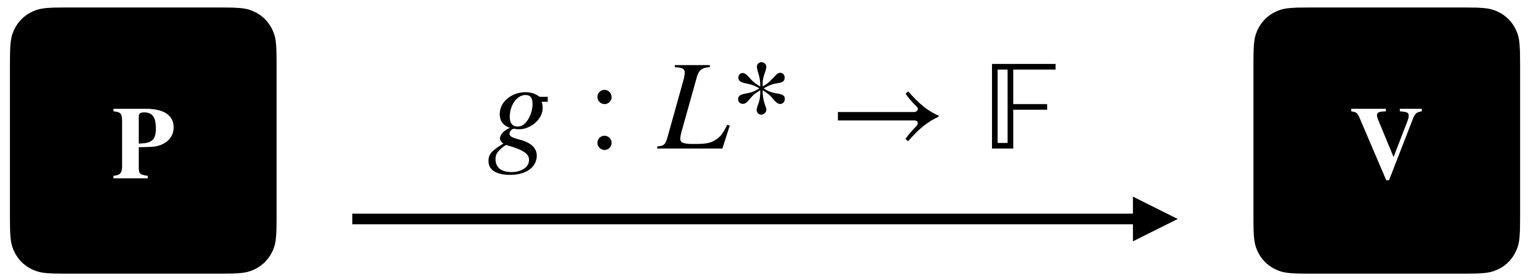
Subprotocol to force unique

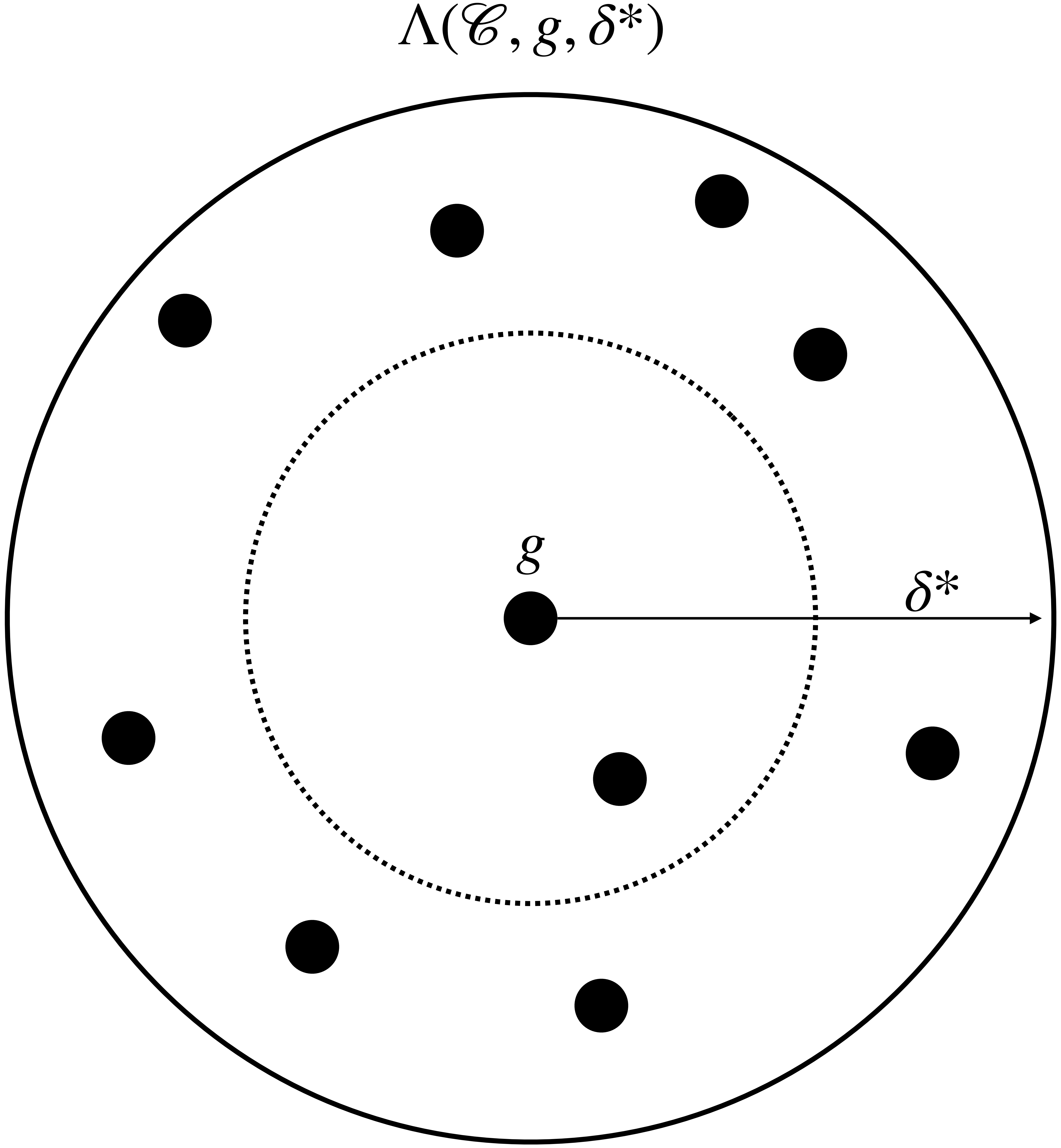




Out Of Domain

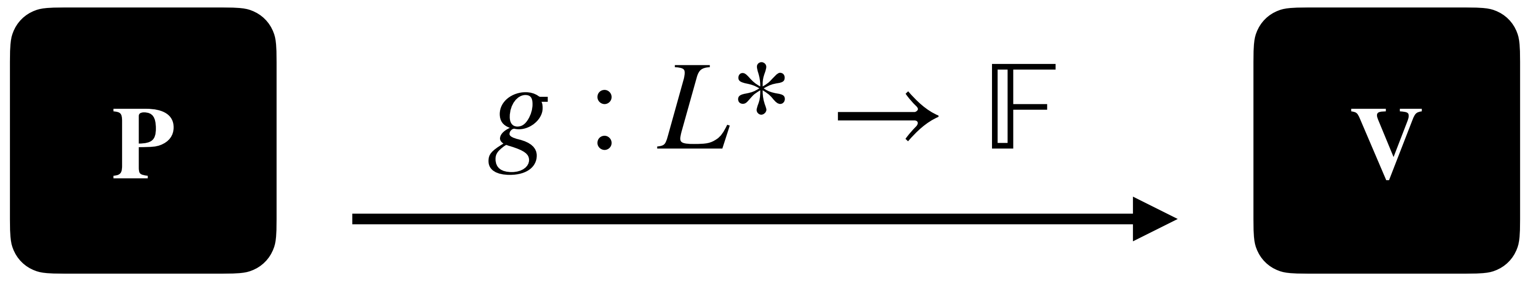
Subprotocol to force unique

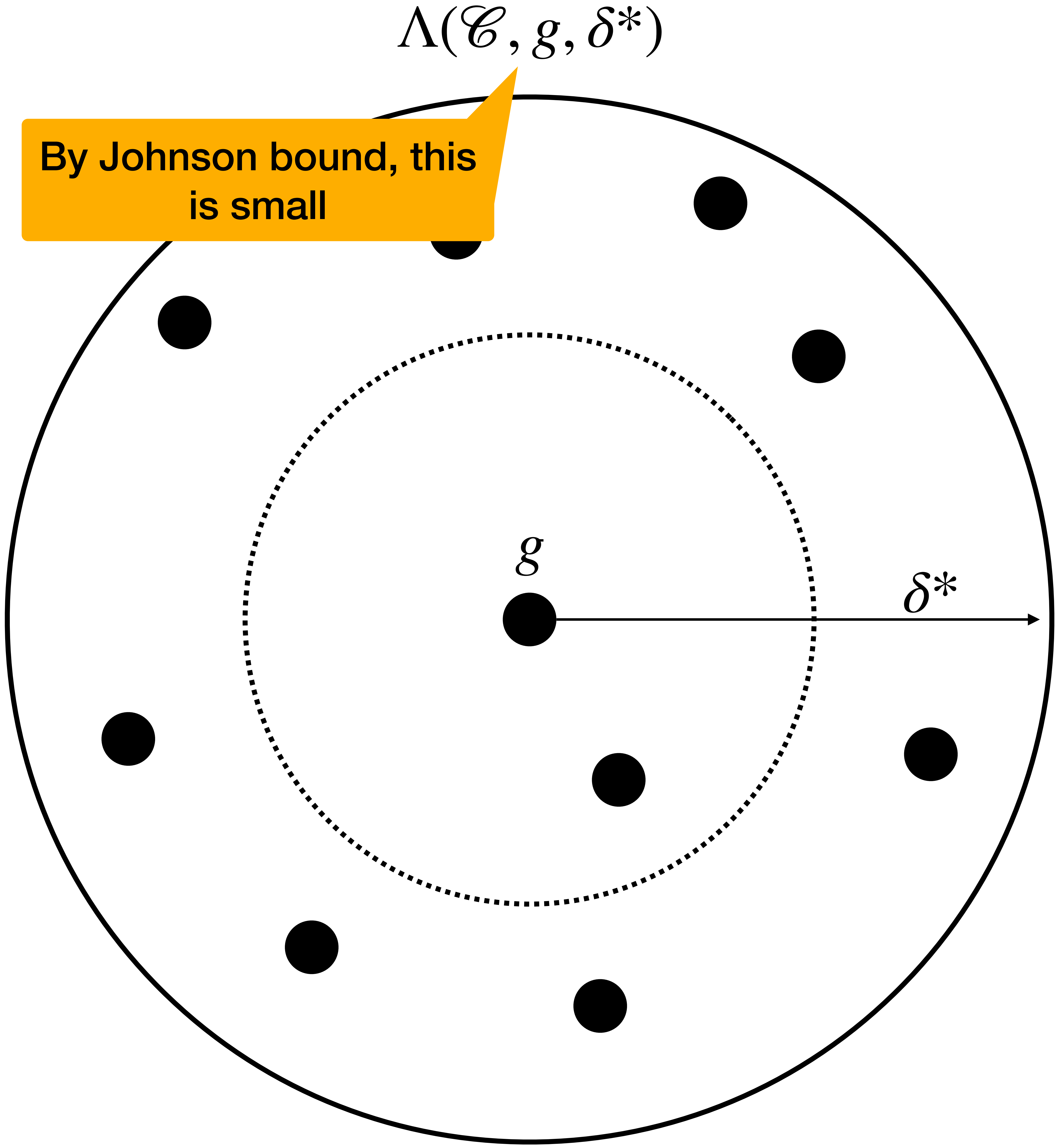




Out Of Domain

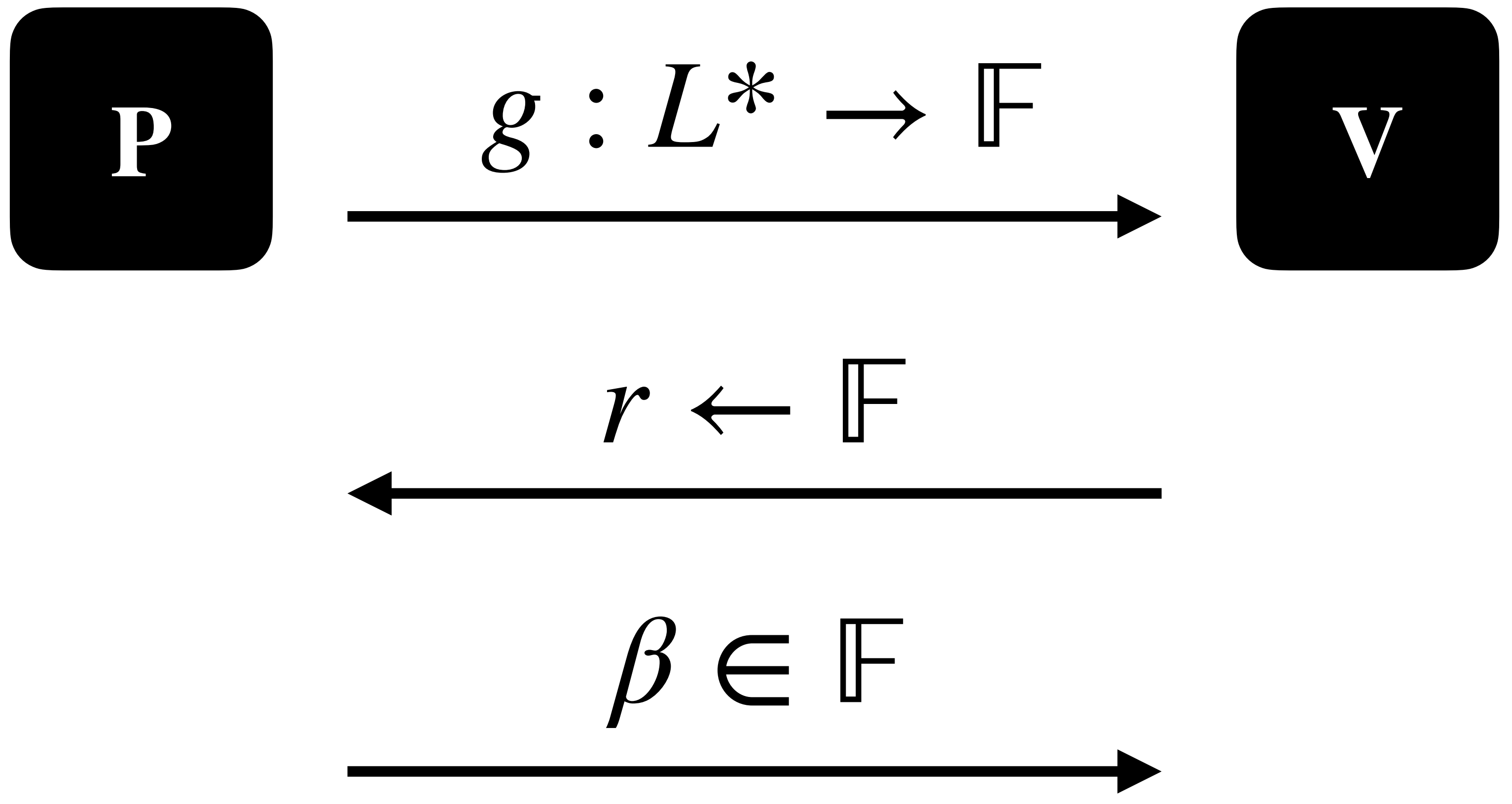
Subprotocol to force unique

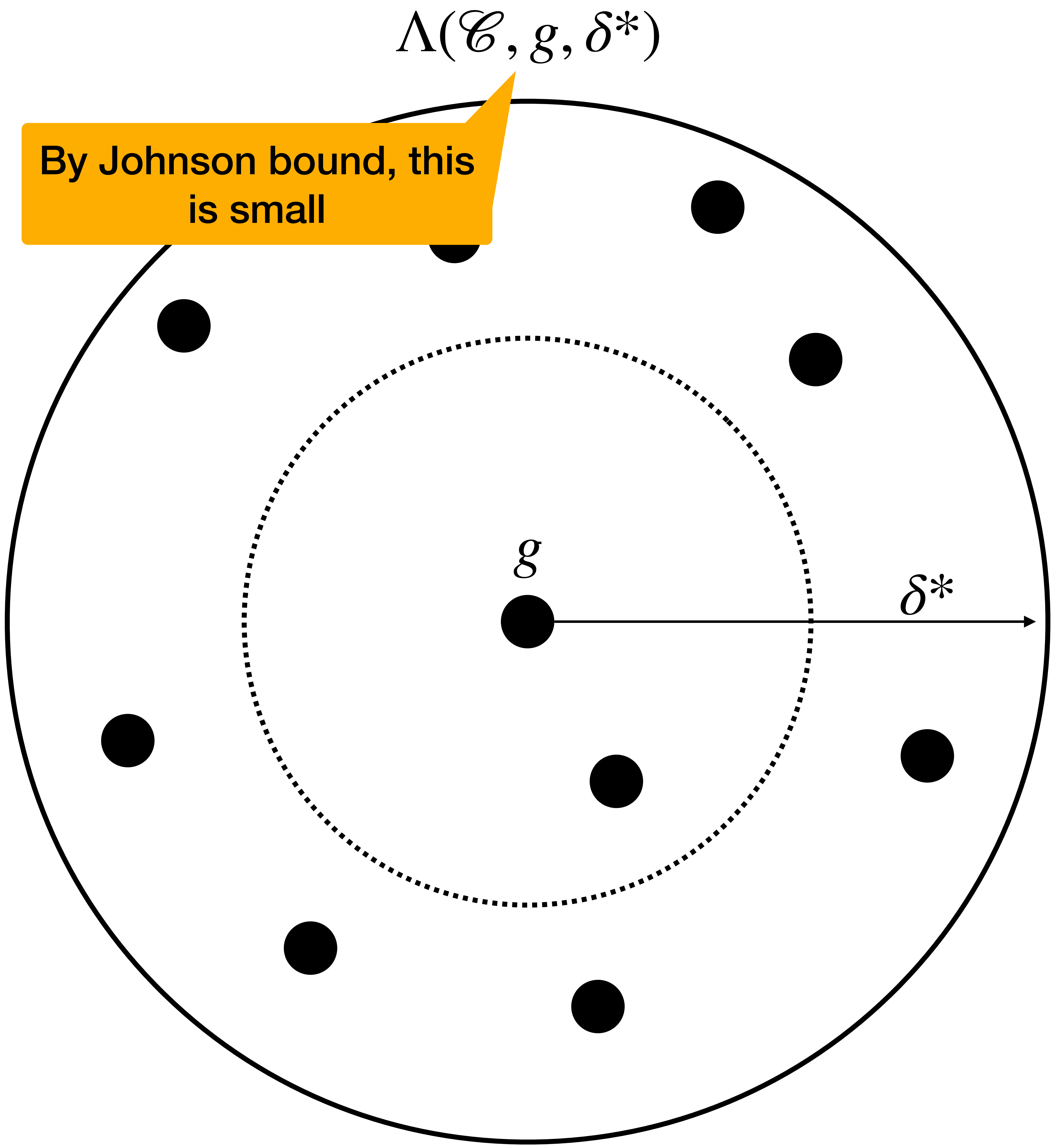




Out Of Domain

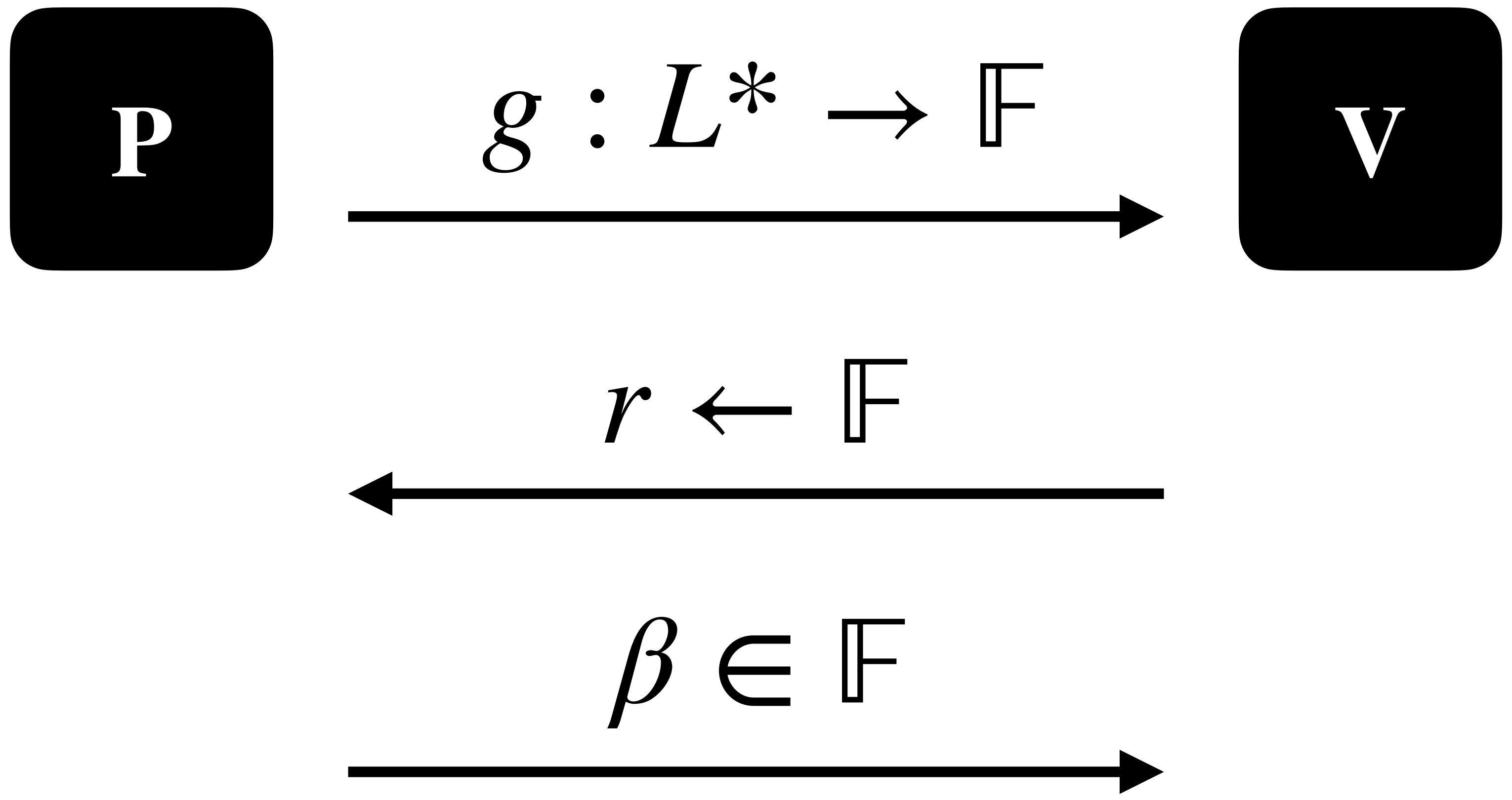
Subprotocol to force unique



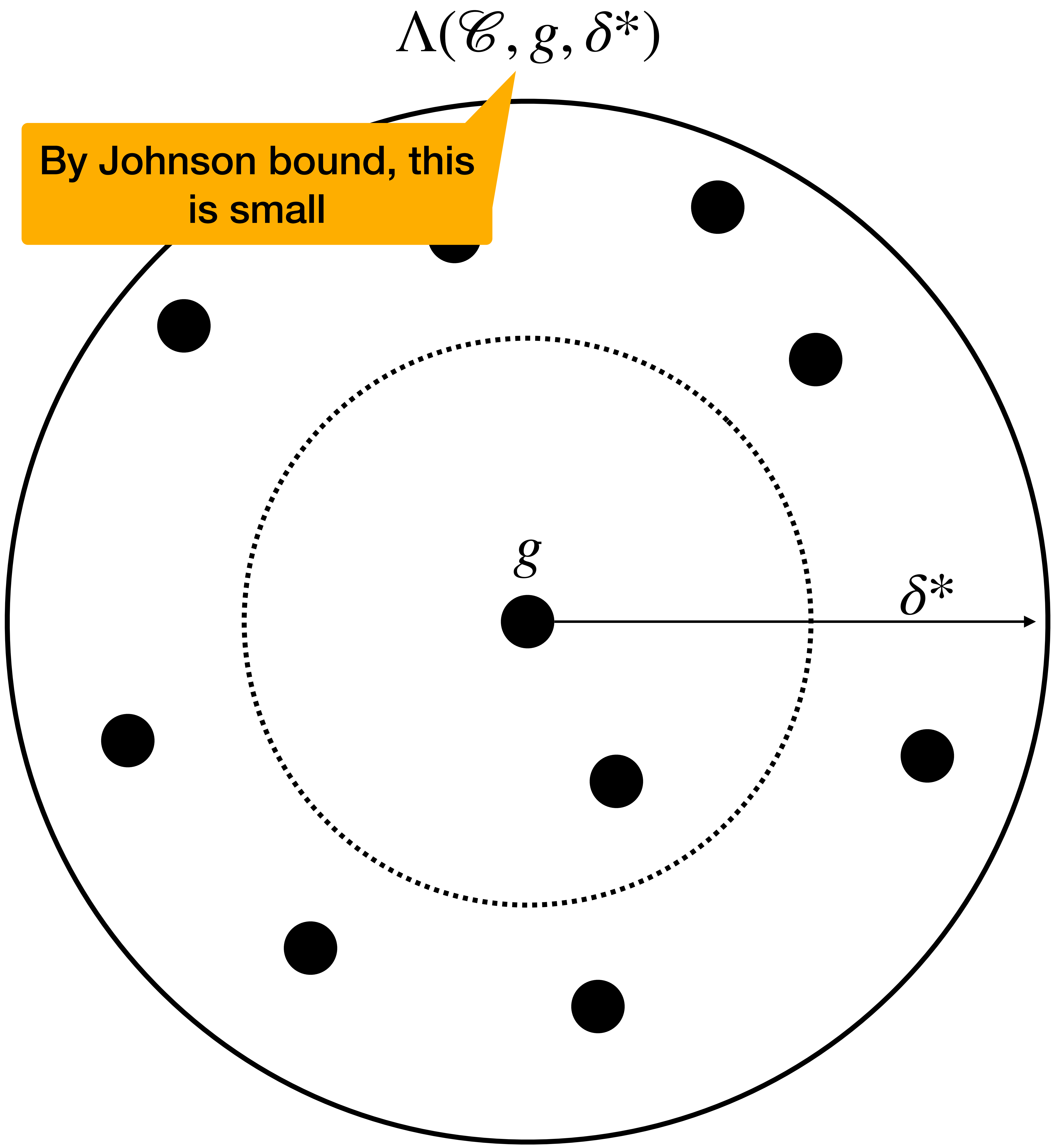


Out Of Domain

Subprotocol to force unique

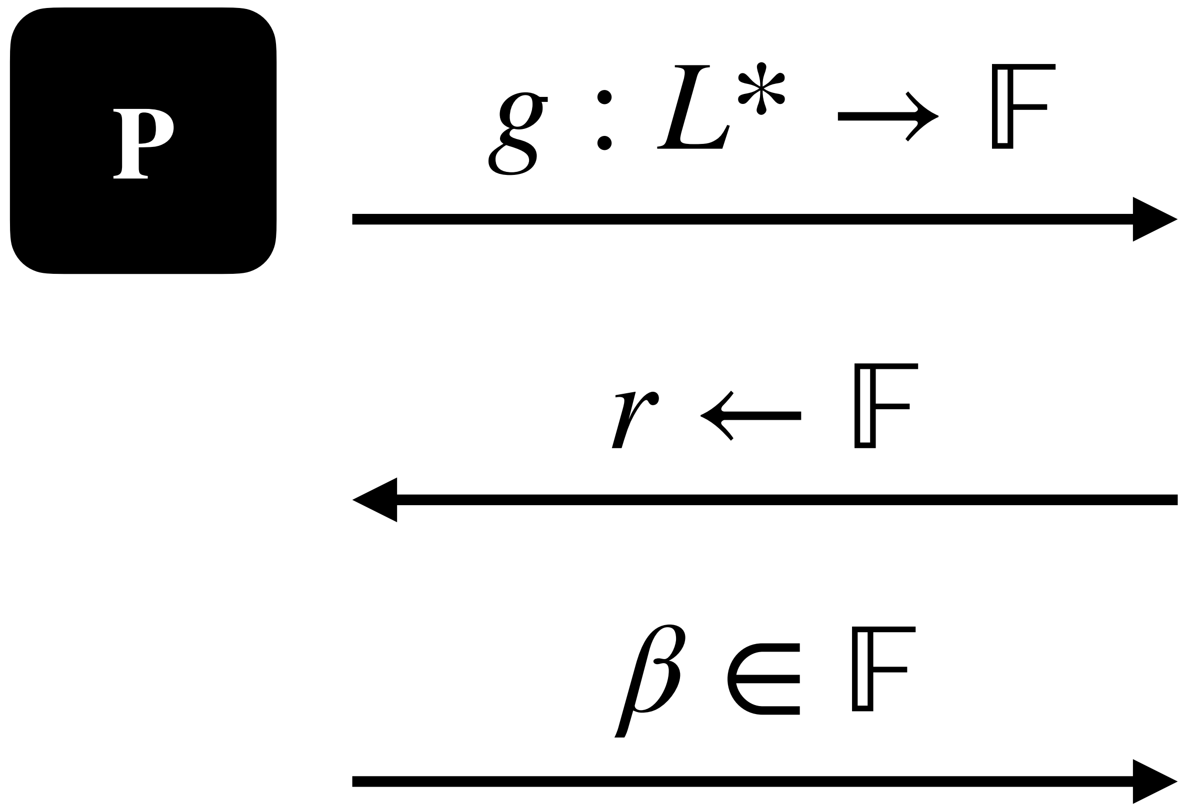


- By fundamental theorem of algebra of w.h.p. no pair $\hat{u}, \hat{v}$ with $\hat{u}(r) = \hat{v}(r)$
- Prover "chooses" which codeword $\hat{u}$ it "commits" to



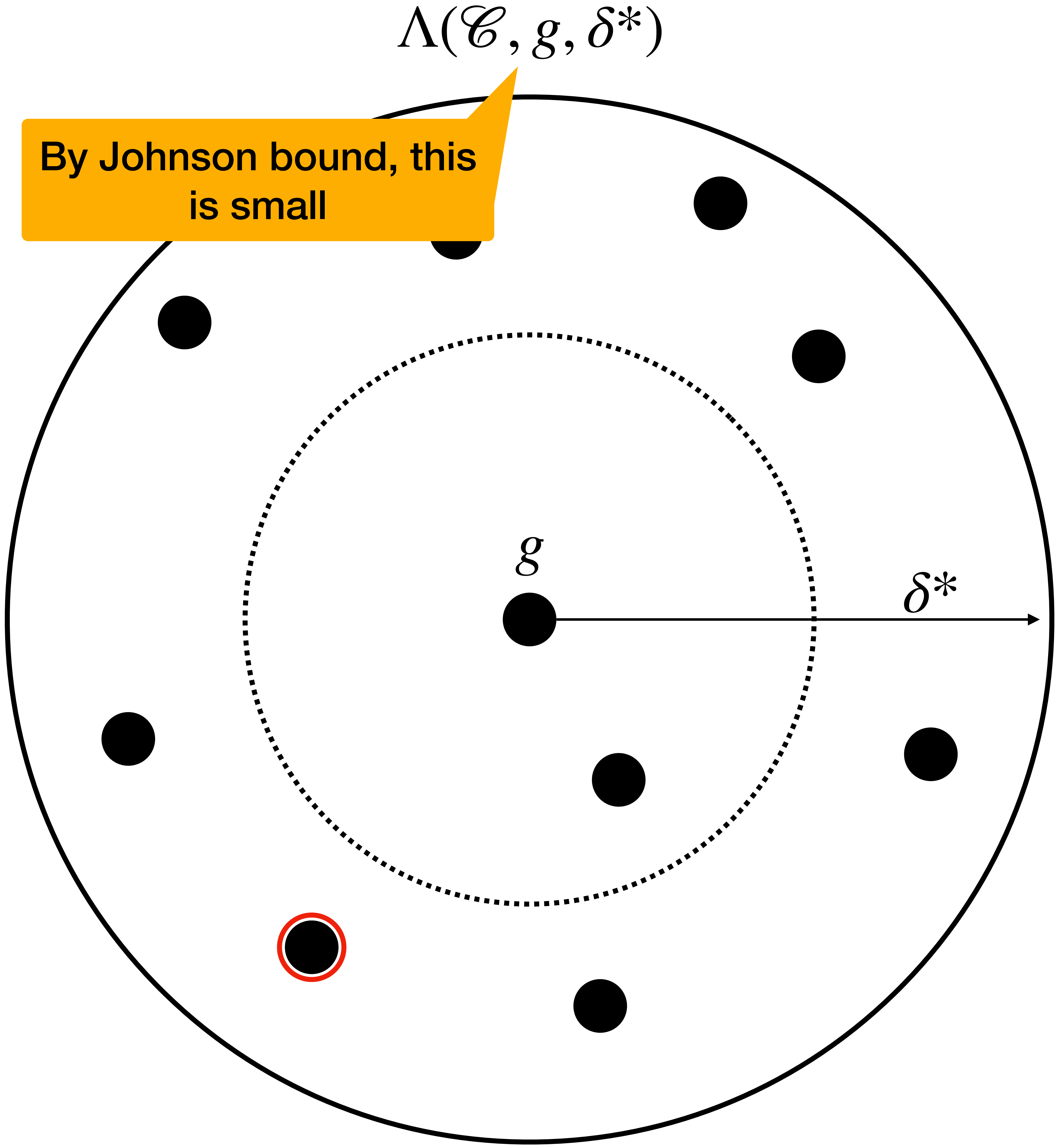
Out Of Domain

Subprotocol to force unique



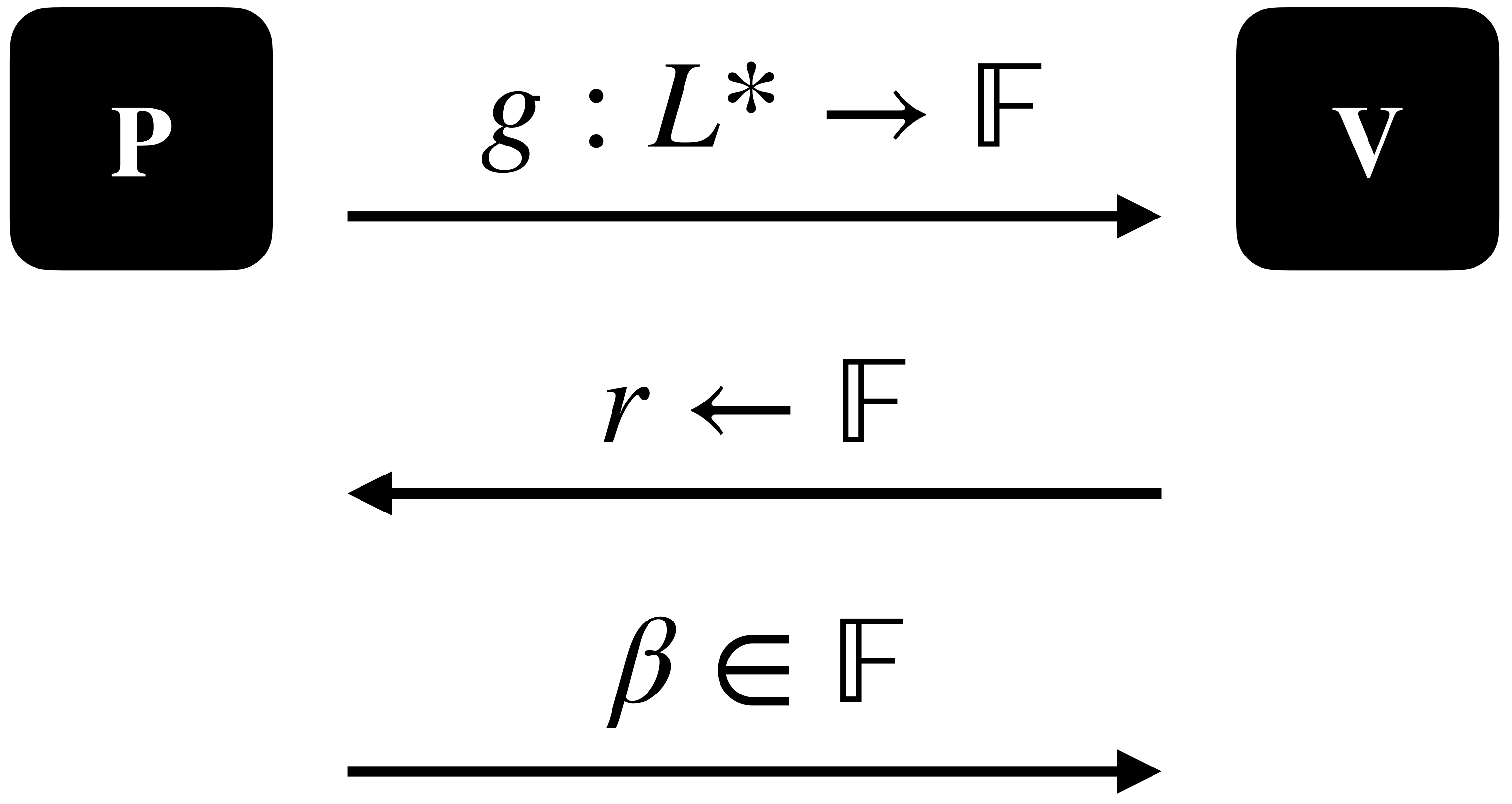
- By fundamental theorem of algebra of w.h.p. no pair $\hat{u}, \hat{v}$ with $\hat{u}(r) = \hat{v}(r)$

- Prover "chooses" which codeword $\hat{u}$ it "commits" to



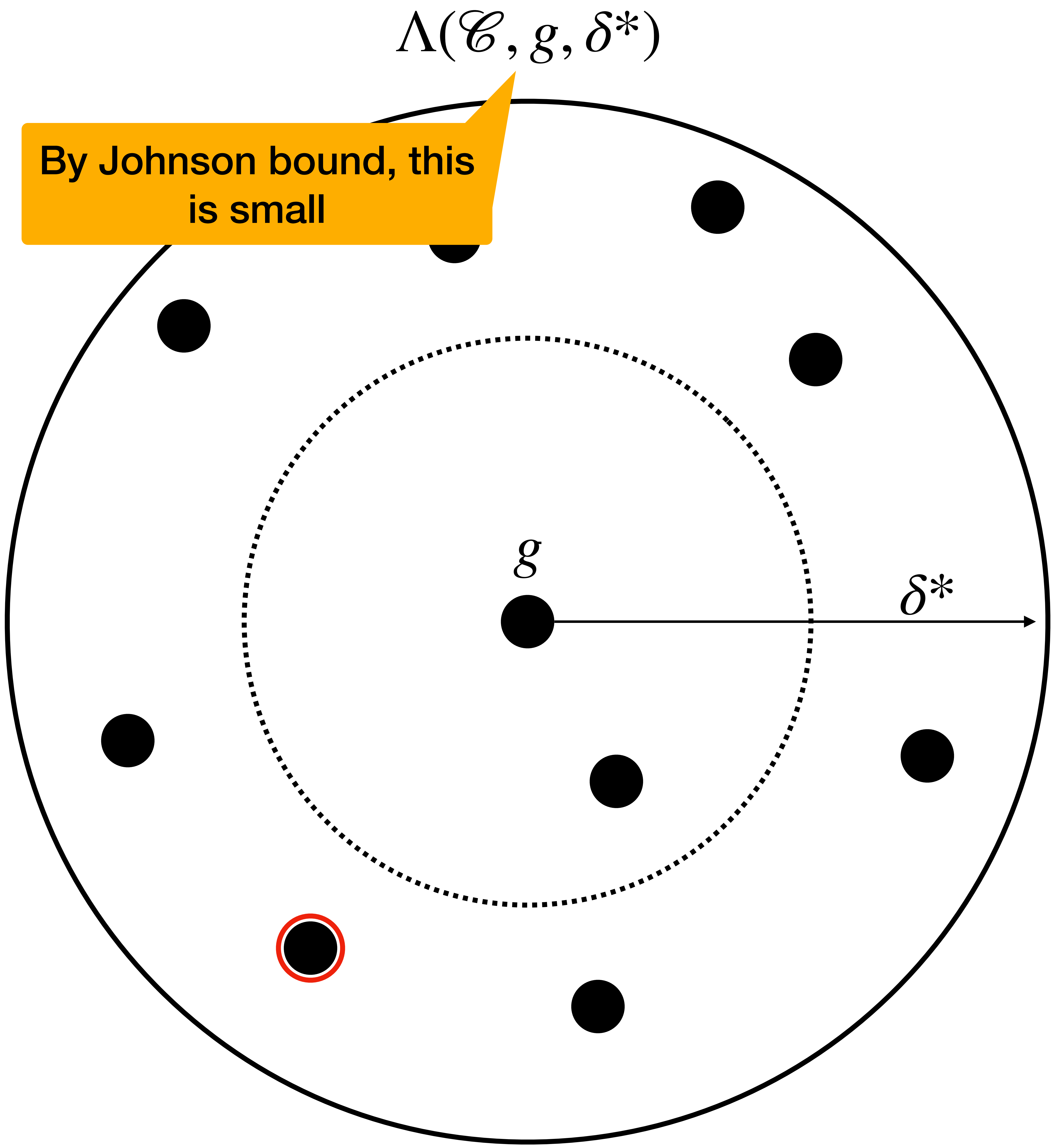
Out Of Domain

Subprotocol to force unique



- By fundamental theorem of algebra of w.h.p. no pair $\hat{u}, \hat{v}$ with $\hat{u}(r) = \hat{v}(r)$
- Prover "chooses" which codeword $\hat{u}$ it "commits" to

Add to list of constraints to **enforce**!



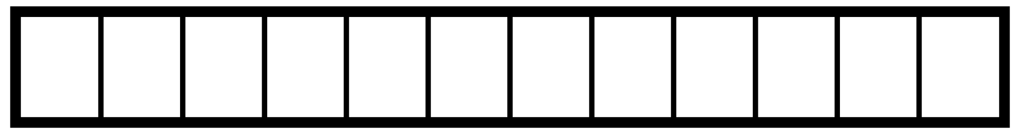
Batching

Pick your favourite sumcheck batching

Batching

Pick your favourite sumcheck batching

$g : L \to \mathbb{F}$



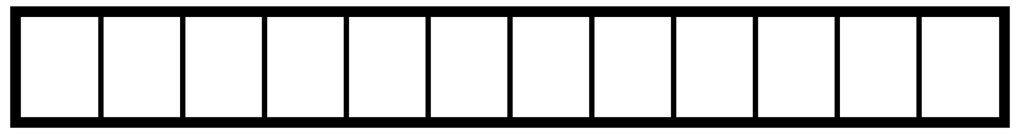
Sumcheck claims on g:

$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching

Pick your favourite sumcheck batching

$g : L \to \mathbb{F}$



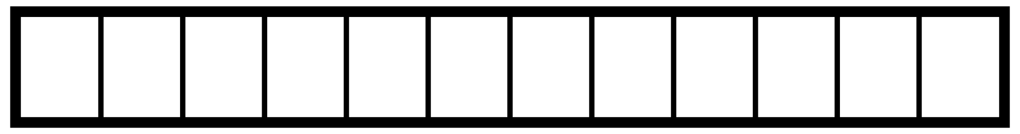
Sumcheck claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching

Batching

Pick your favourite sumcheck batching

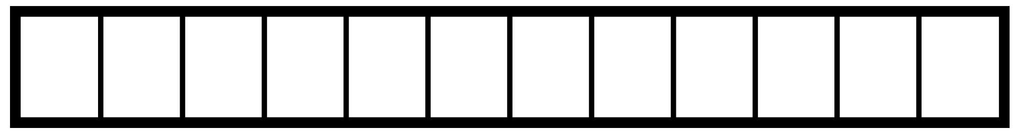
$g : L \to \mathbb{F}$



Sumcheck claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching

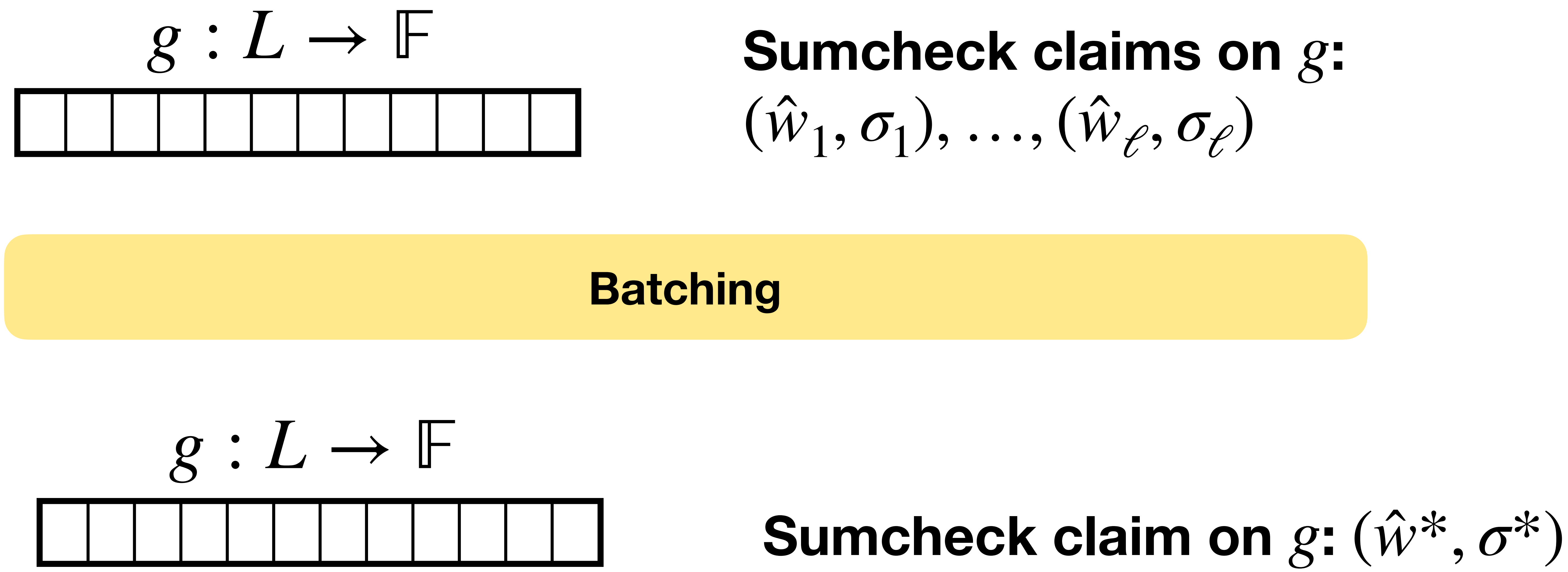
$g : L \to \mathbb{F}$



Sumcheck claim on g: $(\hat{w}^*, \sigma^*)$

Batching

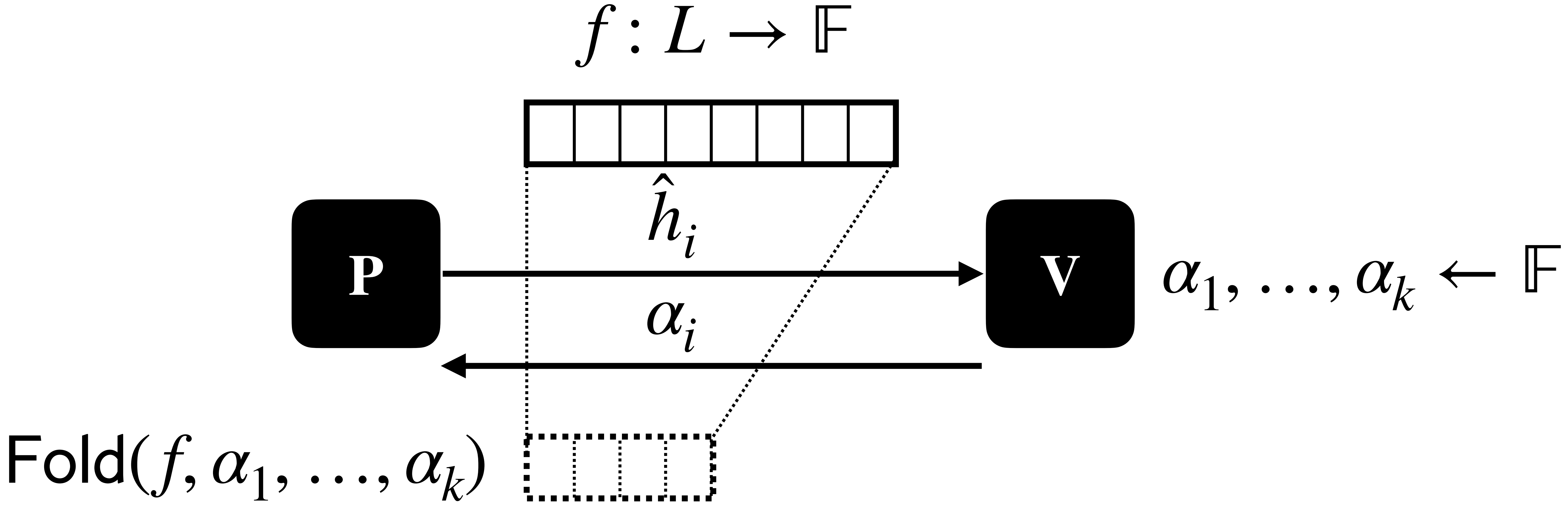
Pick your favourite sumcheck batching



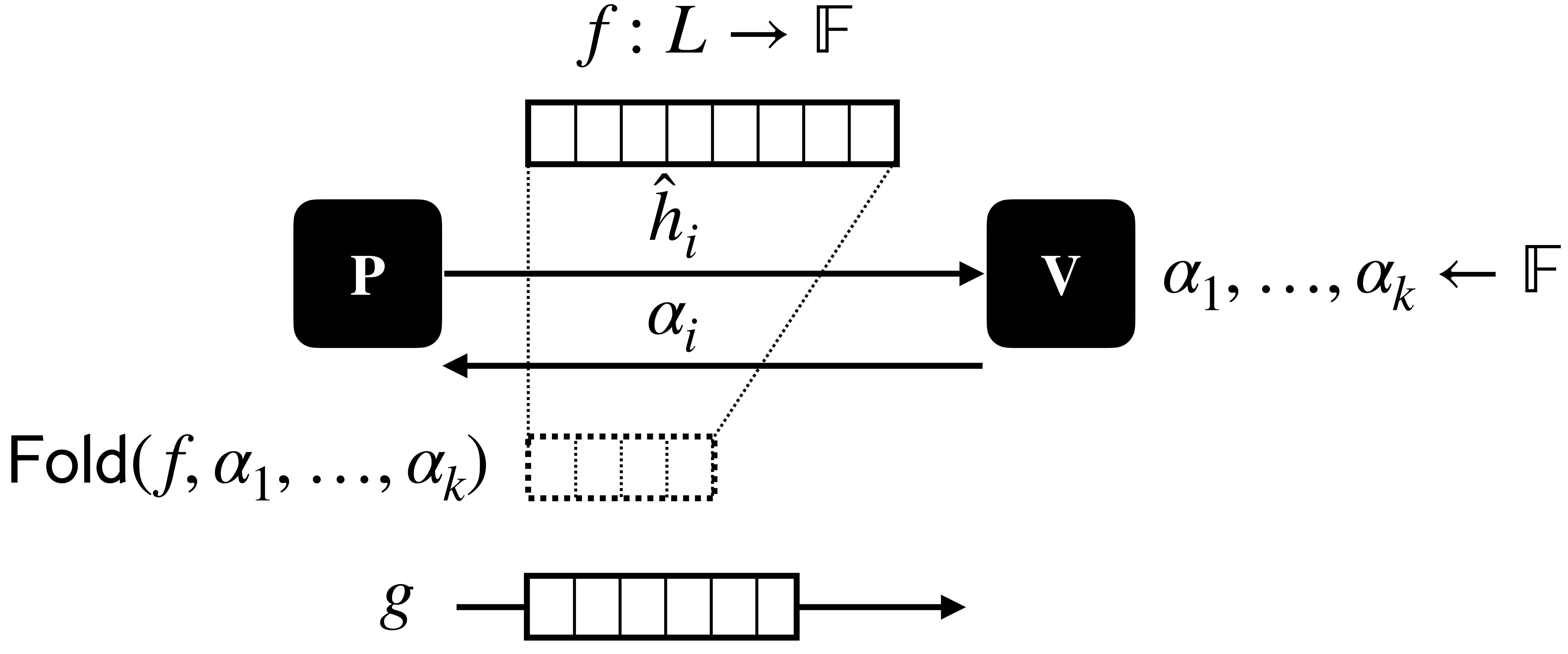
Many ways this can be done: **we chose random linear combination.**

WHIR 🌪️

WHIR

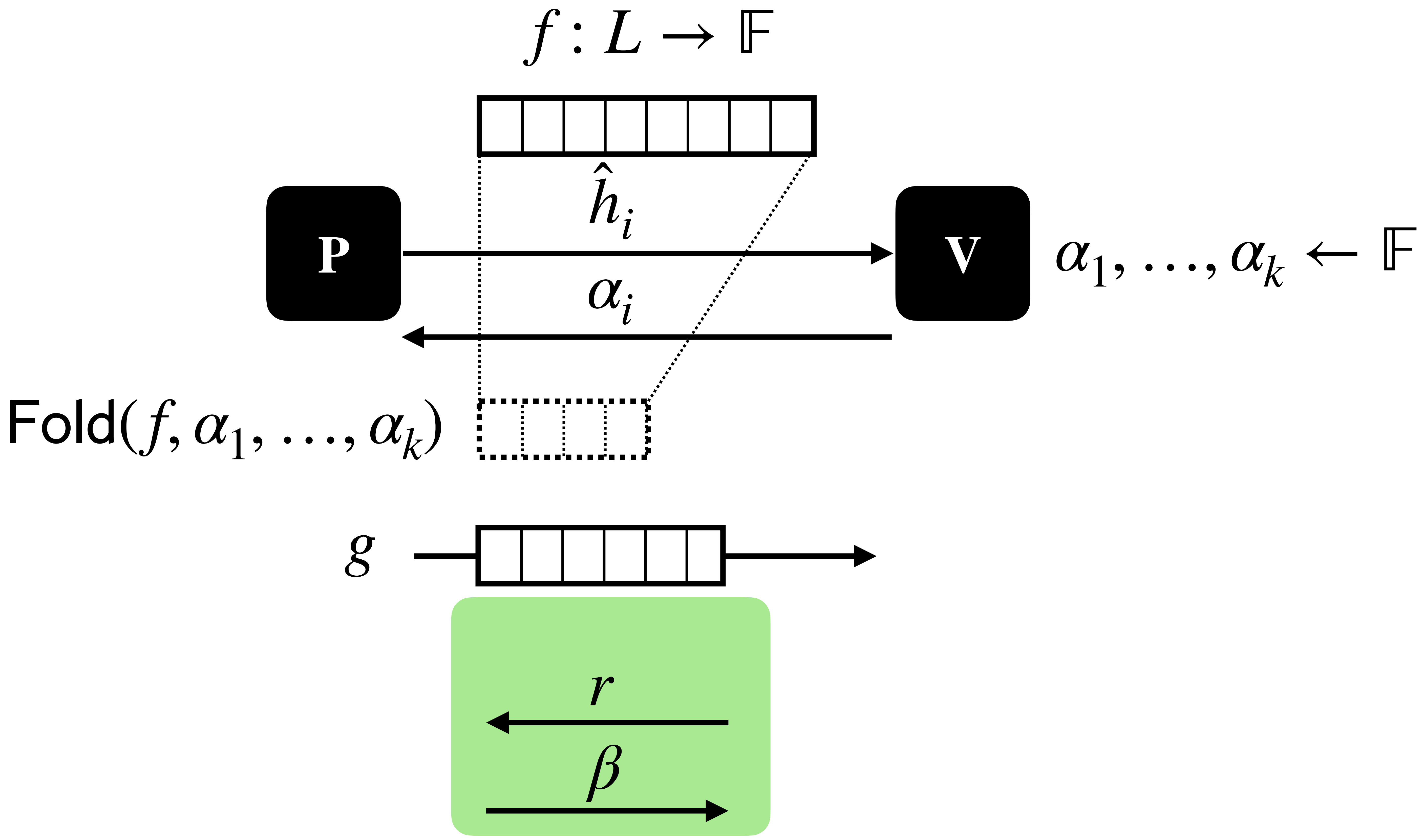

$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
$\mathsf{Fold}(f, \alpha_1, \ldots, \alpha_k)$

WHIR 🌪️



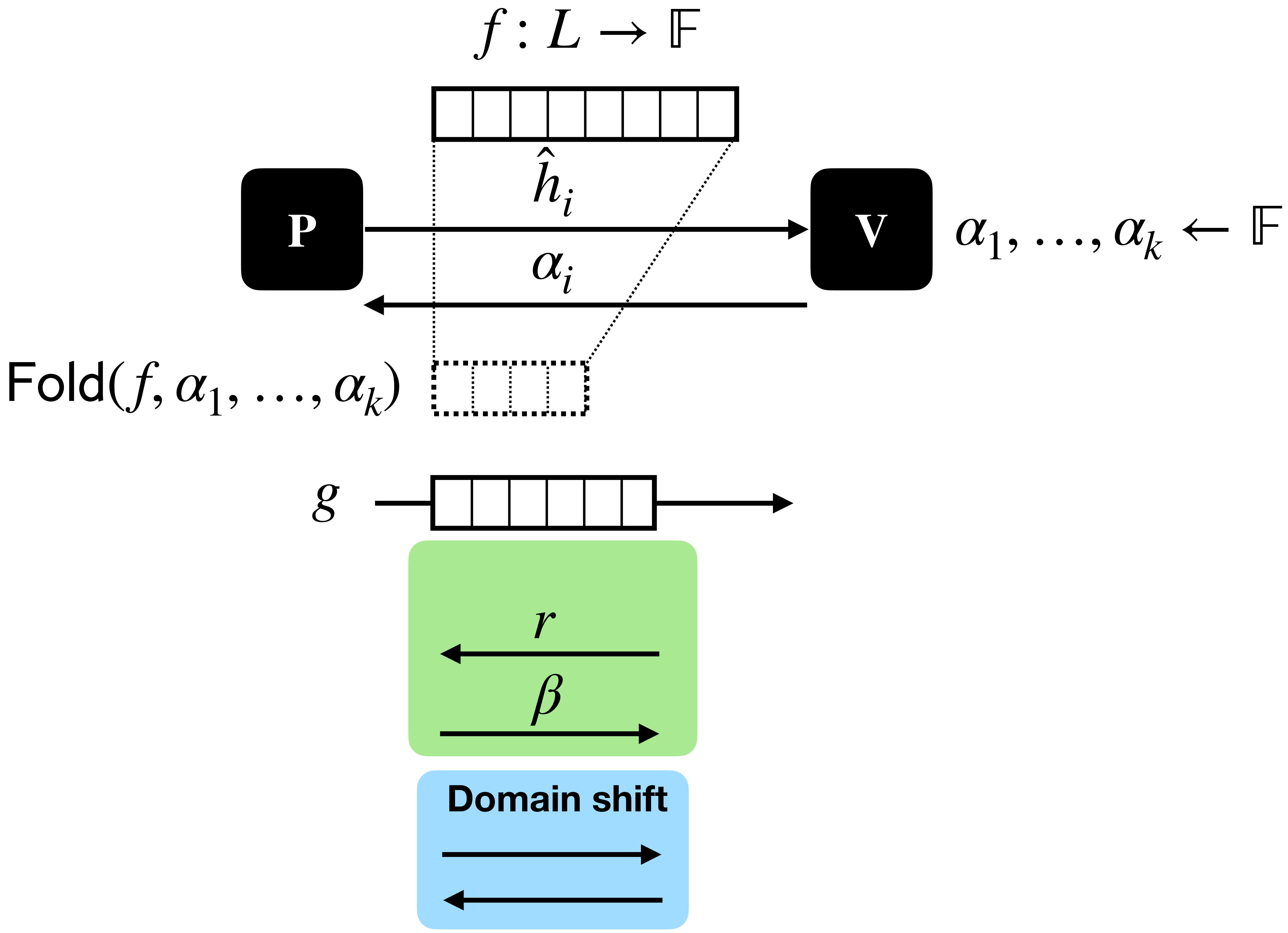
$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g

WHIR 🌪️

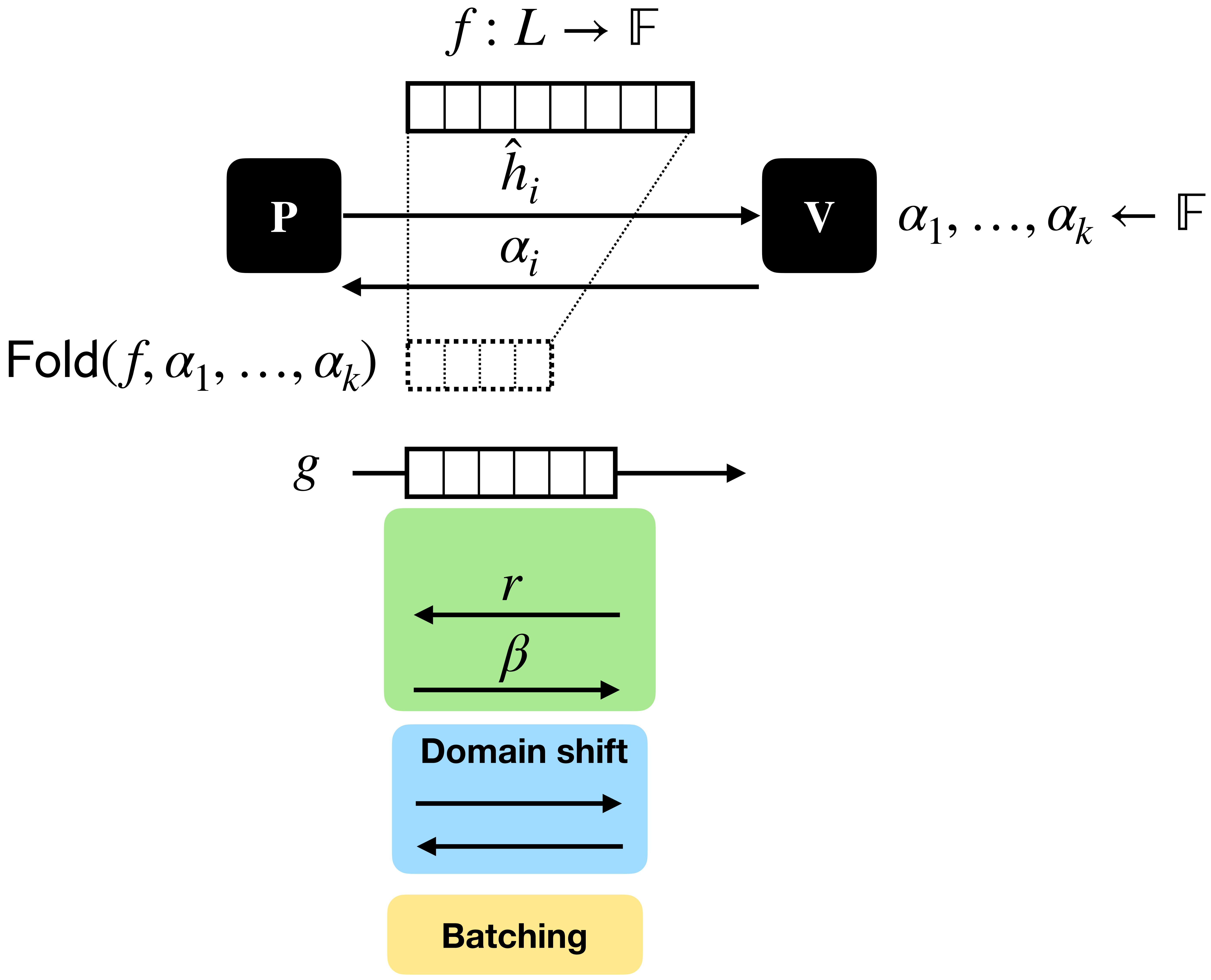


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β

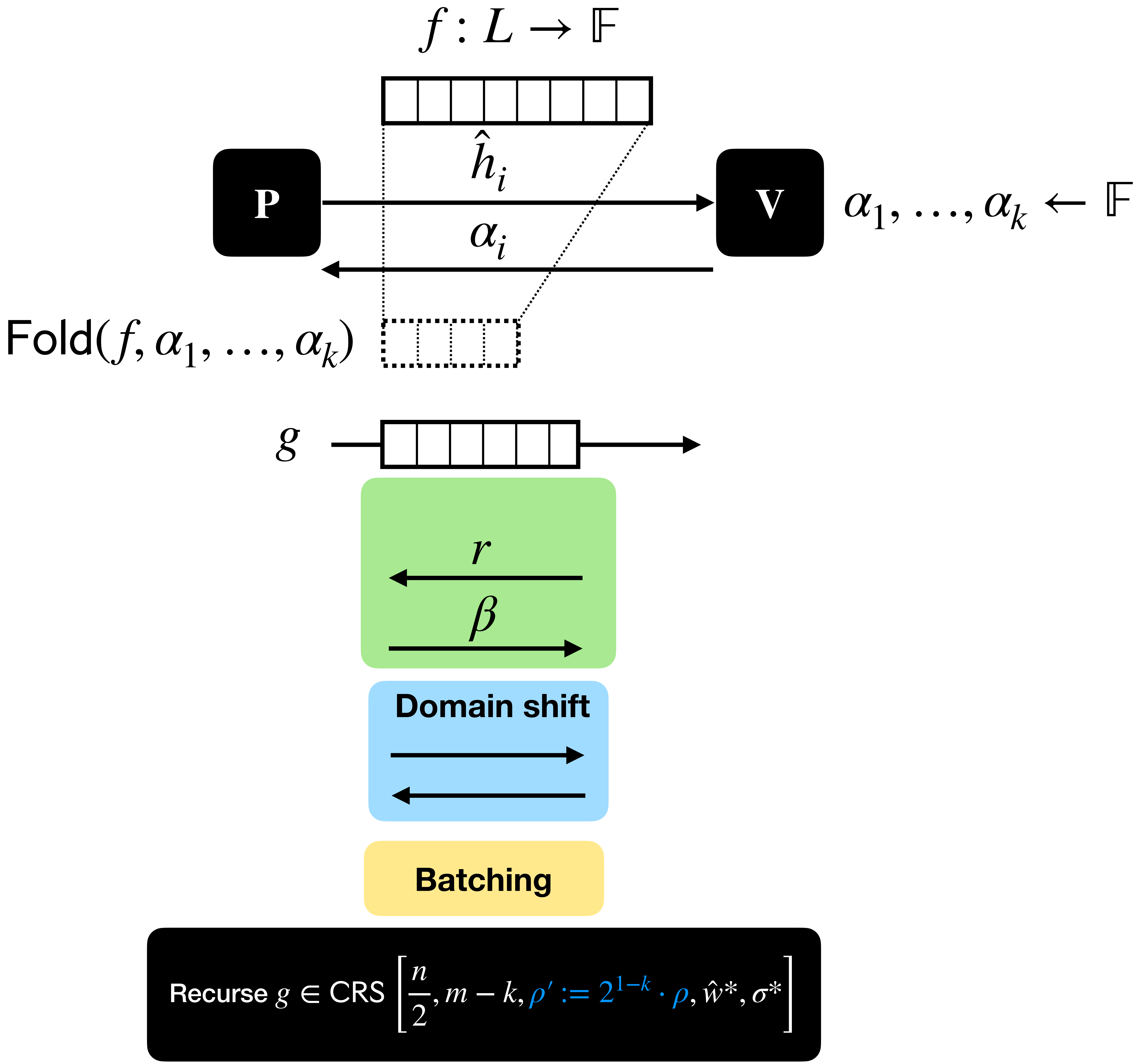
WHIR 🌪️


$f : L \to \mathbb{F}$
P
$\hat{h}_i$
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift

WHIR 🌪️


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
α_i
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift
Batching

WHIR 🌪️



$f : L \to \mathbb{F}$
P
$\hat{h}_i$
V
α_i
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift
Batching
Recurse $g \in$ CRS $\left[\frac{n}{2}, m-k, \rho' := 2^{1-k} \cdot \rho, \hat{w}^*, \sigma^*\right]$

Application: Σ-IOP

High soundness compilation using constrained codes

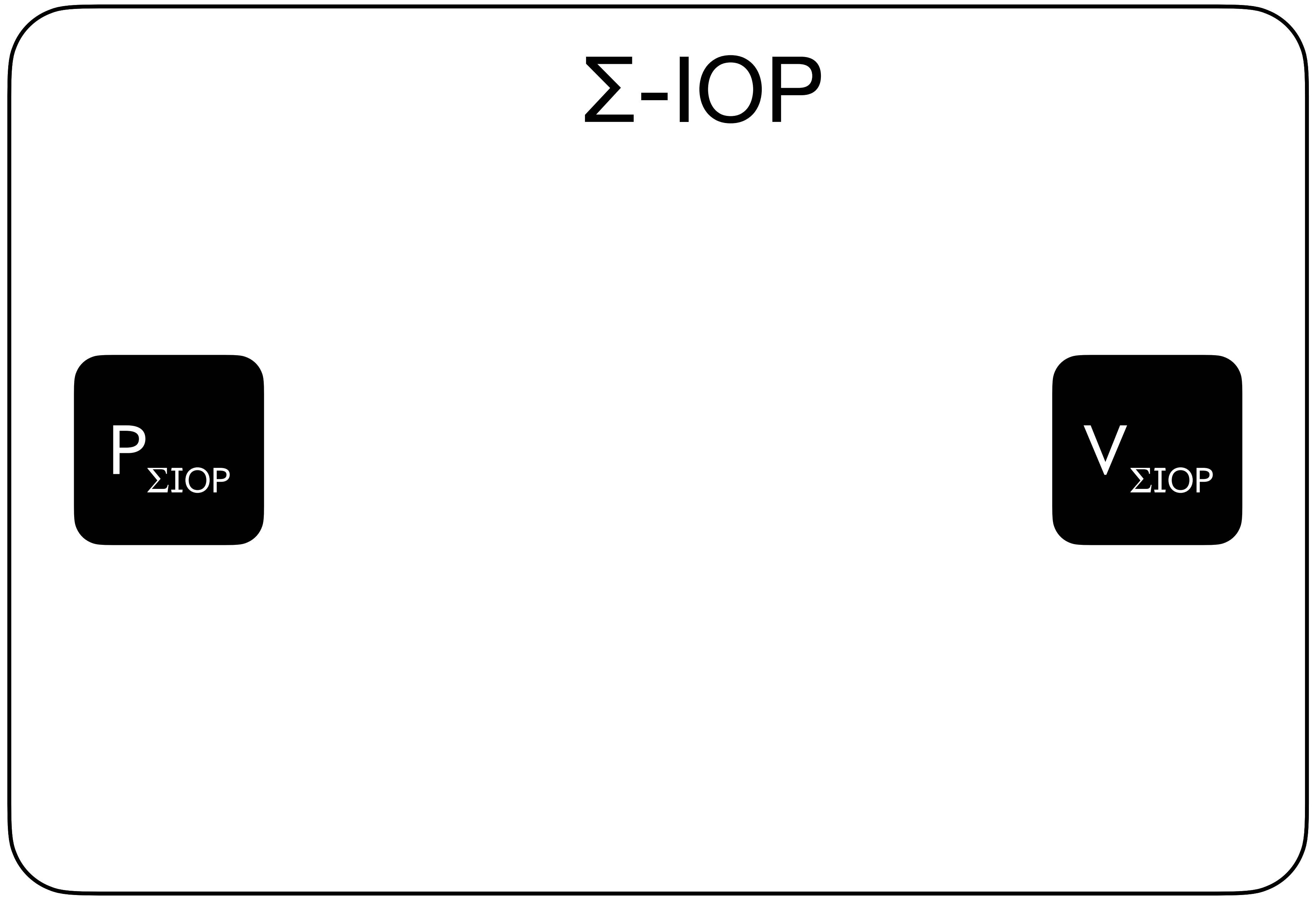
Application: Σ-IOP

High soundness compilation using constrained codes

Σ-IOP

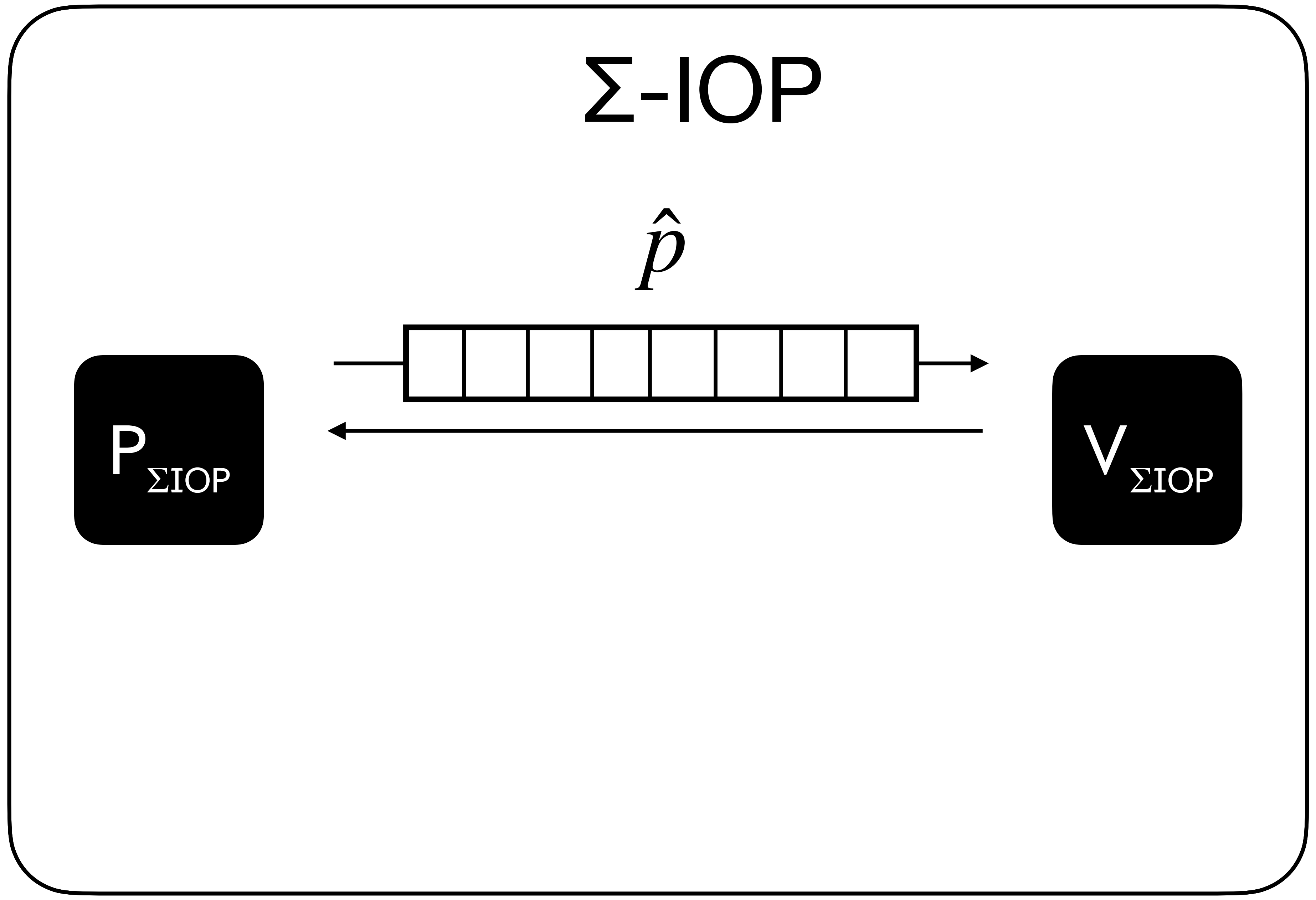
Application: Σ-IOP

High soundness compilation using constrained codes



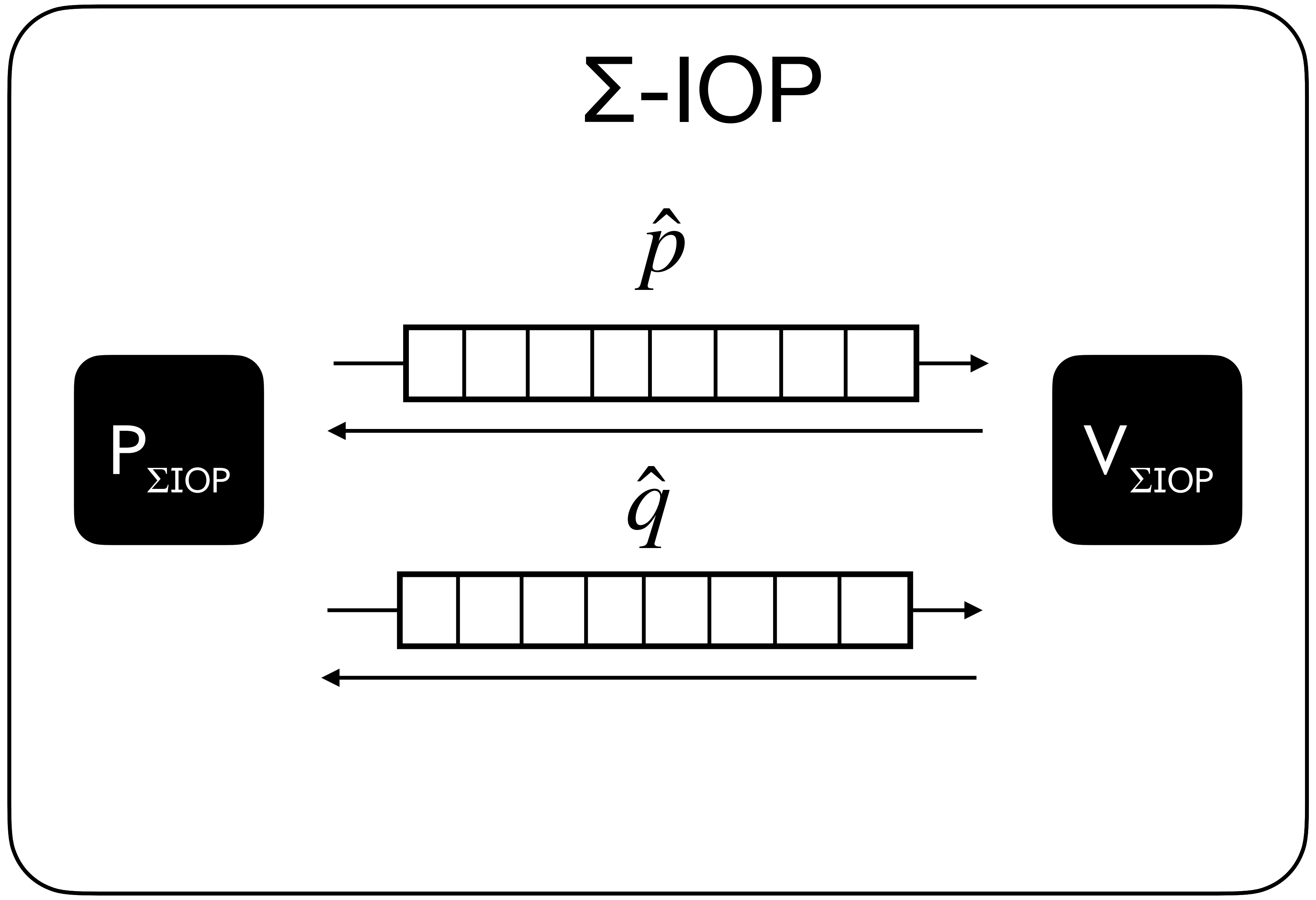
Application: Σ-IOP

High soundness compilation using constrained codes



Application: Σ-IOP

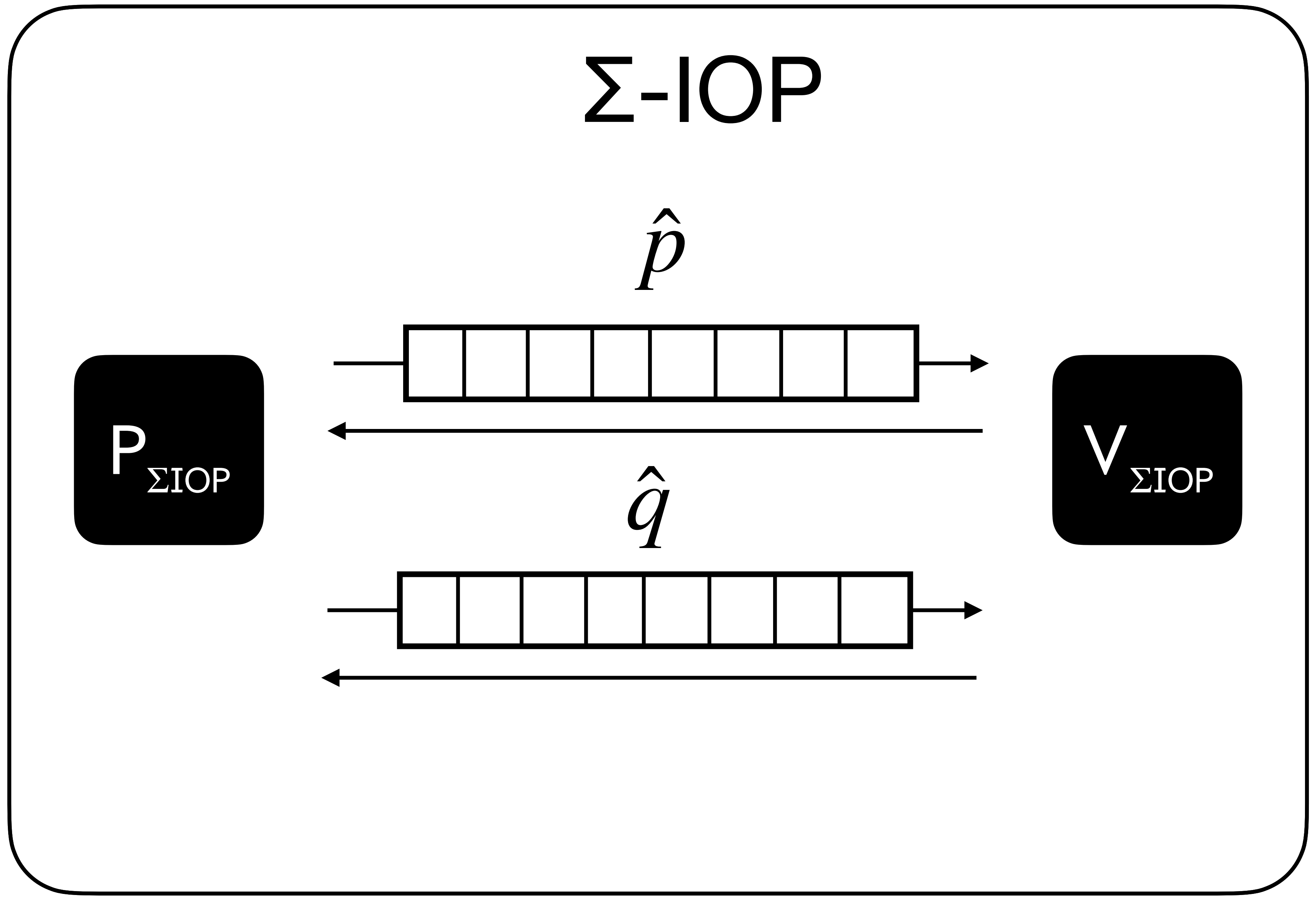
High soundness compilation using constrained codes



Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes



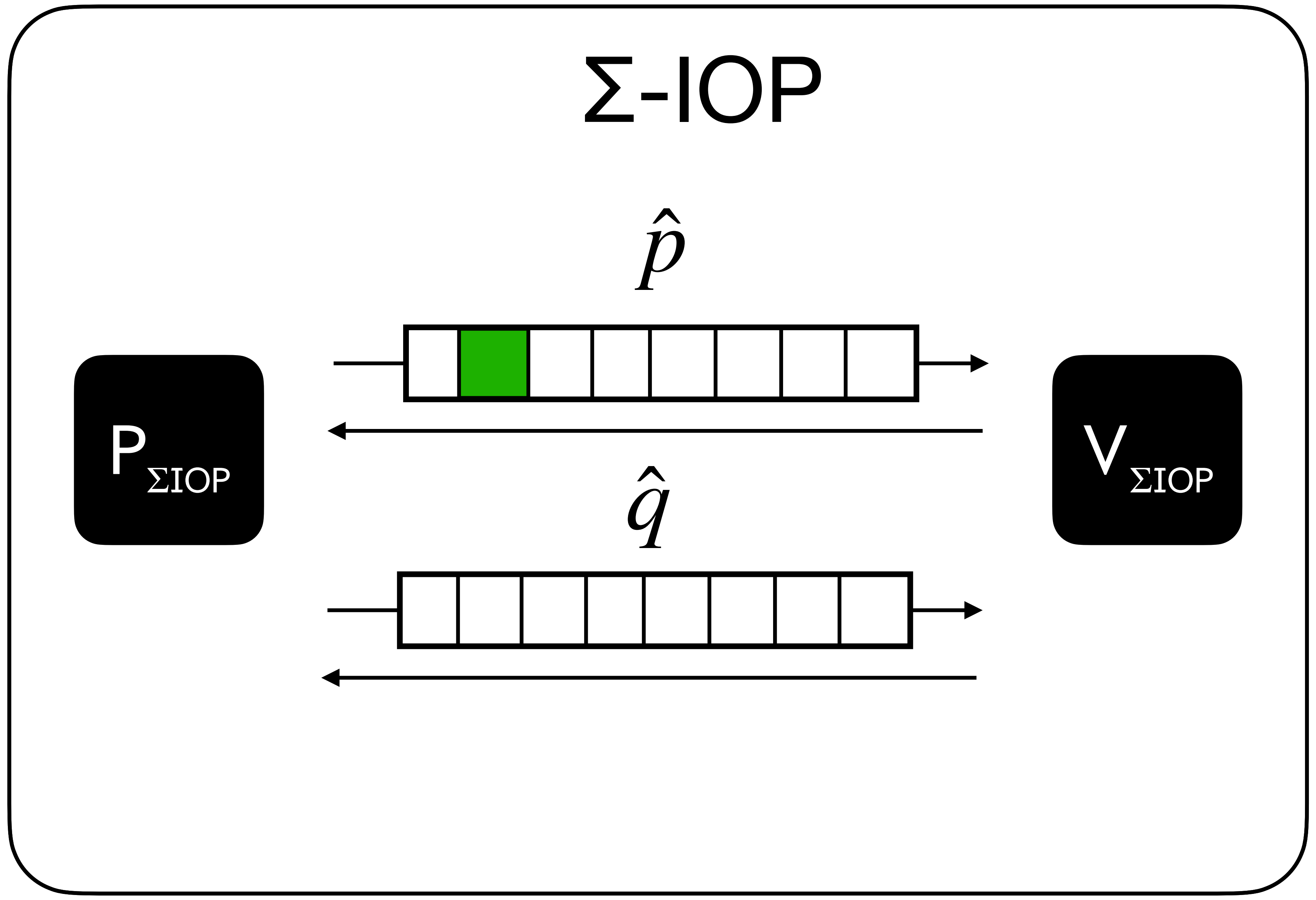
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



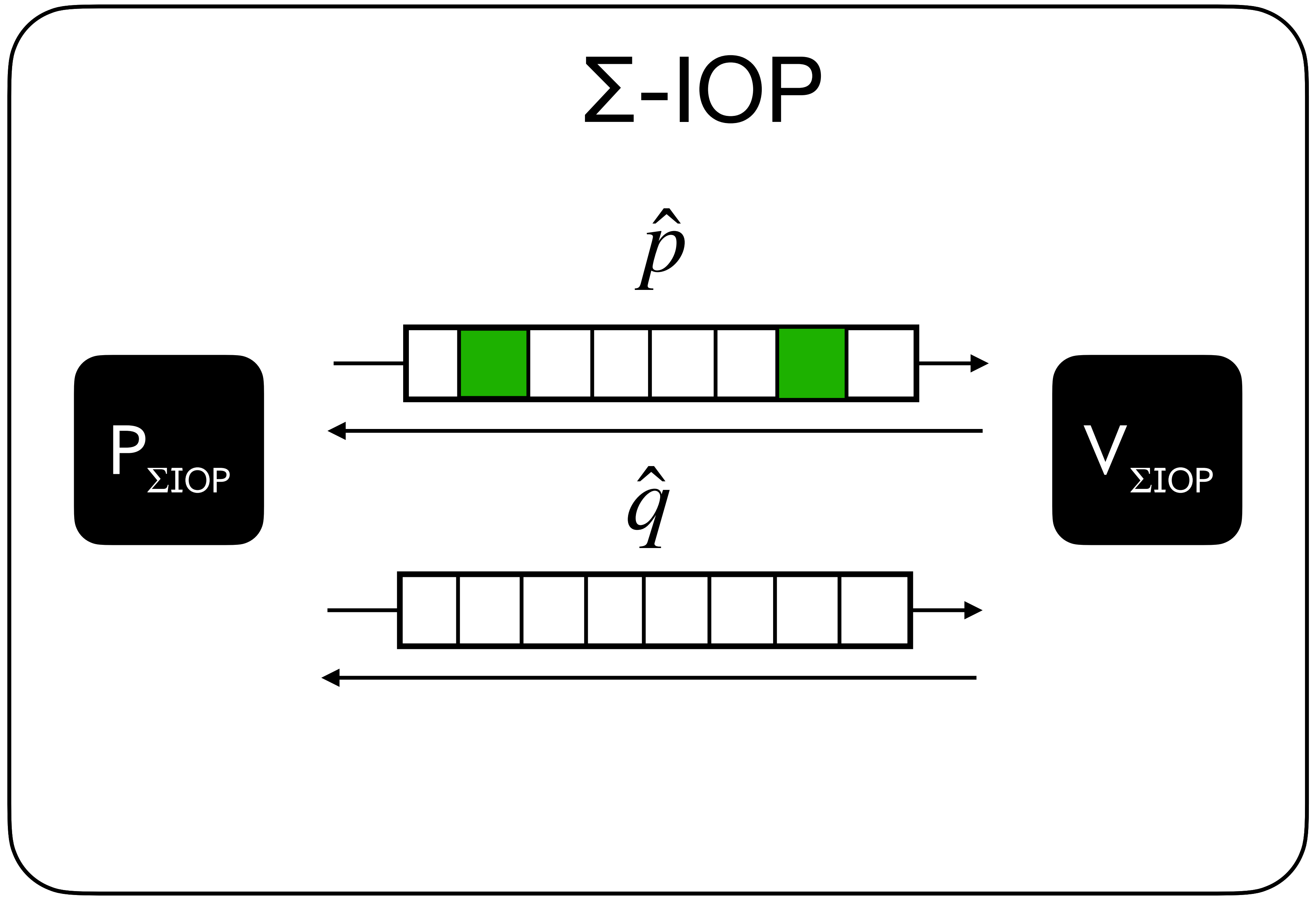
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



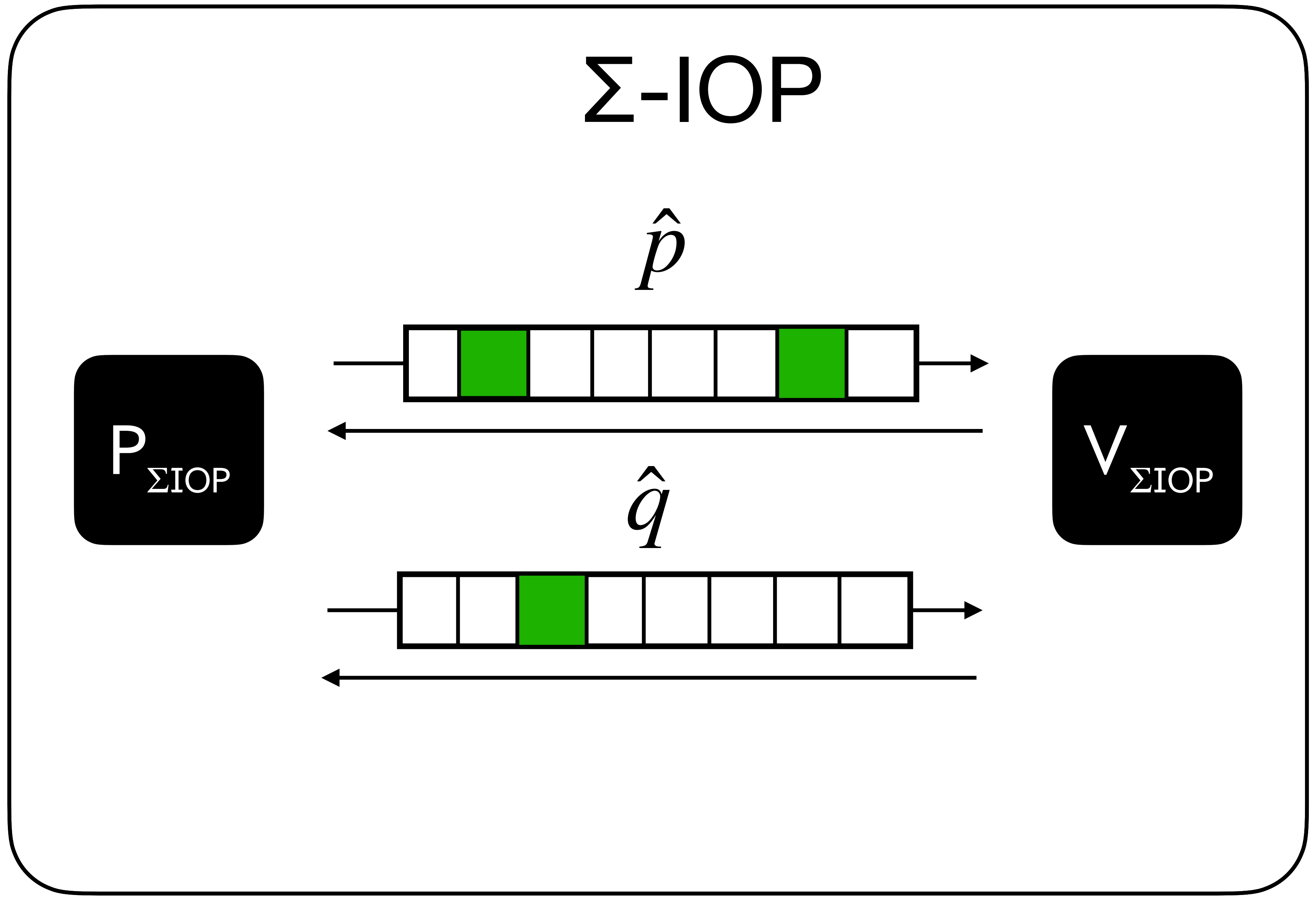
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

Generalizes univariate and multilinear PIOPs at no extra cost!

High soundness compilation using constrained codes



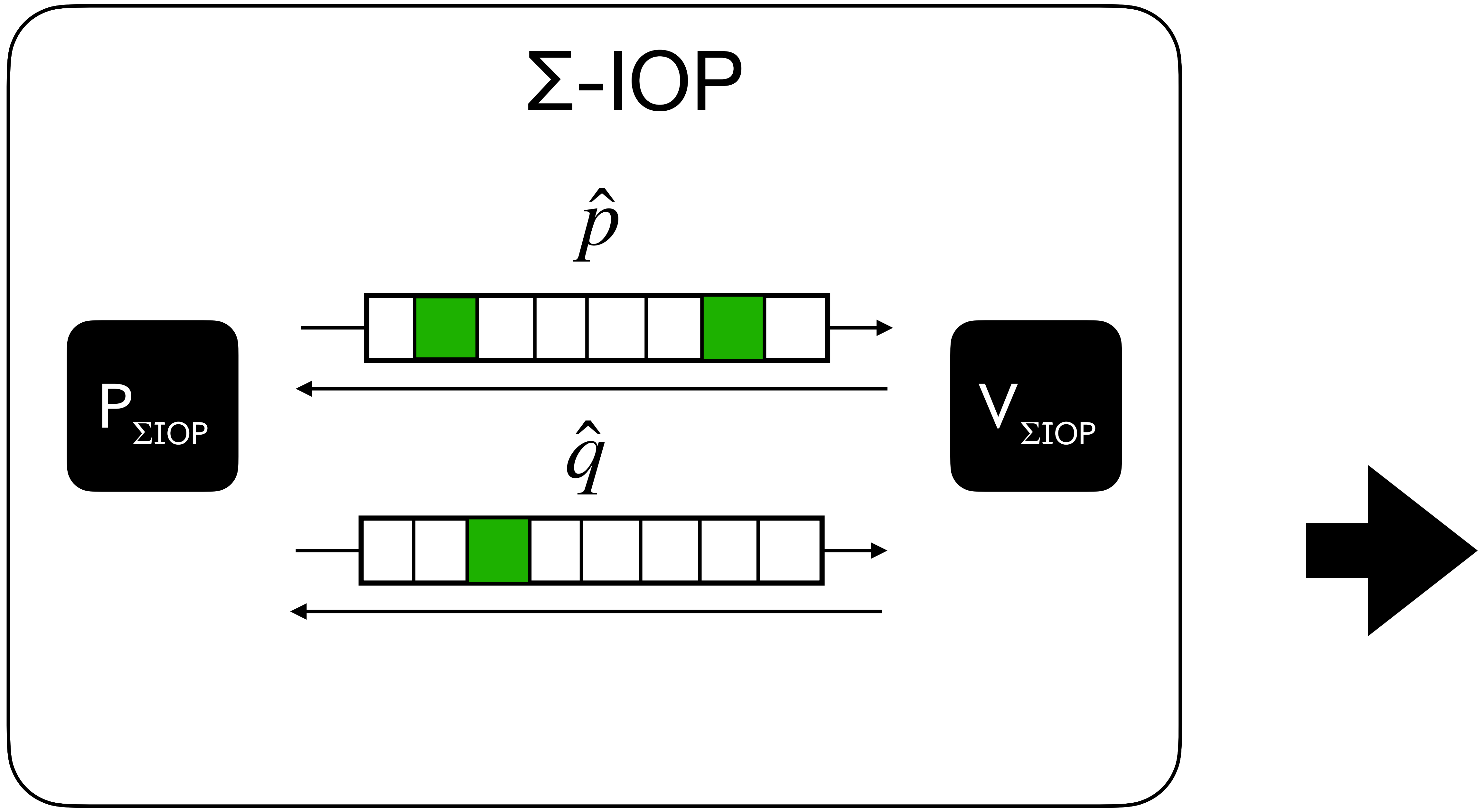
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



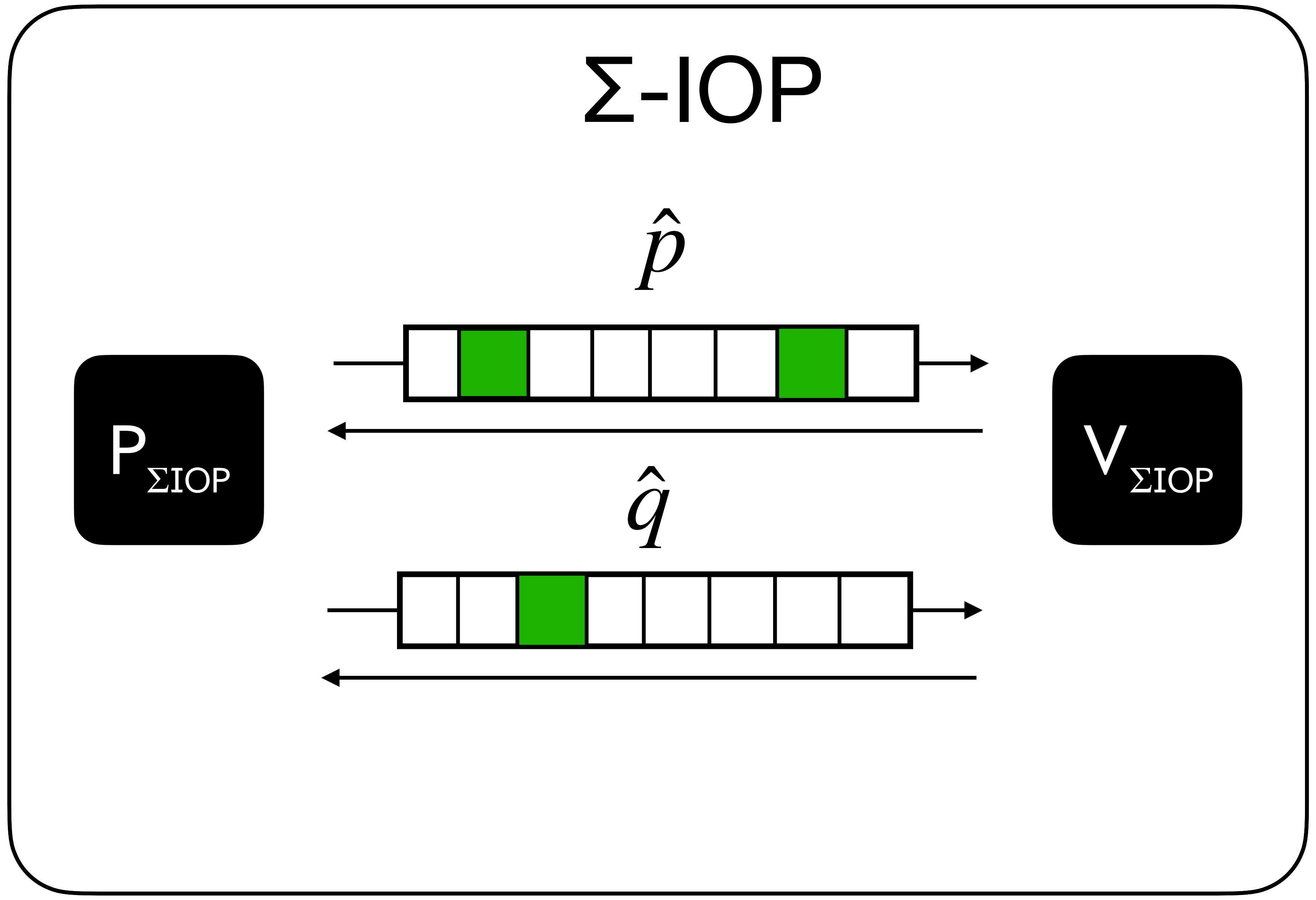
Verifier can ask **sumcheck queries**

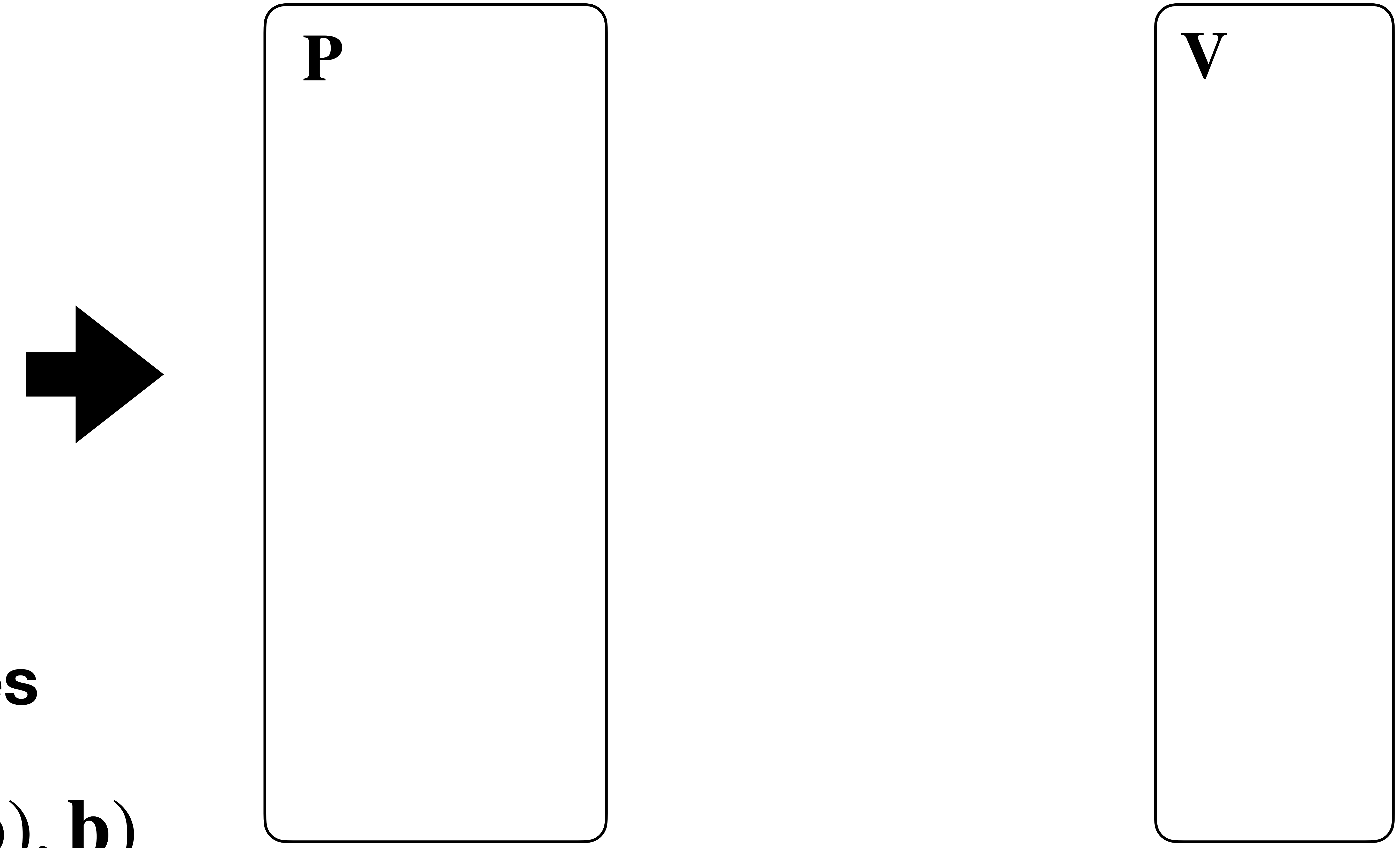
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes





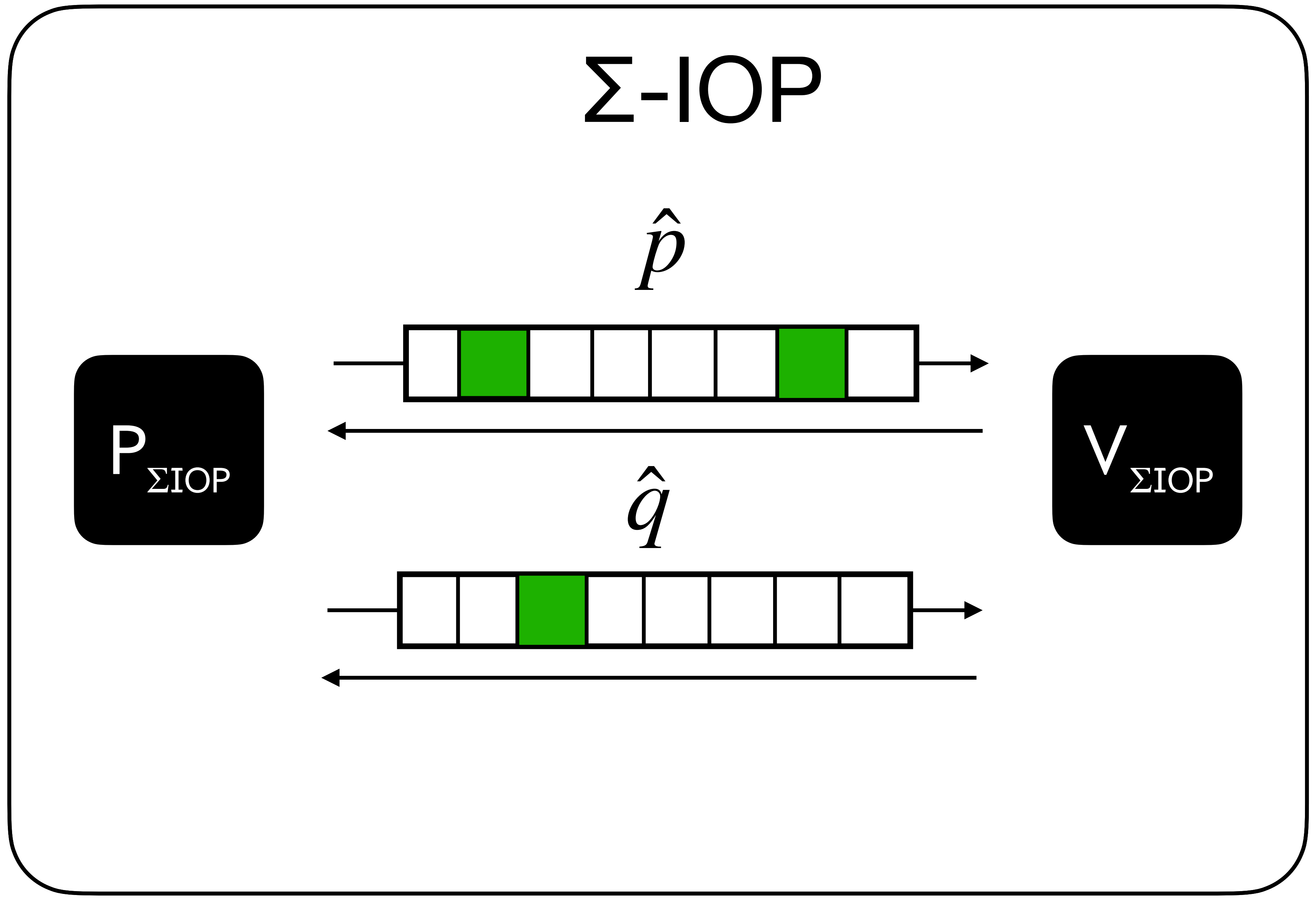
Verifier can ask **sumcheck queries**

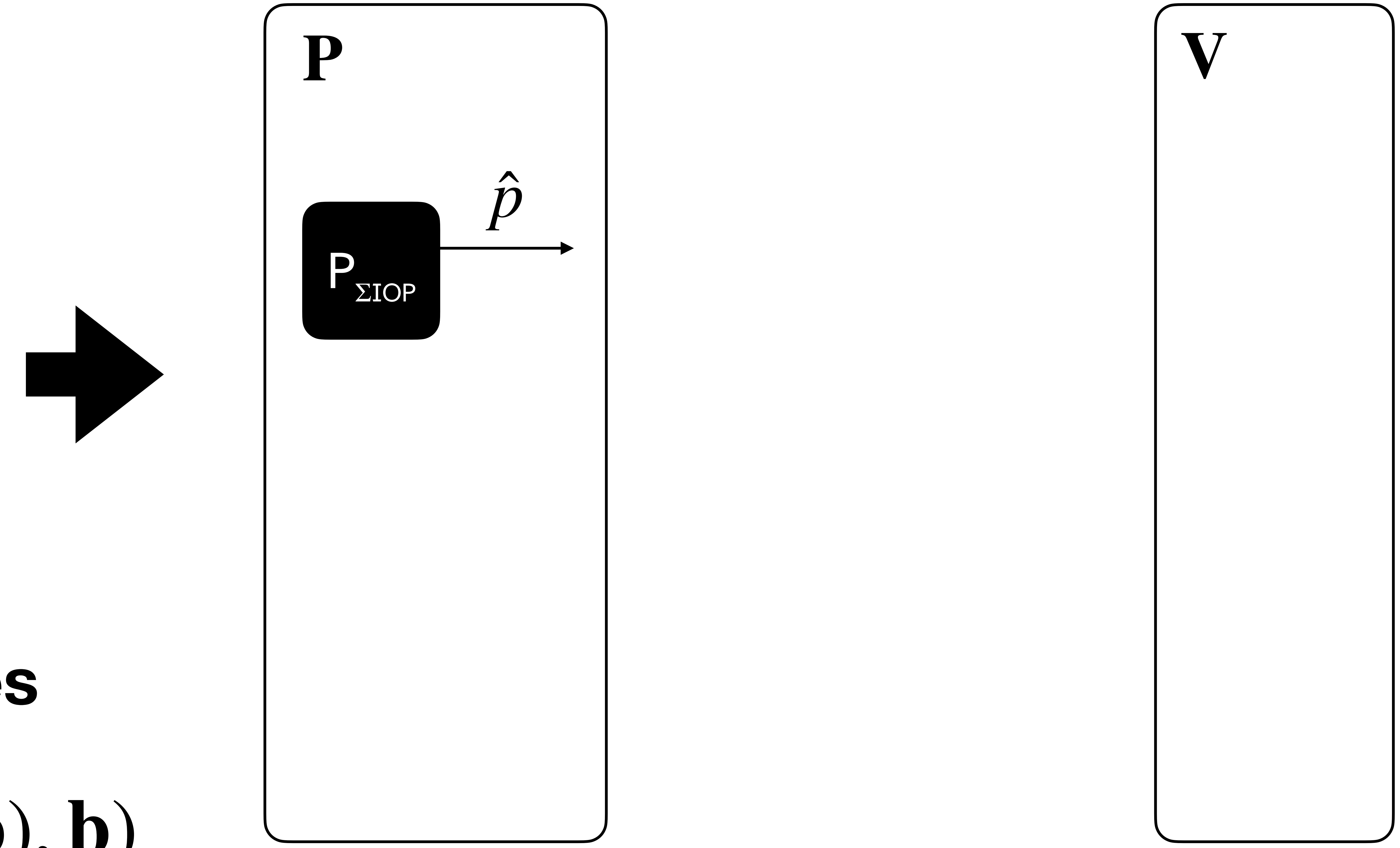
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!





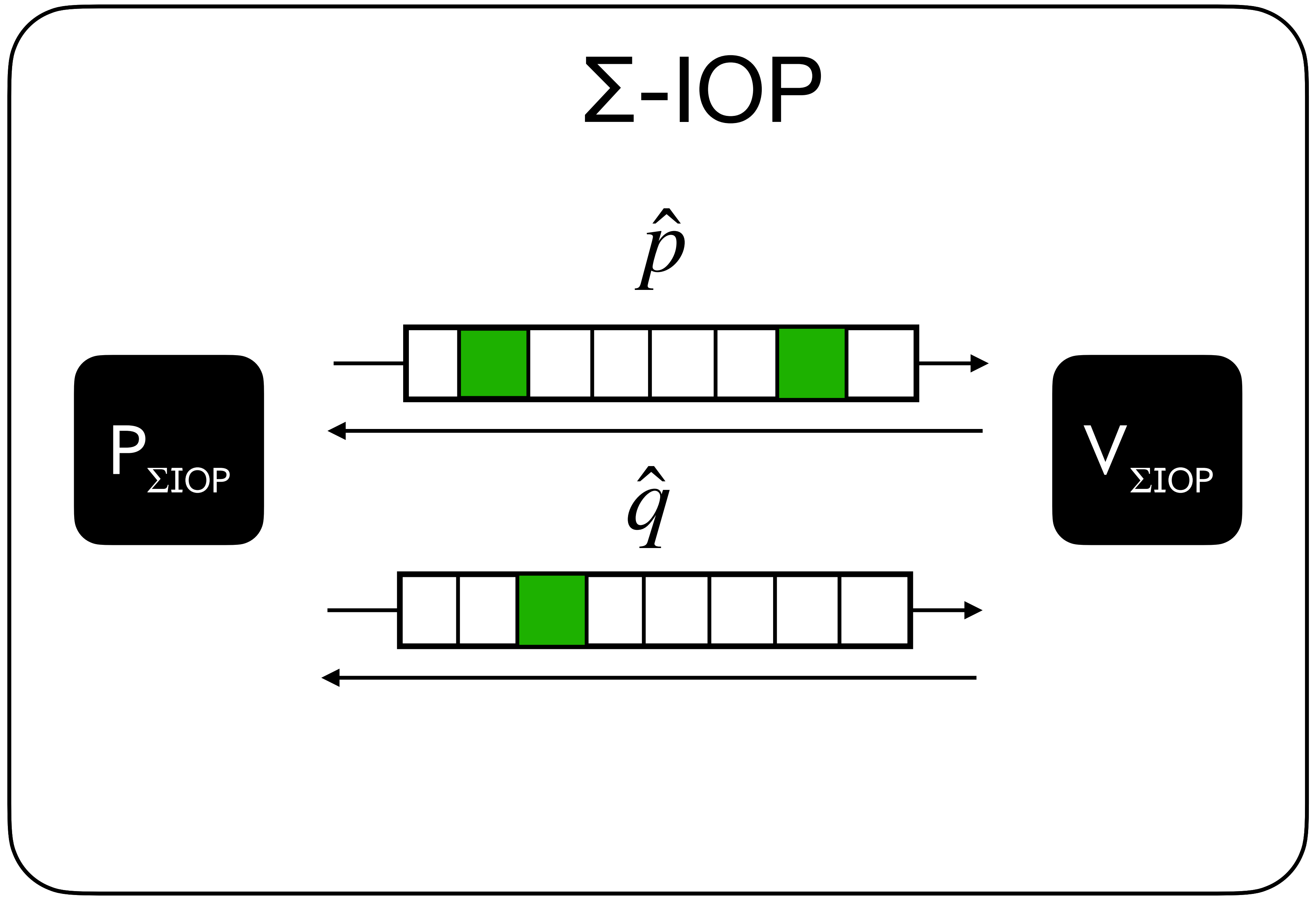
Verifier can ask **sumcheck queries**

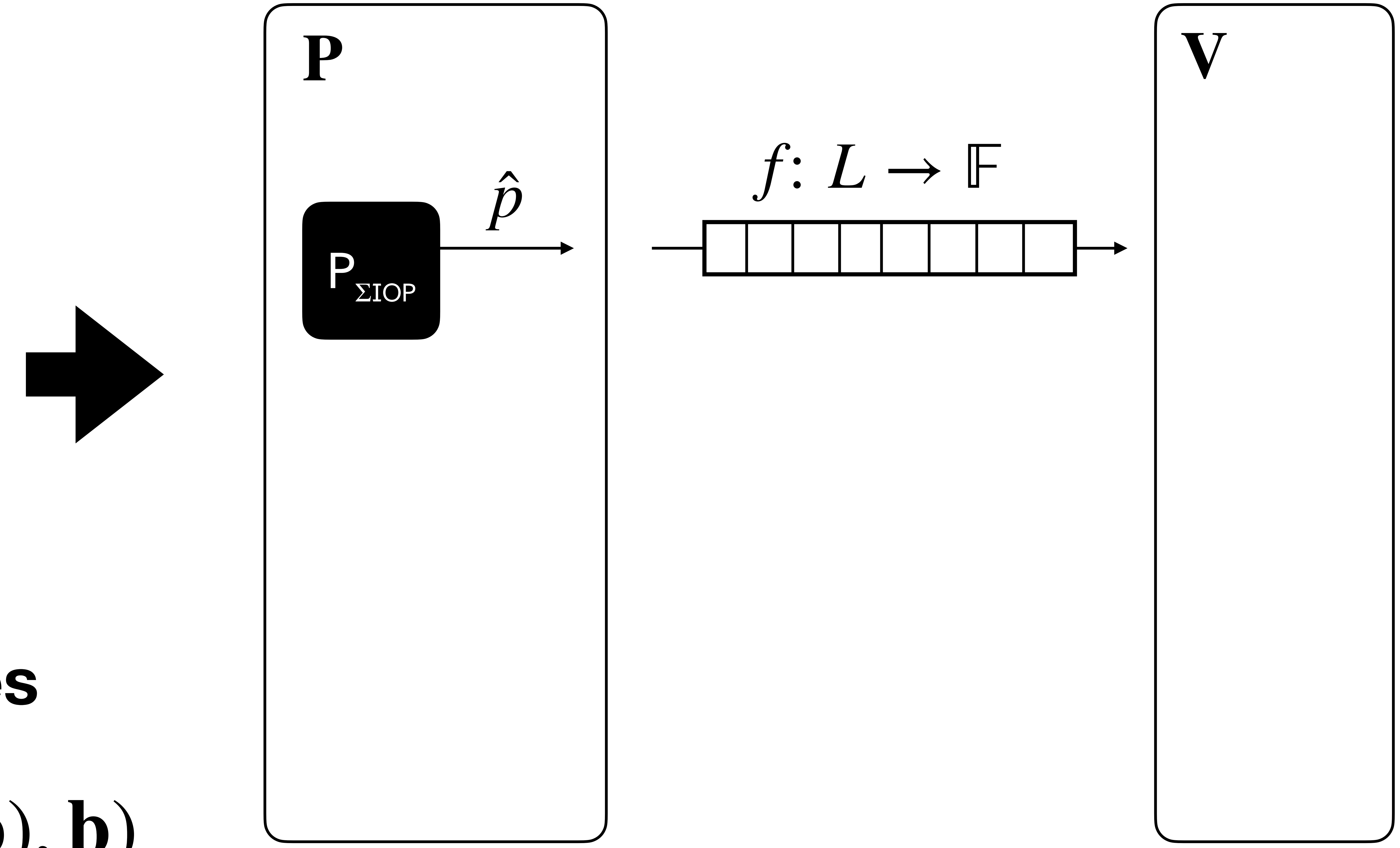
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes





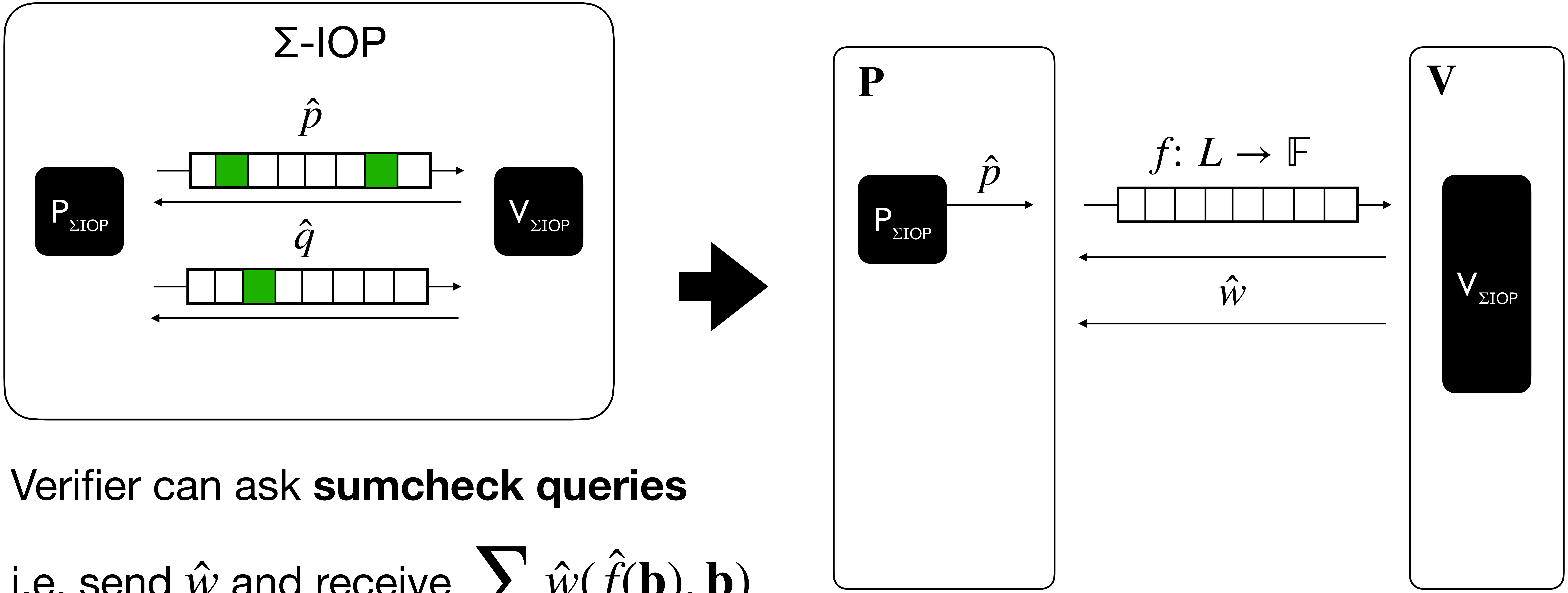
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



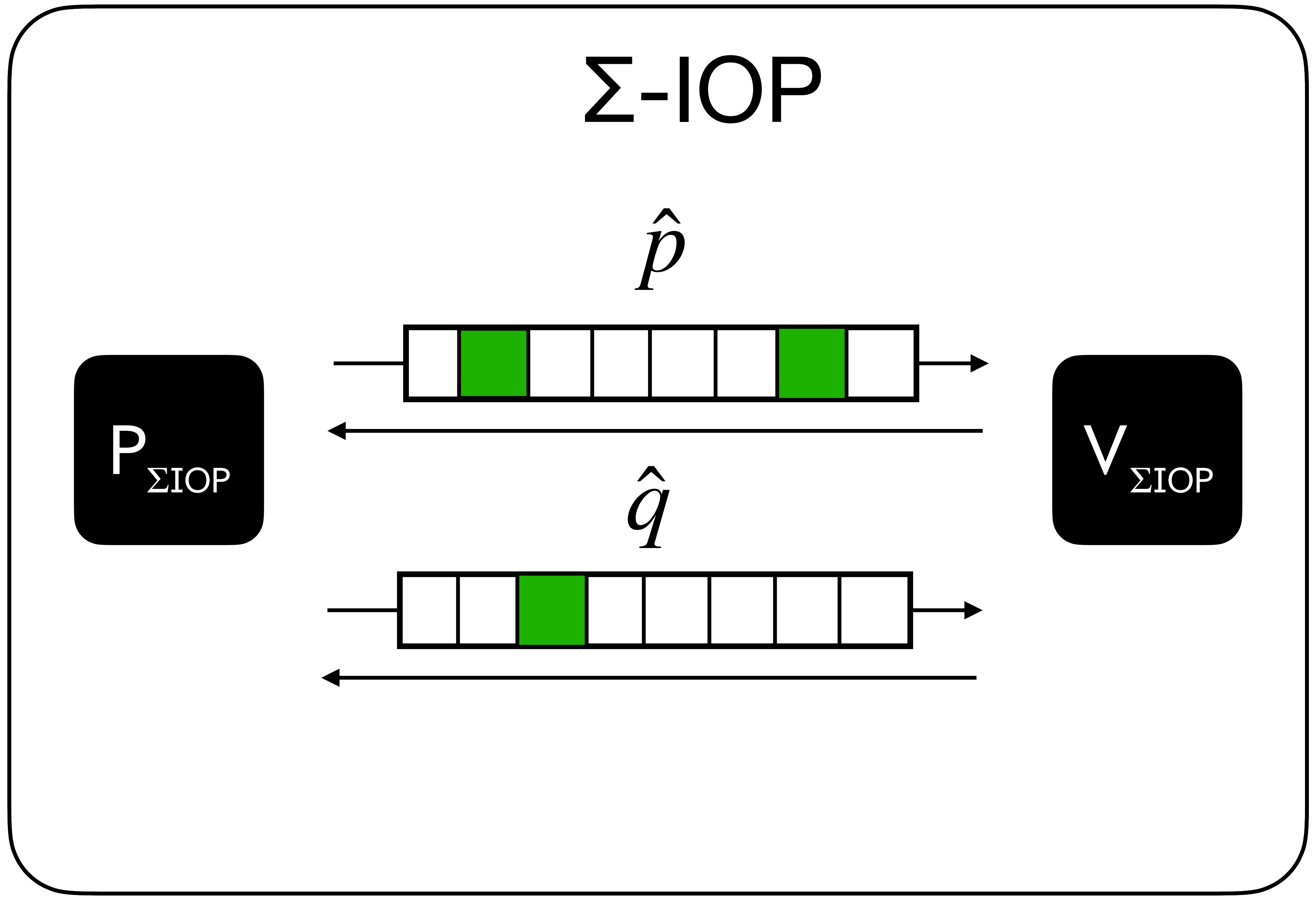
Verifier can ask **sumcheck queries**

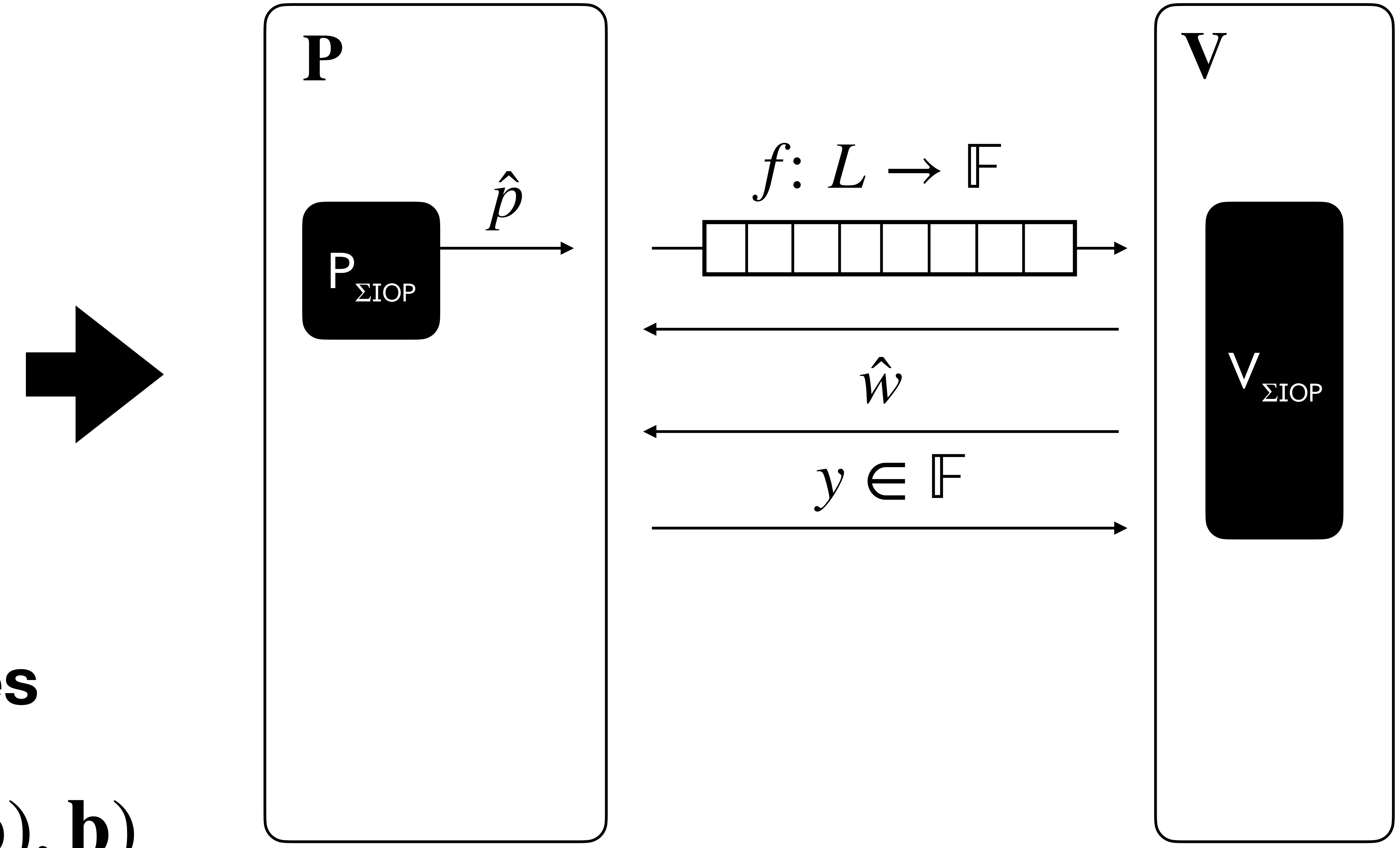
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes





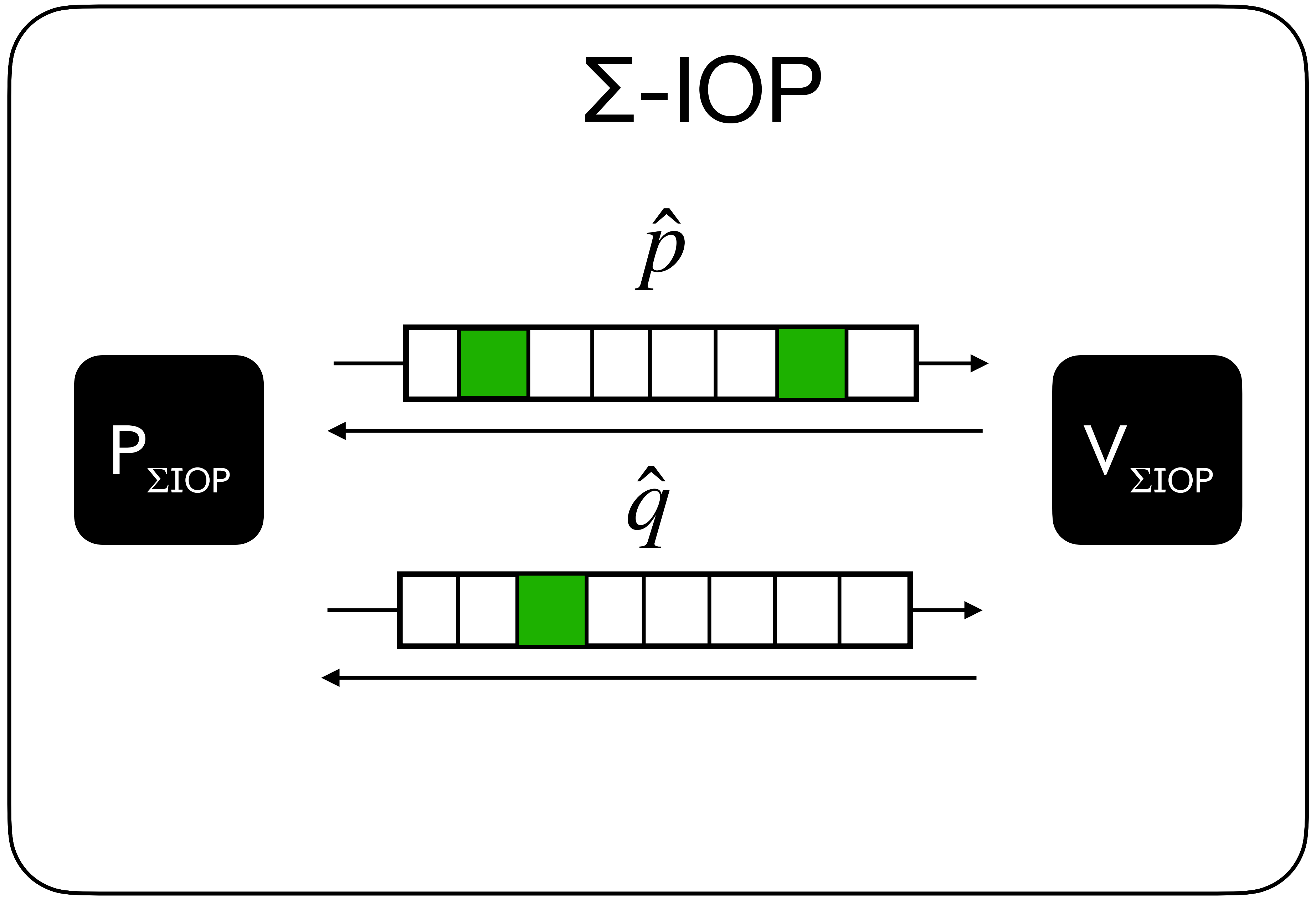
Verifier can ask **sumcheck queries**

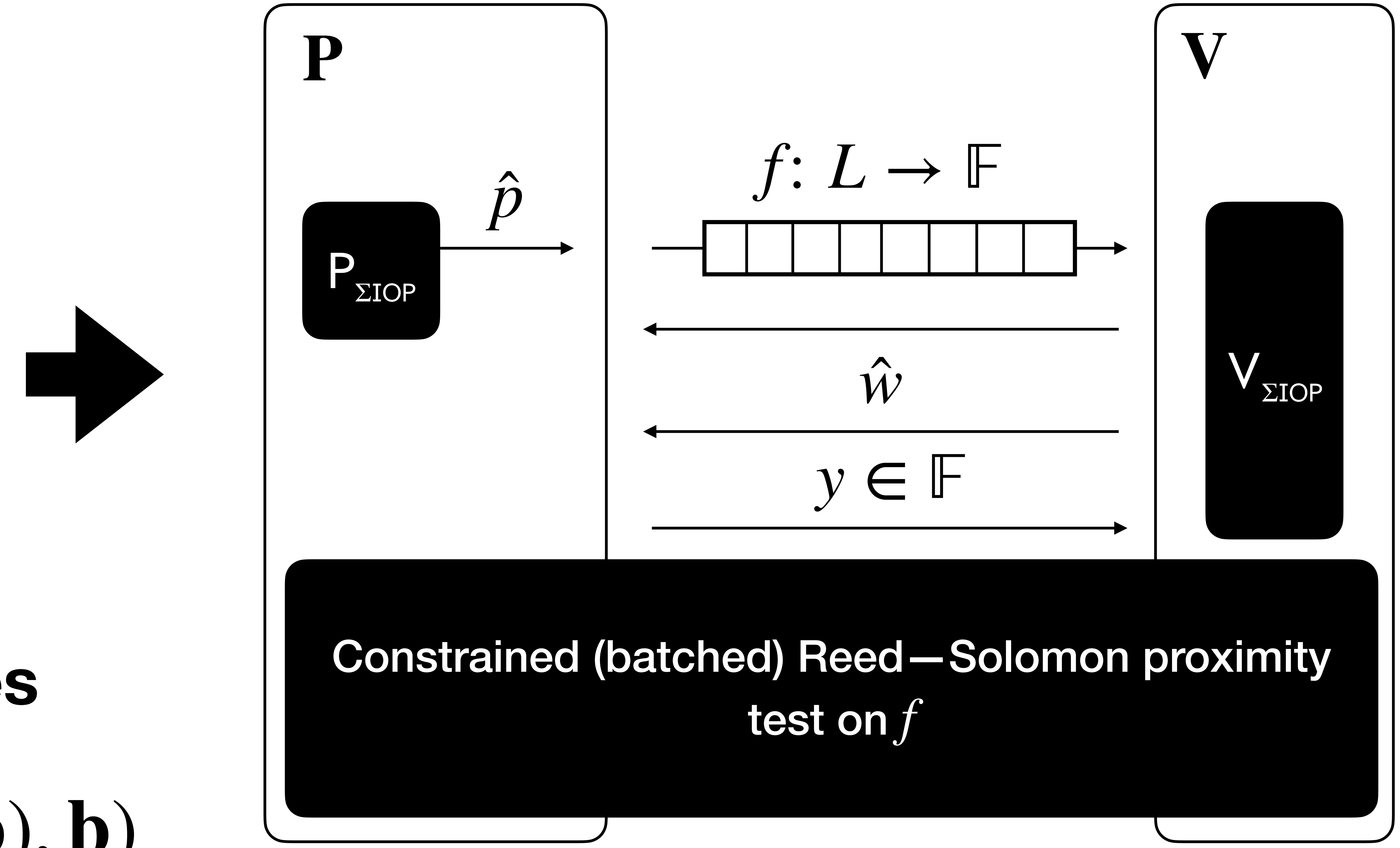
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes





Verifier can ask **sumcheck queries**

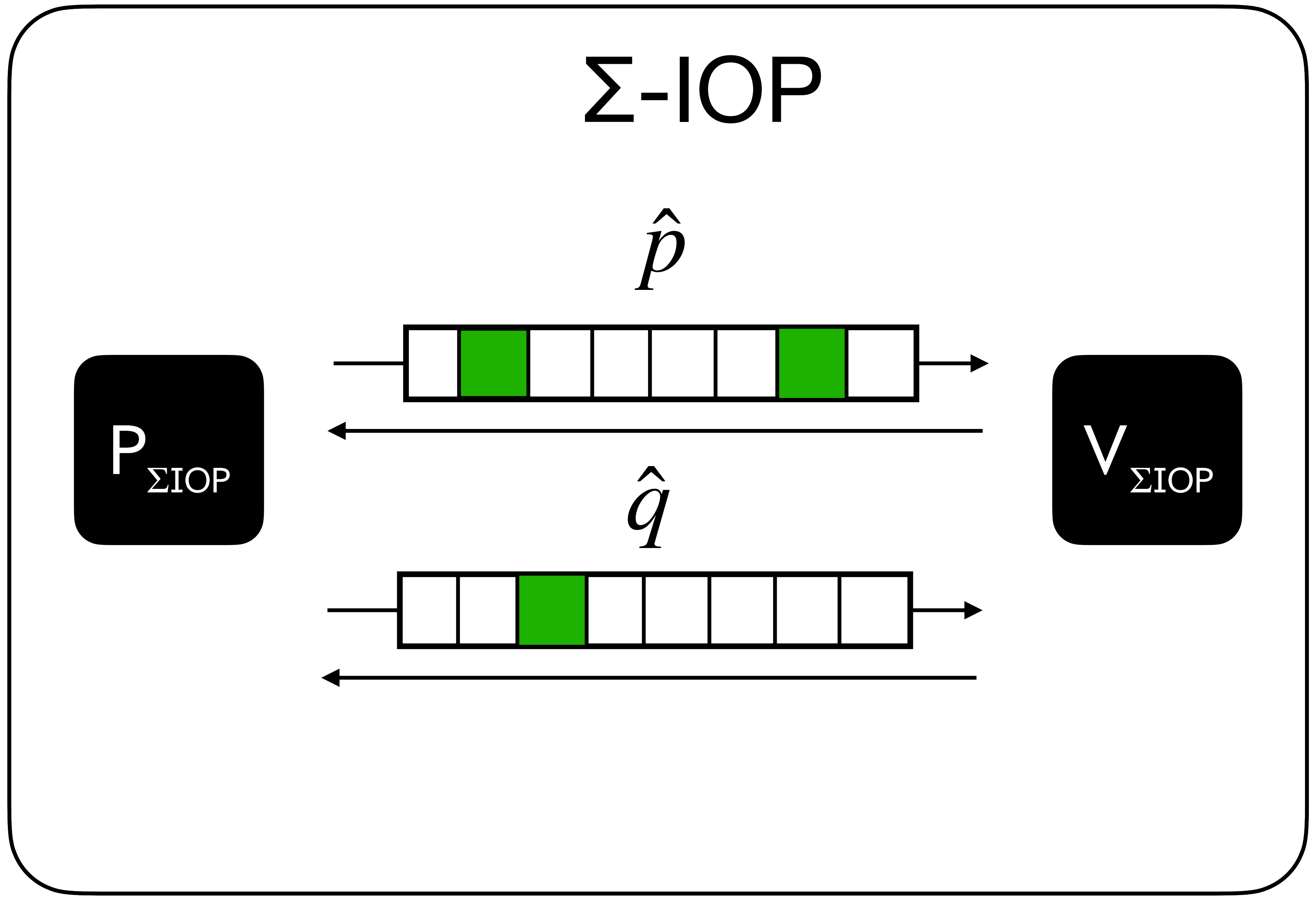
i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

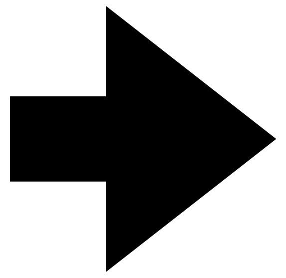
Application: Σ-IOP

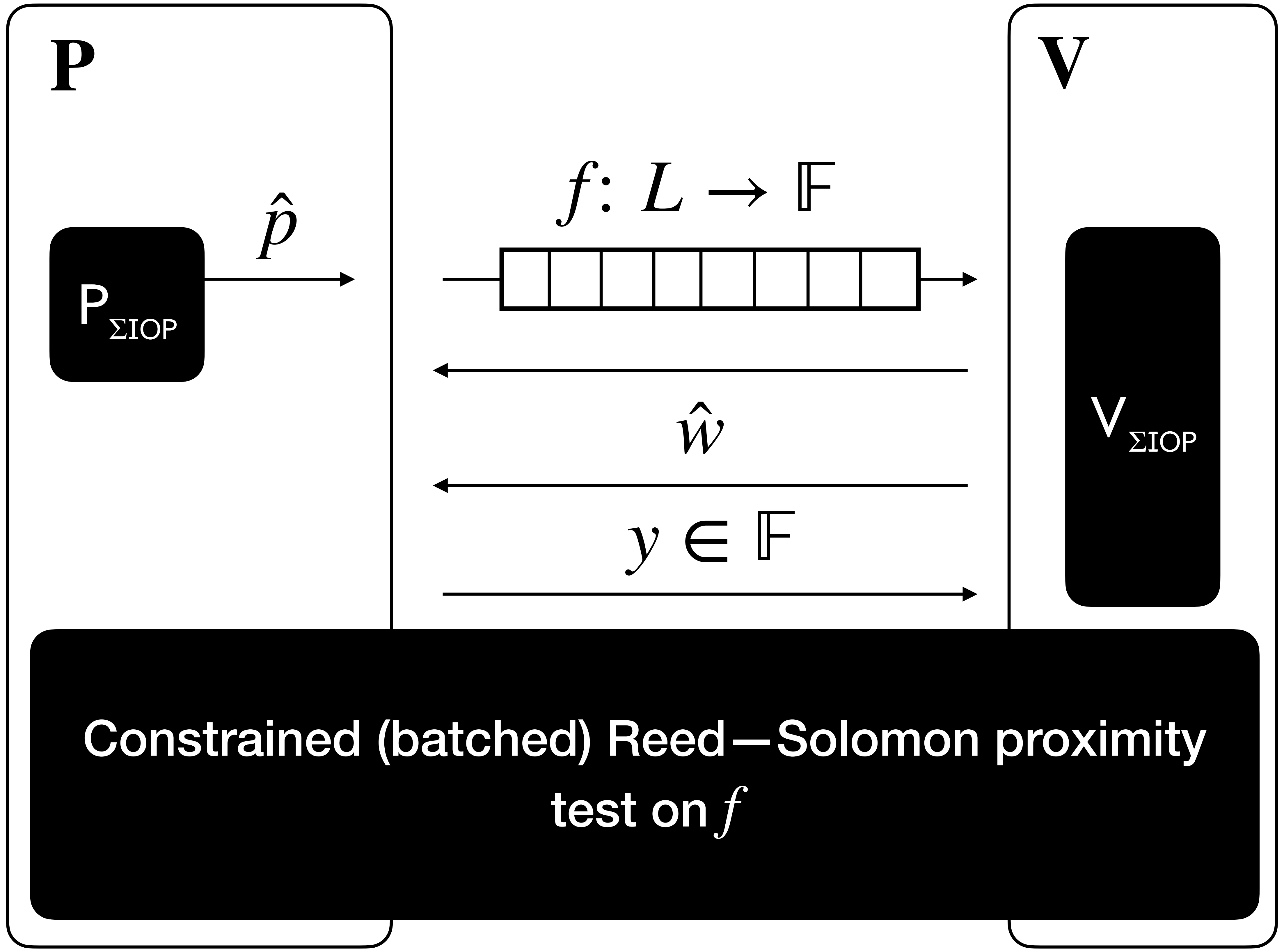
High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!

Q: Can we use this to do more **efficient arithmetizations?**







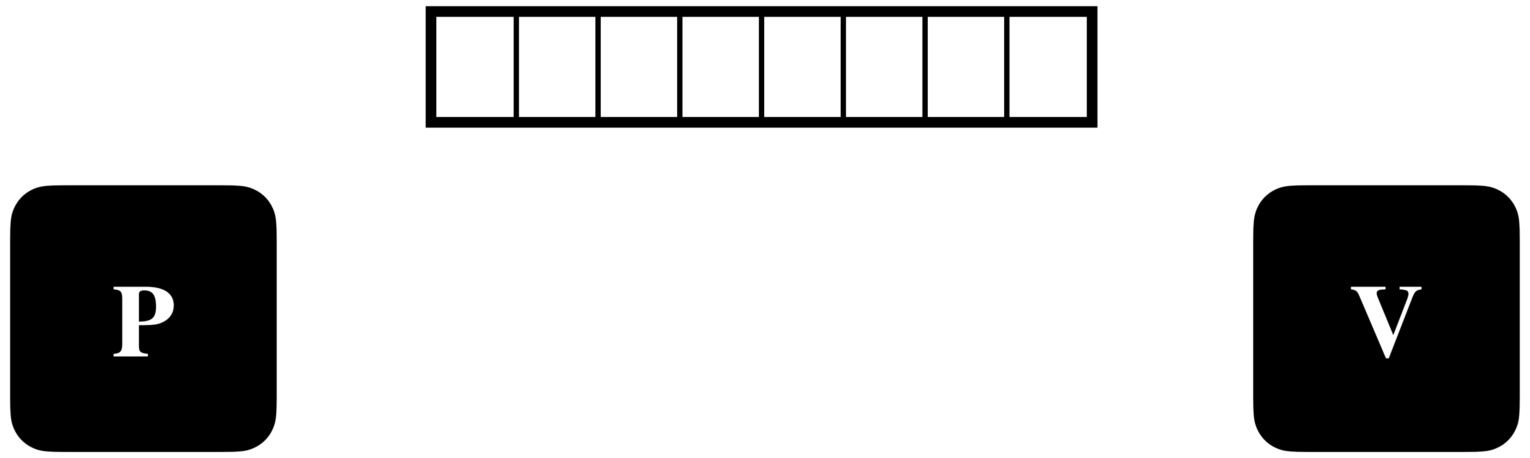
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

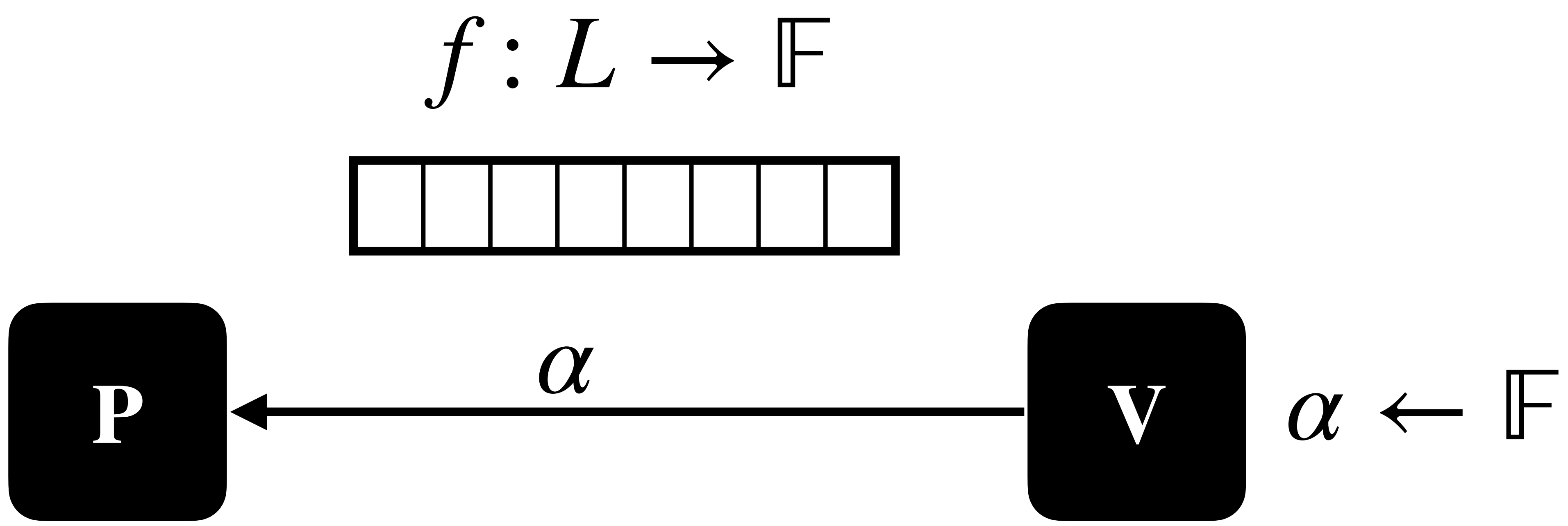
Review: FRI iteration

Review: FRI iteration

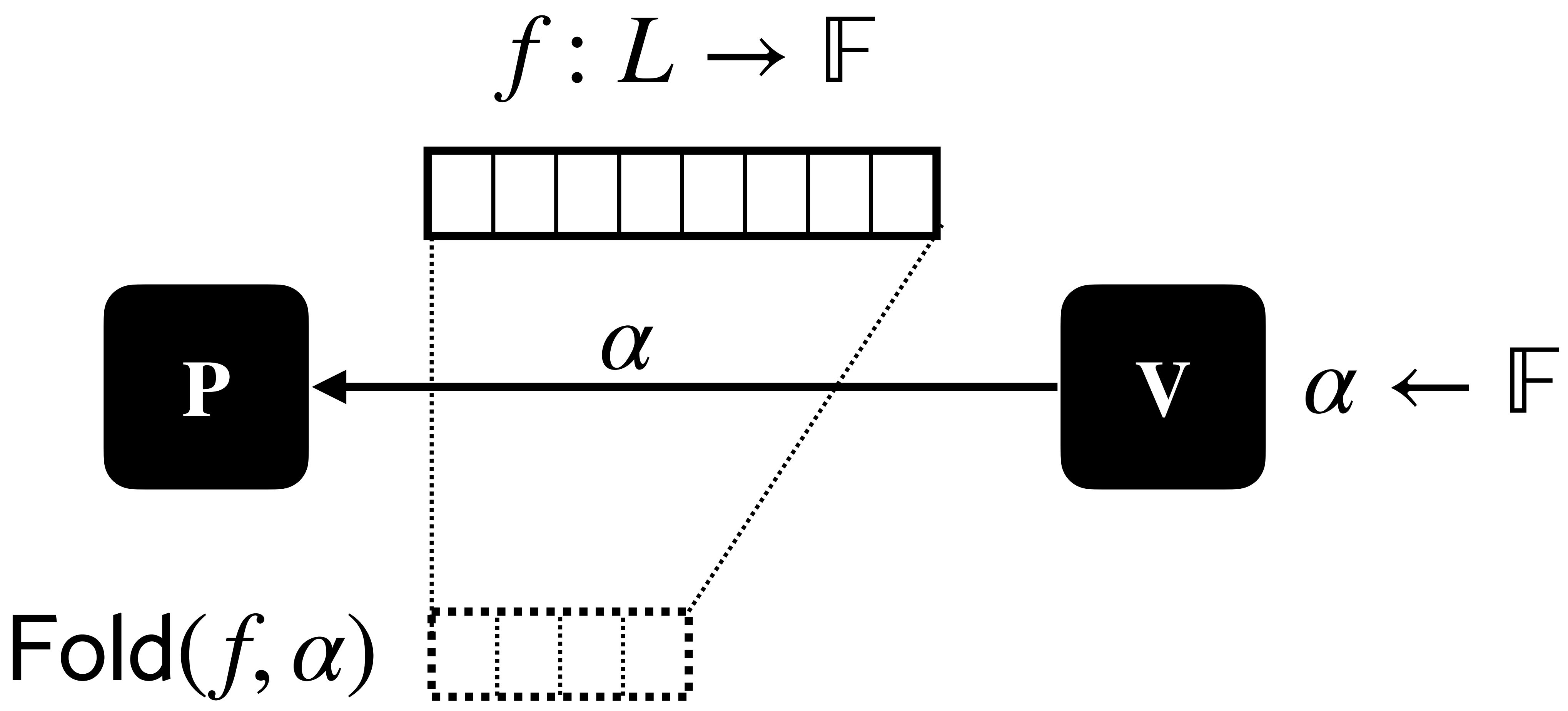
$$f : L \to \mathbb{F}$$



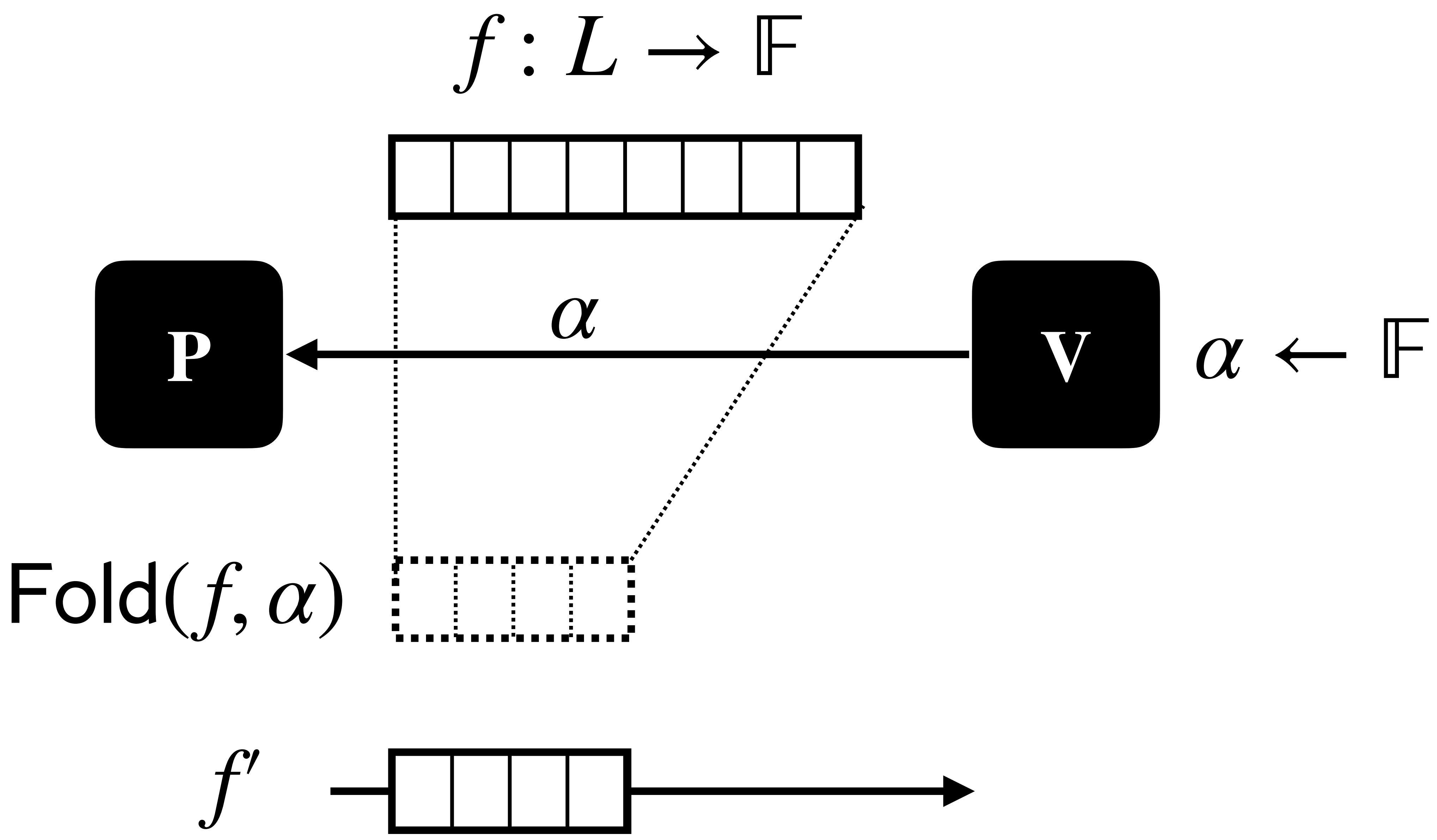
Review: FRI iteration



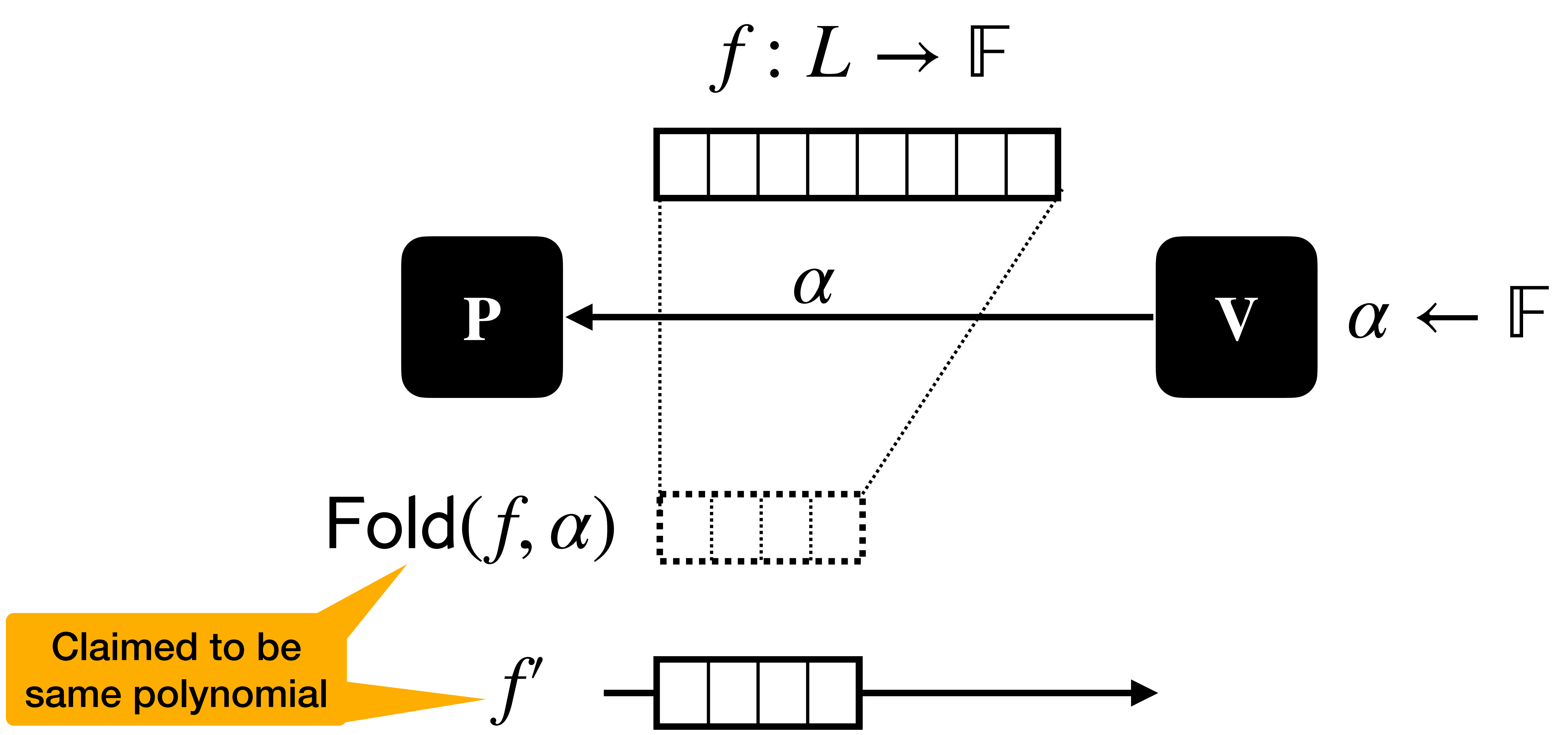
Review: FRI iteration



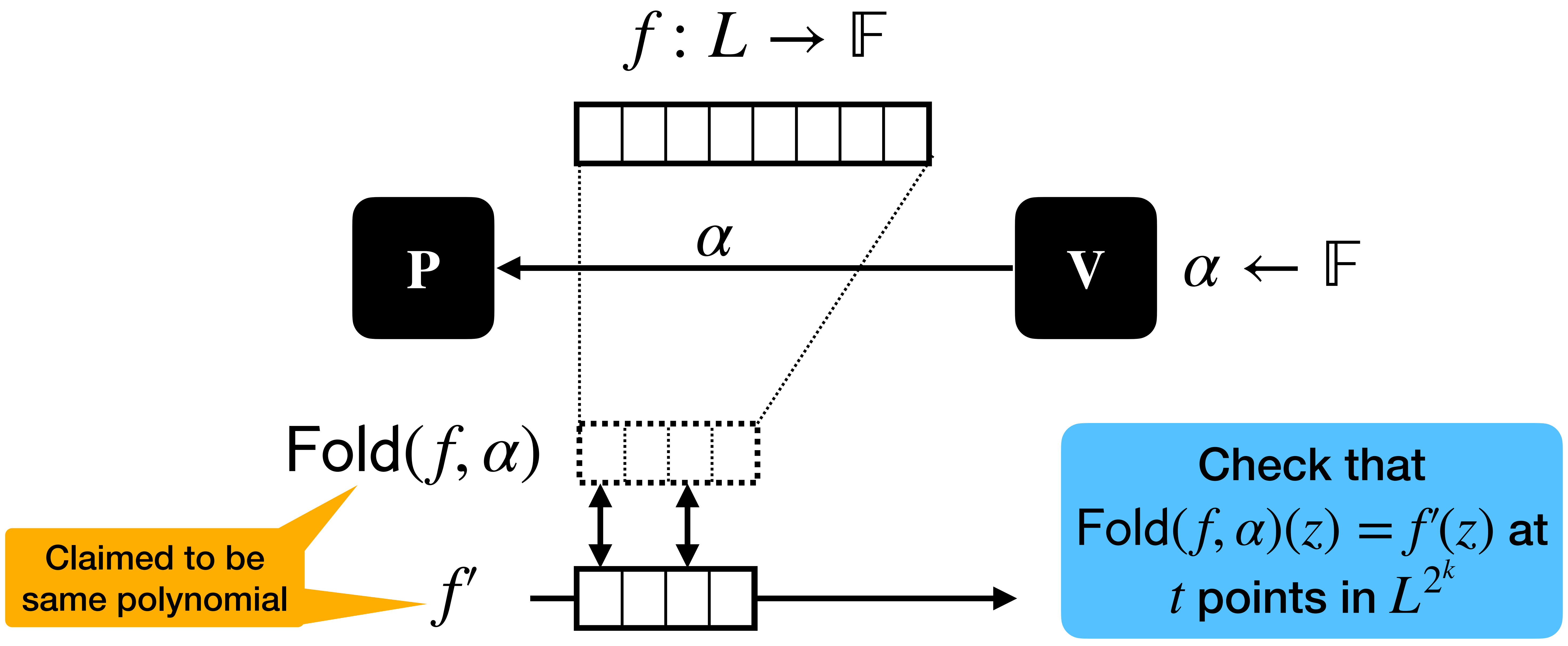
Review: FRI iteration



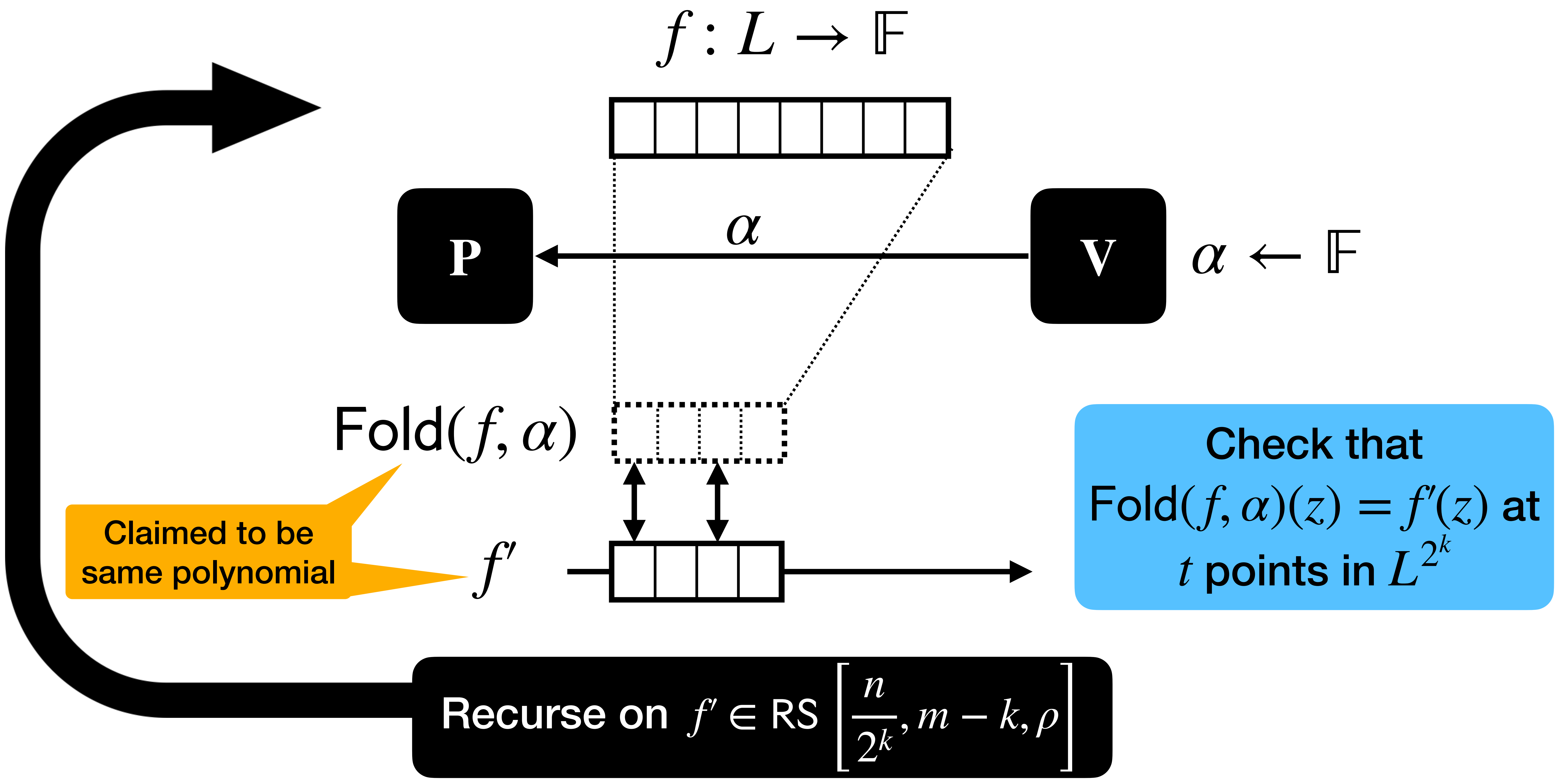
Review: FRI iteration



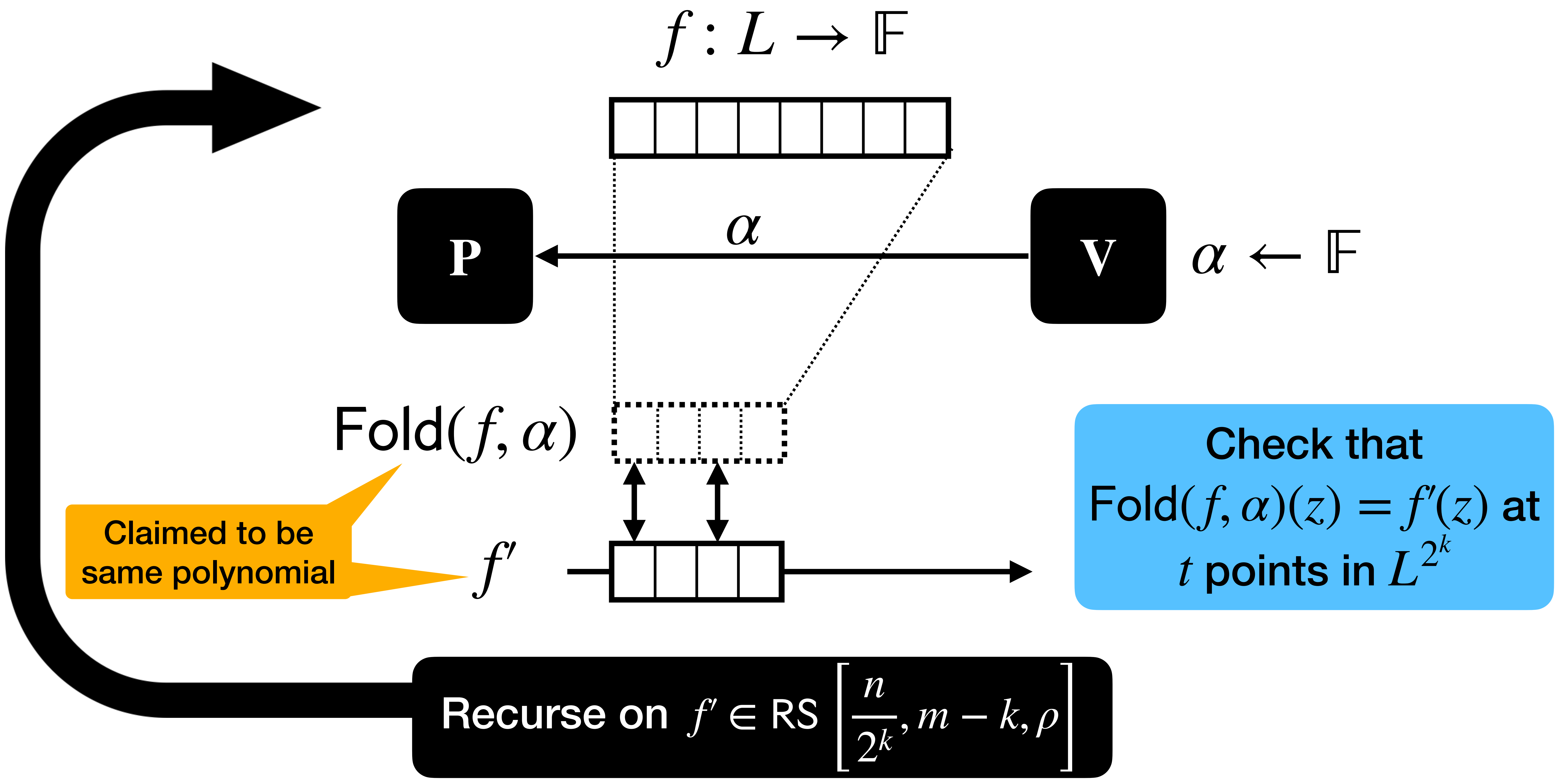
Review: FRI iteration



Review: FRI iteration

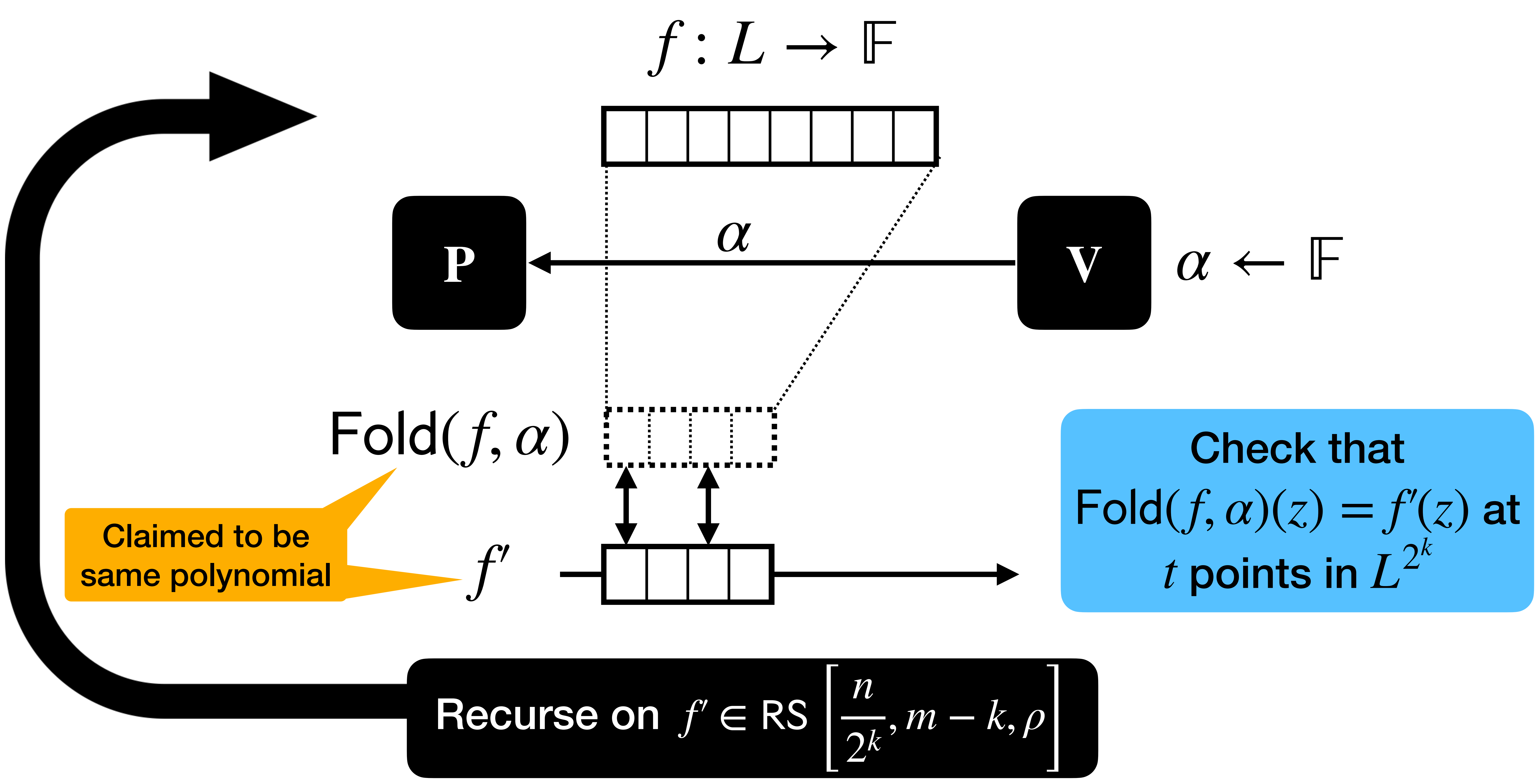


Review: FRI iteration



Disclaimer: in full FRI consistency checks are correlated between rounds.

Review: FRI iteration



Soundness:

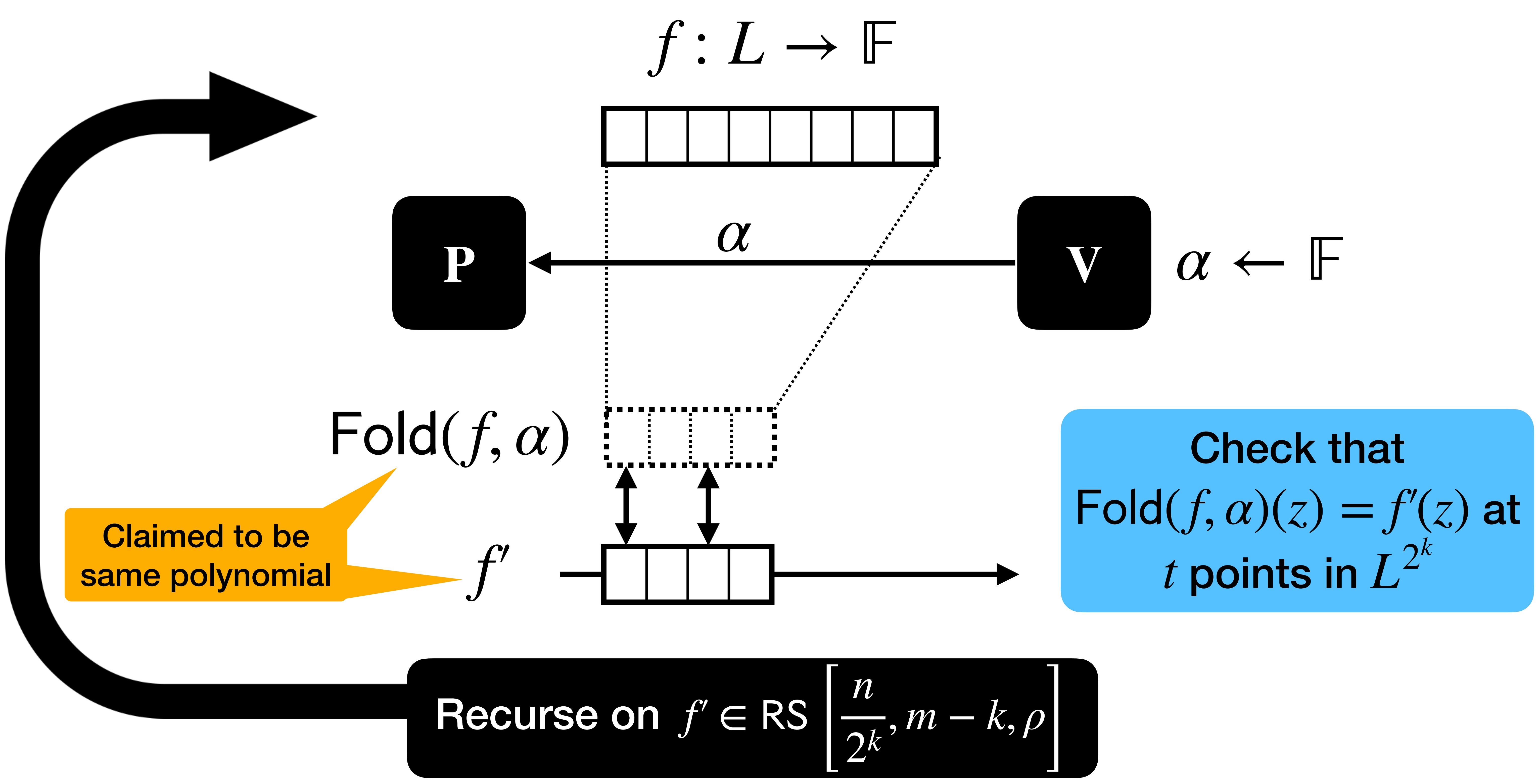
Suppose that $f' \in$ RS$[n/2^k, m-k, \rho]$.

If f is δ-far from RS$[n, m, \rho]$,

Fold(f, α) must be δ-far from RS$[n/2^k, m-k, \rho]$

Disclaimer: in full FRI consistency checks are correlated between rounds.

Review: FRI iteration



Soundness:

Suppose that $f' \in$ RS$[n/2^k, m-k, \rho]$.

If f is δ-far from RS$[n, m, \rho]$,

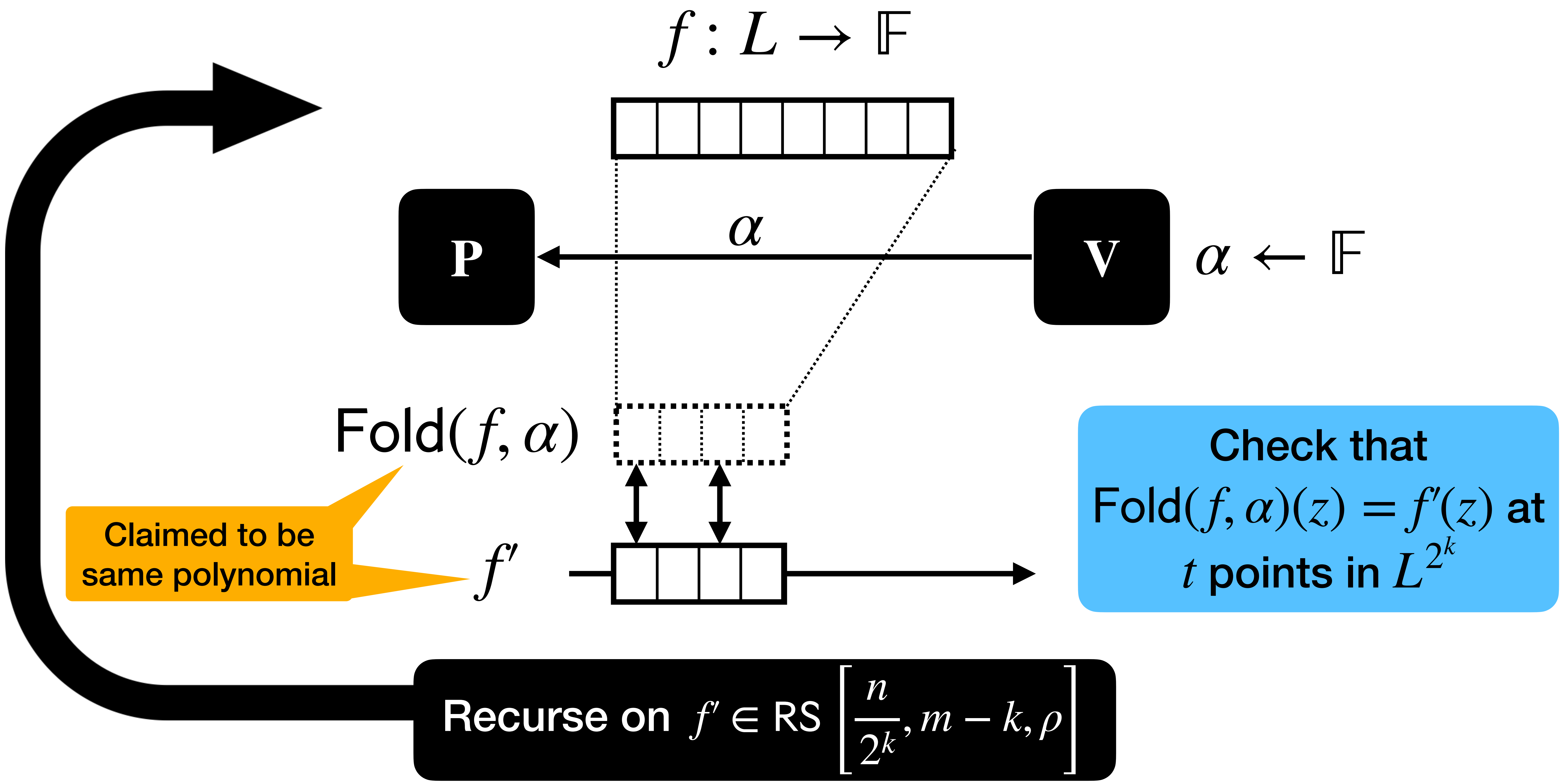
Fold(f, α) must be δ-far from RS$[n/2^k, m-k, \rho]$

Then, f' and Fold(f, α) differ on a δ-fraction.

Soundness error is $(1-\delta)^t$

Disclaimer: in full FRI consistency checks are correlated between rounds.

Review: FRI iteration



Soundness:

Suppose that $f' \in$ RS$[n/2^k, m-k, \rho]$.

If f is δ-far from RS$[n, m, \rho]$,

Fold(f, α) must be δ-far from RS$[n/2^k, m-k, \rho]$

Then, f' and Fold(f, α) differ on a δ-fraction.

Soundness error is $(1-\delta)^t$

To get soundness error $\varepsilon_{\mathsf{RBR}} \leq 2^{-\lambda}$:

set $\delta := 1 - \sqrt{\rho}$ **and** $t := \dfrac{\lambda}{-\log\sqrt{\rho}}$

Disclaimer: in full FRI consistency checks are correlated between rounds.